

**SOCIAL STATISTICS, NATIONAL ACCOUNTS  
AND ECONOMIC ANALYSIS**

**INTERNATIONAL CONFERENCE  
IN MEMORY OF SIR RICHARD STONE**

**Certosa di Pontignano, Siena, ITALY  
October 17-20, 1993**

**SISTEMA STATISTICO NAZIONALE**  

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## FOREWORD

*This volume is comprised of the papers presented at a Conference promoted by the National Statistical Institute in Siena at Certosa of Pontignano October 17-20 1993. This Conference was dedicated to the memory of Sir Richard Stone, awarded the Nobel prize in Economics in 1984, died in December 1991; it was organized in collaboration with the Economics Department of the University of Siena with financial contributions by Eurostat and the Bank of Italy.*

*The objective of the meeting was to provide a forum for statisticians from official bodies, academic circles as well as some of Stone's most renowned students, where the themes developed throughout his long career as a scholar of economics and statistics could be discussed. During the latter years of his life Stone devoted his attention to social statistics, above all to the construction of an integrated system of economic and social statistics. The conference offered the occasion to focus international attention on problems regarding the integration of various areas of applied statistics, particularly with respect to statistics concerning the social area.*

*At the close of the Conference, taking a cue from indications that had emerged during the various sessions and the final discussion, the idea was launched for the creation of a study group which would enable the dialogue to continue between official statisticians, scholars and international organizations on matter concerning several themes of social statistics. This undertaking, first promoted by Istat, Statistics Sweden, Statistics Norway and Eurostat has since developed with the contribution of other statistical institutes and international agencies, such as the Conference of European Statisticians, leading to the founding of what is now known as the "Siena Group for Social Statistics". The Group held its first meetings in Stockholm (1994) and Oslo (1995).*

*Richard Stone chaired an important conference at Istat in September 1988, at a time when the task of revising the series of National Accounts implemented in the 1970's was fully under way. The methodology considered during this revision revolved around the goal of balancing a series of input-output tables in accordance with techniques proposed by Stone and his students. On the one hand, this methodology completed the system of the General Revision of the National Accounting undertaken in 1987, on the other hand, it also considerably conditioned*

*the practices of those involved in National Accounting during the years to follow. This required (and still continues to require) that the balancing of estimation of the main aggregates be seen as a branch of economic activity. For this reason there are very close ties between the Italian National Statistical Institute and Stone's work.*

*This volume closely follows the structure of the sessions of the Conference and begins with brief – but significant – opening remarks by Professor Alberto Zuliani, President of Istat, by Lady Giovanna Stone and by Richard Goodwin who was a lifelong friend of Stone. The volume concludes with a contribution by Paolo Garonna, Director General of Istat and a student of Stone's, giving a summary of the considerations which emerged in the round-table conference where the participants were, among others, L. Barreiros (Eurostat), G. Guteland (Statistics Sweden) and J.C. Milleron (United Nations) and during which the idea of creating the "Siena Group" was first launched.*

*A revival of research into the themes which Stone treated with such perspicacity would appear to be the best possible manner of honouring the memory of an outstanding economist who was always capable of directing his attention to both theory and empirical analysis without overlooking the need to formulate concrete proposals for improving of statistical systems. As Angus Deaton appropriately remarked in February 1992,*

*"Dick and Giovanna offered us, not only a lifestyle, but a professional morality too. There was a "right" way to do economics. It had to make analytical sense, it had to be matched up to the data, and the results had to be useful for something".*

Enrico Giovannini  
Istat, Italian National Institute of Statistics

# INTRODUCTION

Alberto Zuliani

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*Istat, Italian National Institute of Statistics*

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It should be enough to remember the motivation given on the occasion of the assignment of the Nobel Prize,

"...for having made fundamental contributions to the development of national accounts and hence greatly improved that basis for empirical economic analysis"

or that affirmed by Sir Claus Moser during the commemoration held at King's College in February 1992

"And so we remember Dick Stone as one of the outstanding social scientists for our time, a man who, through his work, advanced scholarship and underpinned policy throughout the world. We remember a man who, through his intellectual strengths, integrity, logical grasp and insights, helped to transform the world of statistical thinking and application, as well as of economics" and econometrics. And we remember him for his unfailing encouragement to all those he worked with or encountered at meetings. We admired him, and loved him, for his interest, his gentleness and his warmth in friendship"

to make superfluous all explanations of the reasons leading a National Institute of Statistics to promote this Meeting. Actually, there is a deeper bond that exists between the ISTAT and R. Stone that, as indicated in Lady Stone's letter, was shared also by him. This was that the ISTAT was the first Official Institute of Statistics to systematically apply the method of the national account balancing proposed by Stone more than forty years ago. And there is no doubt that today, five years after the Conference that he held at the ISTAT just on this theme, it can be affirmed just to what extent the ISTAT has benefited from the invitation with which he ended his report

"I have tried in this lecture to show how far we have got in adjusting an initial set of national accounts by the method of least squares, originally proposed nearly fifty years ago, so as to satisfy the constraints to which the true values are subject. The method has been much elaborated since it was initially suggested and number of

applications have been made. In spite of this more experience is needed and especially the experience that could be gained by the adoption of the method by official statisticians".

The interest in these subjects, and more generally in the many spheres in which Stone made an important scientific contribution, not only in theoretical but also in applied terms, on the other hand is largely demonstrated by the number and the quality of the works which will be presented in these days. Studies of the National Accounting foundations and methods, works on the interrelations between account schemes and economic and econometric analysis, new proposals for the balancing of complex account systems, contributions aiming to permit an integrated analysis of social and economic phenomena will alternate, signalling the ever greater importance today of the themes studied by Stone as a pioneer and, at the same time, his ability to transmit to his pupils those gifts of methodological rigor and the attention to applied research characterising all his work.

It is enough just to look at one of his biographies to be sincerely impressed by the breadth of arguments to which Stone dedicated his attention: construction and revision of national economic account systems, social accounting, the detection of economic activity indicators, econometric models building, the analysis of consumption demand, and educational problems, are only some examples of his versatility. Moreover, besides intense academic activity, Stone demonstrated a constant commitment to spreading new analytical techniques to Official Institutes of Statistics and International Organisations by seeking to fill the fracture often present between both university researchers and others.

In a moment, Richard Goodwin will speak to us on these and other aspects of Stone's life. But I would like to briefly recall the importance that the adoption of the approach to national account balancing proposed by Stone has taken, in the sphere of National Accounting. The paper presented in the first session on this theme will focus on the more technical features. Here, instead, I would like to consider the elements "training" human capital which can be produced by this approach, coming from methodological rigor which characterises the general lines of the proposal itself.

In extremely synthetic terms, the problem confronted by national accountants when they derive estimates of various aggregates from basic data, is how to harmonise and integrate them. The heterogeneity of sources and of single data base calculation methods, and the differences of the detected aggregates of a definitional nature make this phase unavoidable, to be dealt with by approaches characterised by different sectorial details and different methodologies. In particular, that proposed by Stone, Champernowne and Meade in 1942 provides for the balancing of different estimates of the same aggregate and is founded on an explicit evaluation of the reliability of the single estimates. Such evaluation, in formal terms, contributes to defining the variance-covariance matrix of the entire system of equations which are

the object of the balance, on which basis the various returned discrepancies are "distributed".

The methods, the interpretations and the algorithms proposed to carry out such an operation vary as a function of the typology of accounts to balance, and of the presence or not of a dynamic characteristic of the economic variables being studied, but the idea, as often occurs with "great" ideas, is relatively simple. In the Italian case, for example, such an approach is currently used to balance single annual input-output tables at current prices and in more than 40 productive branches (the tables have been available since 1970), starting from which one then derives estimates at constant prices not balanced within an analogous accounting scheme.

The point that I would like to underline is that such a methodology tends to formalise a behaviour common to national accountants, that is of harmonising different estimates, though it is not limited to returning that which could be considered a "practice" of little theoretical content to an attractive sphere on the statistical-methodological level, but to forcing the accountants themselves to be extremely rigorous in their hypotheses formulated to balance the accounts and to document the motivations of the choices adopted, that is to reduce to a minimum that element of discretionality inherent in such an activity.

Regarding this, one can remember how, again in the Italian case, in the presentation both of the first estimates made for the '80s after the general revision realised in 1987, as in the historical series reconstructed for the '70s, it was decided to emphasise the relevance of the approach followed, publishing tables that summarised the differences revealed between the initial estimates of the various aggregates and of the definitive ones obtained through the balancing, giving information on the structure of variance-covariance matrix used, and thus, noting the precise valuations (at least in relative terms), even though subjective, of the trustworthiness of the single sources and of the methods of calculating used.

This is an example of the approach that, in my opinion, an Official Institute of Statistics should have: maximum transparency and methodological rigor, without hiding the difficulties encountered, because only in this way it is truly possible to individuate the fields for investment to improve statistical knowledge. It is the construction of a "historical memory" of the statistical apparatus on which to base operative choices requiring that real limits are made explicit, and in this there is no doubt that the documentation and information necessary to practice the procedure of balancing in question constitute fundamental parts of such a "memory".

In the history of National Institute of Statistics it can happen that, notwithstanding its wide dimensions, "a generational rupture" of some importance occurs, to which "cultural ruptures" that allow to proceed more rapidly in the processes of innovation are associated. The computer revolution is a typical example of such a phenomenon, having changed not only the approach of single researchers to statistical data, but also the organisation of statistical work. And then, in the new

generations of national accountants have fully learned the message that, as I have sought to underline, the techniques of balancing proposed by Stone contain within them, extending the range also to areas not directly concerning them.

I believe on the other hand, that the sensation that leads one to hold the approach profoundly "educational" can be confirmed not only by the attention that Stone gave to the world of official statistics, which he knew the functioning of to its base, its gifts and its defects, but also from the scientific "success" reached by so many of his students. On other occasions in fact, some of them, in particular some those present, have born witness to the scientific community of how the bond that Stone established with those he worked with, went beyond strictly technical aspects, the sign of a continuity of commitment that could not but be reflected in the subjects confronted and the solutions proposed.

Also in homage to this spirit, we preferred to organise this Conference in a welcoming and relatively isolated environment, to consent to a greater ease and continuity of dialogue in these days between participants in this meeting, some of whom have accepted to travel thousands of kilometres in just a few days to honor the memory of their teacher.

I would like at this point, to express my warmest thanks to the University of Siena and in particular to the Political Economy Department that have shared the organisation of this encounter with ISTAT. I thank in addition the Bank of Italy and EUROSTAT for the financial contribution made, wishing that in the future there can be other and equally fruitful occasion for collaboration.

Before concluding please allow me to express my disappointment at the absence of several people who certainly would have wanted to be present today. In particular James Meade and Giovanna Stone were prevented from being here in person for health reasons, but I would like to underline the interest and affection with which they have followed development of this initiative from the beginning, demonstrated moreover by the affectionate address that Giovanna Stone sent us and that we exchange from the heart.

## INTRODUCTION

Lady Giovanna Stone

Since to my great regret I cannot come to this conference, I am glad to avail myself of Professor Garonna's kind invitation to send a few words in writing. I am neither an economist nor a statistician nor a mathematician so this is not going to be a scientific paper. Merely a personal message to the participants and a brief account of what my husband was working on when he died, which I thought might be of interest since it is still unpublished.

First of all I would like to tell the organisers how delighted I am by their initiative. It is a mark of respect for my husband's memory and of ongoing interest in his work which touches me greatly. And so does the presence of the participants, all busy people with many other things to do. Some of them have been his collaborators and are friends of long date, and they will understand my feelings very well. Others I know only through the references to their work in my husband's papers or through their correspondence with him or with me, and it adds considerably to my regret that I shall not meet them personally on this occasion. But if any of them ever come to Cambridge I very much hope they will pay me a visit.

My husband had, not unnaturally, a particular liking for Italy, and it was a cause of great satisfaction to him that the Italian Statistical Office should have been the first to apply to the national accounts the adjustment method he had proposed some forty years earlier. Of course he was aware that it was only a start, and he welcomed all attempts at developing and perfecting it. He was also aware of other methods, independent of his, which had been applied elsewhere, and was very interested in them. Perhaps if he had been younger and in better health he would have continued to take an active part in these developments, but with so many clever people working in the field he thought he could bow himself out and devote his remaining energies to a very different subject which had always held a great attraction for him, the origins and progress of quantitative statistics.

The idea of actually writing a book about it arose from an invitation he received from the Banca Commerciale Italiana to deliver the 1986 Mattioli Lectures and subsequently expand them into a book. The underlying theme of these lectures, which are an annual event, is the history of economic thought, and my husband chose as his topic twelve pioneers of economic, demographic and social statistics in England from the seventeenth century to the end of the nineteenth. They are a very

diverse and in some cases surprising lot. I shall list them in order of appearance.

William Petty, a typical representative of what was perhaps the most exciting period in the history of English science: physician, surveyor, cartographer, writer on economic and demographic matters, naval engineer, founding Fellow of the Royal Society and, above all, originator of the national accounts.

Gregory King, officially a herald, unofficially a one-man central statistical office; besides his well-known *Natural and Political Observation of England*, the mass of economic and demographic statistics that fill his one surviving notebook, dated 1695, is staggering.

Charles Davenant, publicist and Member of Parliament, who made a special study of public finance; his other claim to fame is that he may have been a grandson of Shakespeare, his father William D'Avenant, the playwright, being rumoured to have been an illegitimate son of the poet.

William Fleetwood, a liberally minded and politically active bishop of the Church of England, who in 1707 devised the first index-number of prices.

Arthur Young, one of the principal protagonists of the English agricultural revolution of the eighteenth century, who constructed the first production accounts based on value added.

Patrick Colquhoun, cotton merchant, magistrate, writer on poverty and crime, who drew up the accounts of the British Empire at the end of the Napoleonic Wars.

John Graunt, a London draper, whose *Observations upon the Bills of Mortality*, published in 1662, is recognised as the foundation of demographic statistics.

Edmond Halley, the astronomer, who calculated the first scientifically-based life table to obligate a friend.

William Farr, a doctor turned statistician, who in his long career at the General Register Office brought vital statistics to a high degree of excellence; he was a vigorous campaigner for public hygiene, and one of his campaigns resulted in the disappearance of cholera from England.

Frederick Eden, a humanitarian nobleman, who carried out the first large-scale quantitative survey of poverty in England.

Florence Nightingale, a reluctant member of what might be called the Victorian jet set, who transformed not only the civilian hospital service but also the living conditions of the British soldier in war and in peace.

And finally Charles Booth, a Liverpool shipping magnate, whose seventeen-volume survey of the London poor marked an important step towards the Welfare State.

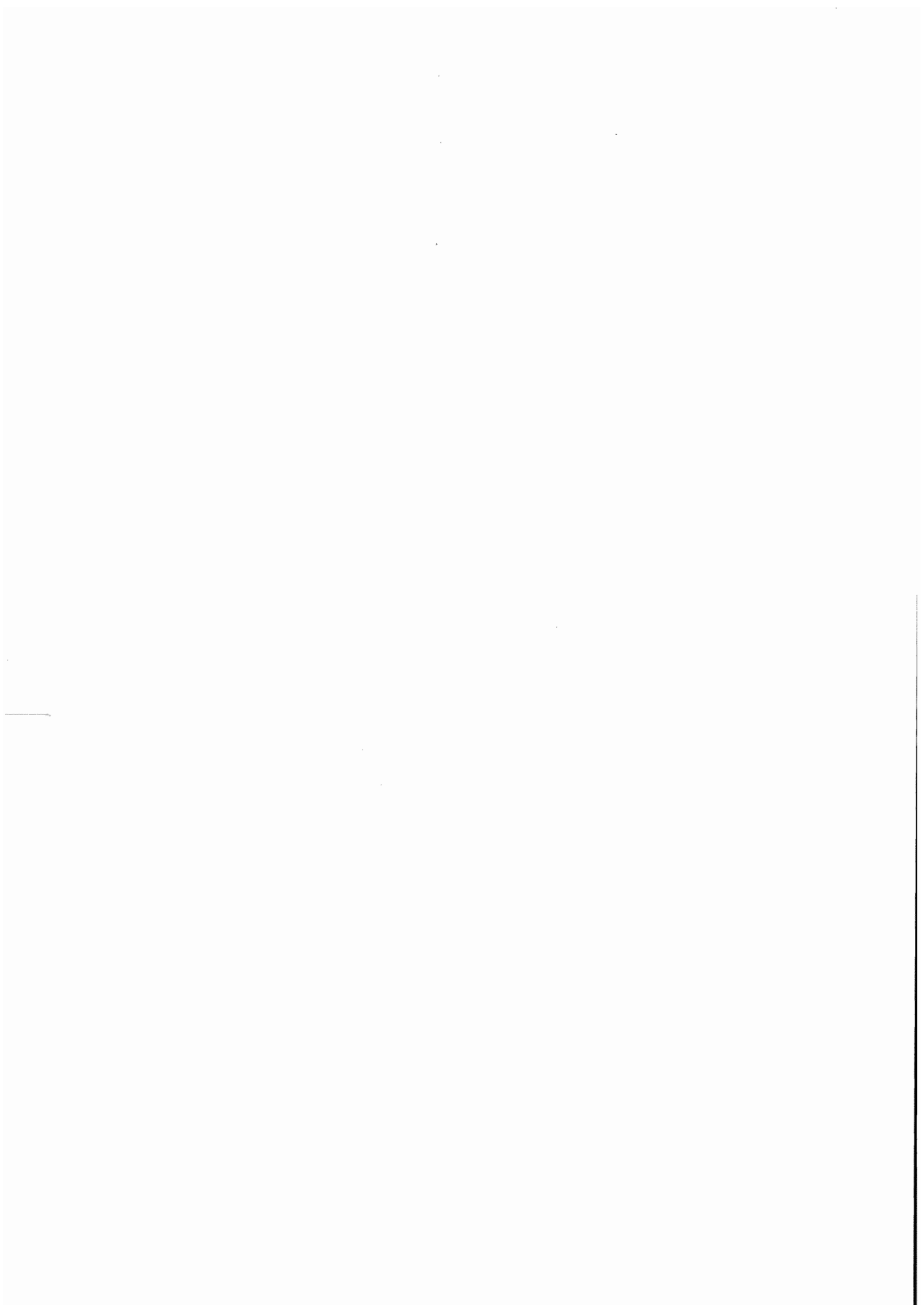
In the book each of these characters has a chapter to him or herself. Each chapter opens with a pen portrait, followed by an account of the protagonist's contributions to statistics illustrated by a number of tables. When my husband died he had finished the first nine chapters, in fact they were already in proof. The last three were unfinished, but he had carried them far enough for me to think I could complete them. I had followed his work closely and knew his intentions. The substantive parts, including the tables, were all there, and all I had to do was to provide



some of the connecting links. This I have done, and the last three chapters are now with the printers. The book, which is entitled *Some British Empiricists in the Social Sciences*, is part of a series published by the Banca Commerciale under the imprint of the Cambridge University Press and is due to appear, with luck, next year.

There is only one more thing I want to say. I have chosen to write in English for obvious reasons, but I would like to assure our Italian hosts that I have not forgotten my Italian.

In conclusion, I would also like to express my gratitude to Professor Garonna and his colleagues for the courtesy they have shown in consulting me over the organization of this conference and keeping me up to date on the progress of this fine initiative. I would also reiterate my sincere regrets for not being able to participate in person.



## IN MEMORY OF SIR RICHARD STONE

Richard M. Goodwin

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*University of Siena – Italy*

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In the American academic tradition of "publish or perish", I perished when in 1950, Harvard's economic department failed to renew my appointment. With the foolish recklessness of youth, I decided not to look for another appointment in the US, wisely, as it turned out, since, in the Mac Carthy period, I would have been again sacked because of my previous membership in the Communist Party.

Instead I applied for and received, after some delay, a Fulbright Fellowship for a year abroad. I had fortunately been invited by Jan Tinbergen to a conference in Driebergen Holland. There I had the good fortune to meet Dick Stone. Not only that, but later the same summer I gave a paper at another conference in Varese, Italy, where, happily, I again met Dick.

There, sitting over a cup of the good Italian coffee, he asked me whether I would be interested in coming to Cambridge England. Without saying that I was unemployed and with no secure academic future in US because of my Communist past, I said I would very much like to come to Cambridge. This was natural because I considered the Cambridge economics faculty to be, more or less, the best in the world.

Hence he, as the head of the new Department of Applied Economics, promised me a one year appointment in it. In the course of my year there I met various people, particularly Richard Kahn and Joan Robinson, whom I had greatly admired for their continuation of the Keynesian tradition. They had perpetuated Keynes's tradition of having a regular discussion group of a number of interesting economists, young and old.

What I did not know, but soon became aware of, was that though Stone had been taught as student by Kahn, the two had grown apart because of Kahn's active hostility to econometrics. By careful discretion, I was able to have a long and happy relation with both Stone and the Kahn-Robinson group, by diplomatic separation of the two groups in my relations with each separately.

I liked to work alone, by contrast Dick Stone had no such narrow approach to life and work. His devotion was to his chosen path and he worked happily with many others. His father, who had been a High Court judge in India, had wanted him to be a lawyer and in deference to his

wish Dick first studied law, gaining a first class, but then, following his own vision, he changed to economics, was taught by Kahn in economics and Colin Clark in statistics becoming a friend as well, and again gained first class honours.

On leaving Cambridge, and again in deference to his father, Stone went to work in London, in the City. But already he began doing applied economics, editing with his first wife, Winifred Jenkins, a journal called Trends. This marriage was dissolved at the beginning of the war. His second marriage was with Feodora Leontinoff, secretary of the National Institute of Economic and Social Research, where Dick had started his work on modelling consumers' behaviour. This marriage lasted until her death in 1956. Later he met and married in 1960 Giovanna Croft-Murray, a member of the distinguished Italian Saffi family of Forlì, one of her forbears having been, with Mazzini, a triumvir of the short-lived but gallant little Roman Republic of 1848-49.

When the war started Stone went into the government service, first in the Ministry of Economic Warfare and in the Central Economic Information Service of the War Cabinet, to which he was transferred in order to help James Meade in the construction of what became the first national accounts, a collaboration that led to a life-long friendship. Keynes, who had just published *How to Pay for the War*, followed Meade and Stone's work closely, and it was thanks to him that their estimates of income and expenditure were published in the Budget White Paper of 1941 and that Stone was able to go on producing the British national accounts until the end of the war. In parallel with this task he was carrying on his work on consumers' demand with the small team he directed at the NIERS.

In the late 1930's the University of Cambridge, at the instigation of Keynes, had agreed to the creation of a Department of Applied Economics, but with the advent of war the plan had been shelved. It was revived at the end of 1944 and Stone, again thanks to Keynes, was appointed the Department's first director, to his great delight, since it was precisely the sort of work he liked and was fitted for. Before taking up the post, however, he spent a few months in 1945 at the Princeton Institute of Advanced Study, where he formulated for the League of Nations the methodology of national accounting which eventually led to the world-wide System of National Accounts, the SNA. (The UN committee that prepared the first model of international accounts in 1951 was chaired by him).

At the DAE he attracted and worked with a remarkable collection of able young econometricians (some of whom are here at this conference). There was always the combination of model-building with the pursuit of the number, the statistics, which gave reality to the concepts. For him, analytic theories should always be paired with the corresponding numbers, making the whole more realistic and hence more powerful. In the early years the emphasis was very much on demand analysis, culminating in the publication of *The Measurement of Consumers' Expenditure and Behaviour in the United Kingdom, 1920-1938*, by common consent one of the classics of econometrics. This work on consumer demand was continued in his later life. Later around 1960, he started work with Alan Brown on the construction of a large,

desegregated model of the British economy, known as the Cambridge Growth Project. The Project, developed under his leadership until his retirement, is still going strong under the direction of Terry Barker (who can tell us more about it).

Actually by the time he started the Growth Project, Stone though still working with members of the DAE was no longer its director, having been appointed in 1955 to a University chair. But it was a research chair, with no lecturing commitments, so he could continue to devote most of his time to the kind of work he liked best. In doing this he was reaching a far wider audience than the Cambridge undergraduate body, since the lessons learned from his researches would be repeated all over the world, as well as in conferences like this one, by people some of whom were involved in the DAE and who carry on his work.

In his study of economics, he came to the interesting and subversive conclusion that economists spend too much time on the concepts and analytics of the subject, with little concern with its substance, in the form of the actual numbers and statistics of its structure. In his life work he set about providing that detailed, numerical actuality of the economy. This concern followed naturally from his work on the war-time economy and its capability to fulfill its maximal obligations. His remarkable achievement was to carry out such detailed and perceptive work on the statistics of national income, output and expenditure, that it actually also enriched basic areas of economic theory and analysis.

Another undertaking of his during the 1960's and 70's was the development of an accounting system of demographic and social statistics, which he considered a necessary complement to the understanding of economic behaviour. This system, which has the human being as the unit of account, shows the stocks and flows of individuals into and out of various activities and phases of life, such as education, employment, illness and so on, and provides a framework for modelling the changes that occur from year to year in these variables. By appropriate transformations it can also be linked to the economic account. His ideas and his models were brought together in the report *Towards a System of Social and Demographic Statistics* (SSDS), published by the UN in 1975.

After his retirement in 1980 he continued to work in his splendid library at home. In 1984 he was very appropriately elevated to the international Economic Nobility. Although physically weakened in his later years by a multiplicity of illnesses, he never ceased working. He had developed a great interest in the typically British, pragmatically-minded, economists of the seventeenth century. He studied in depth the political arithmetic of William Petty, Gregory King and Charles Davenant and others, and found that one could give a reasonably complete account of the economic and social structure then and in the present time and present it in graphical as well as numerical form. When he died in 1991 he had almost finished a book on the pioneers of economic, demographic and social statistics from Petty in the seventeenth century and up to Charles Booth in the nineteenth, which his wife has now completed for publication.

In later life Dick once said to me that it was surprising that, since we were both basically interested in the same range of topics, we had never

worked together. I do not now recall how I answered him, but in retrospect I think I know the explanation and it is not very flattering to my own judgement. I always admired his desire to relate theory and empirical fact and I shared his belief in that.

In my youth I had made some efforts at statistical determination of parameters, but I think I came to the conclusion that one could not really pursue equally analytic concept and structures, along with statistical testing and verification. Hence I chose to look for and construct analytic structures. Also very differently from Dick Stone, I liked to work on my own and understand and build analytical structures; he by contrast believed, quite rightly, that his problems required massive statistical assembly and required a large group of workers, which is what he assembled. Though I cannot defend my choice, it was to work alone and seek my own types of analytical structures, leaving the empirical verification or falsification to other investigators. Typically this basic difference between us never in any way inhibited our enjoyment of one another's company.

For those who have not met and learned to know him well, it is very difficult to give an account of what a special and remarkable person he was, quite apart from the impressive monument that his works constitute. What is the memory one has of this remarkable man? he was reserved but generous of spirit and simple and straightforward of manner. He enjoyed social intercourse and had a warm approach to friends so that it was a great pleasure to be with him and to talk, equally with him and his delightful and vivacious wife.

In their house there was no radio, no TV, no gramophone. He dressed simply but elegantly, on the whole in an earlier style. He always wore a bow tie, which was more common in former times (I often wore one when meeting him, as a friendly gesture). He and his wife admired the Citroen of early 1950's, so when they were no longer made, they searched for, and found, a good example; had it refitted and repainted in black, and never drove anything else!

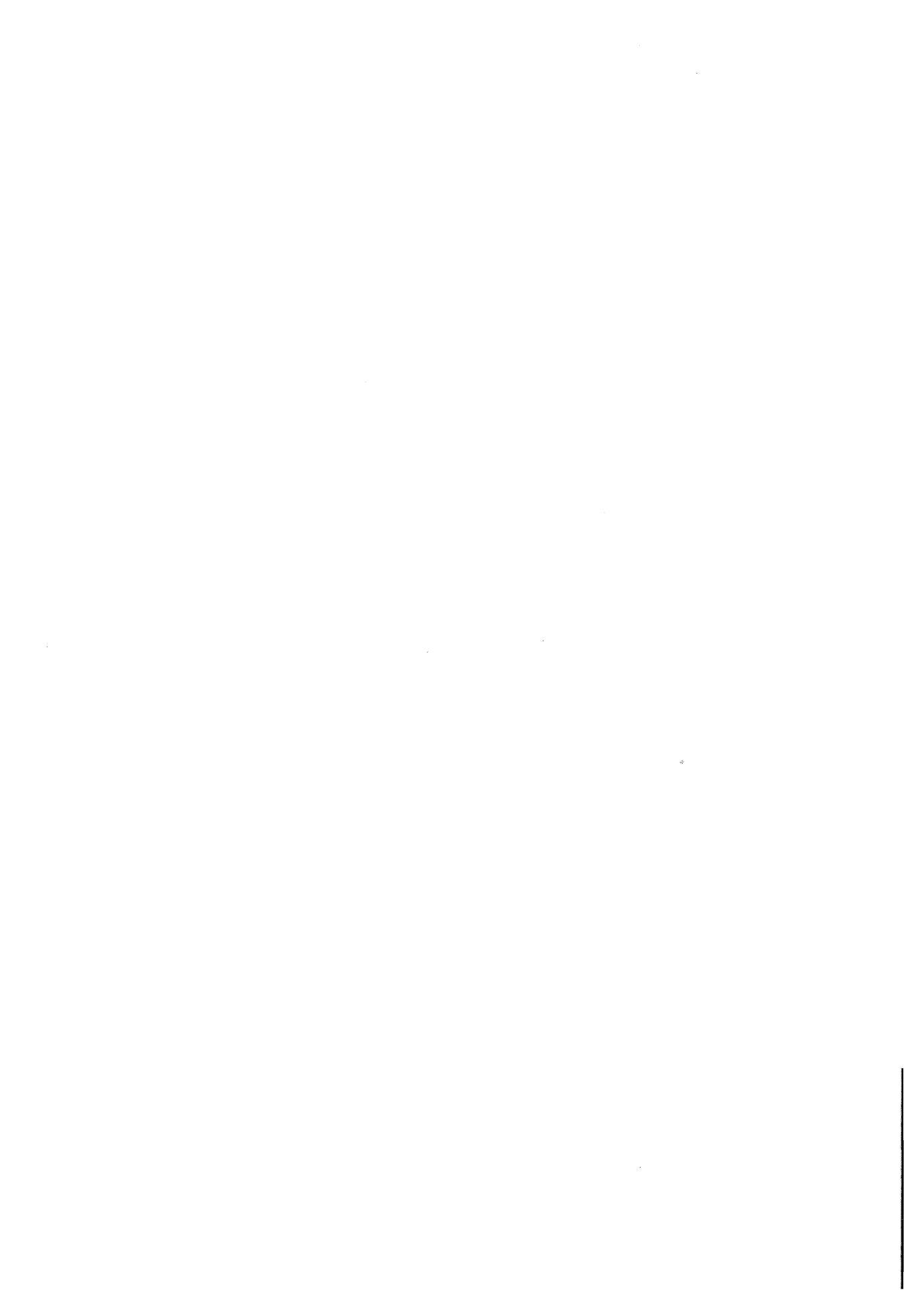
He had a lively and impressive interest in a wide range of things, from romanesque architecture to moths and butterflies. He had a strong feeling for painting. His favourite painters were Tintoretto, Renoir, Monet and Matisse. His tastes in music were consistent with his tastes in painting: he liked Bach, Vivaldi, Debussy and the great men of Jazz, particularly Gershwin and Cole Porter. He had a splendid collection of old English coloured glass, which was in daily use on his dinner table. He had also made a collection of small hand-turned ivory boxes, as well as one of early nineteenth century china. He had, of course, a large personal collection of economic, demographic and social literature.

Knowing and being with him was for me a rewarding experience. He and I were born in the same year; his early death has unhappily robbed us of rare, attractive and special person. He was always elegant in dress and manners, but this in no way diminished his open and friendly discourse with the people he met. He never tried to dominate conversation and to be with him was invariably attractive and rewarding.

For me personally his early death has meant an irreplaceable social and human loss.

**I SESSION**  
**REVISION, EXTENTION AND**  
**DEVELOPMENTS IN NATIONAL ACCOUNTS**

Chairman: Guido M. Rey (*University of Rome III – Italy*)





# RICHARD STONE'S CONTRIBUTION TO NATIONAL ACCOUNTING

Peter Hill

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*OECD, Paris – France*

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Richard Stone was awarded the Nobel Prize in Economic Sciences in 1984 for having made fundamental contributions to the development of systems of national accounts. His major contributions were made during the 1940's and 1950's, although Stone was also one of the principal authors of the 1968 revision of United Nations System of National Accounts (SNA), the version which has remained in use for 25 years and whose own revision has been completed only this year, 1993.

The list of Stone's published works is so large, amounting to about 200 publications over a period of half a century, that it is difficult to single out a few in which may be regarded as more significant, or substantial, than others. Nevertheless, in the context of his seminal contributions to national accounting, a few key early papers may perhaps be mentioned. These include the article in the 1941 *Economic Journal* on "The Construction of Tables of National Income, Expenditure, Saving and Investment" written jointly with J.E. Meade and the article in 1943 *Review of Economic Studies* on "National Income in the United Kingdom and the United States of America". In 1944, the first edition of the very successful little textbook on National Income and Expenditure, also written jointly with J.E. Meade, was published. During the late 1940's and early 1950's Stone also published several important articles on transactions and transaction matrices, a subject treated in more detail later in this paper. However, much of Stone's most influential work was being published in ways other than academic journals.

After the end of the Second World War Stone became involved in the development of the newly emerging international systems of national accounts working both for the United Nations and the Organisation for European Economic Cooperation (OEEC) set up in Paris to administer Marshall Aid under the European recovery programme. Stone made his first contacts with the official experts on national accounts in North America, including Milton Gilbert, George Jaszi and Edward Denison, when he was sent over to Ottawa and Washington in the autumn of 1944 to try to reach agreement on national accounts methodology and presentation. A year later in September 1945 on a visit to Princeton, he

met the head of the Intelligence Department of the League of Nations who asked him to write a report on national income statistics for the League of Nations Committee of Statistical Experts. Thus began Stone's long association with the international organisations involved with national accounts. Stone completed the report by the end of 1945. It was discussed by a group of national accounts experts specially convened for the purpose and chaired by Stone in December of that year. Their report was subsequently published by the UN Geneva in 1947 as the *Measurement of National Income and the Construction of Social Accounts* with a famous appendix written by Stone on the "Definition and Measurement of the National Income and Related Totals". This Appendix seems to have established Stone's pre-eminence in the field of national accounts at a world level at a time of intense activity when a number of people from different countries were also making important contributions to the subject of national accounting.

Stone's work was by no means of an entirely academic nature. He was directly involved, together with James Meade, in the compilation of the first official estimates of the UK national income and expenditure during the early years of the Second World War when he worked as a Civil Servant at the UK Treasury with some guidance from J.M. Keynes. After returning to Cambridge in 1945 he also became the Director of the National Accounts Research Unit established by the OEEC in 1949 at the suggestion of Richard Ruggles. It is interesting to note that the impetus for the development of the first set of accounts conforming to internationally agreed standards came from the policy needs of an international organisation, the OEEC. The Research Unit was initially located in Cambridge under Stone's direction during the three years 1949-51. It brought together a number of economists and statisticians from European countries who were themselves making significant contributions to the development of national accounting: for example, Odd Aukrust from Norway and Jan Marczewski from France. The unit was later merged with the Division of Statistics and National Accounts at the OEEC headquarters in Paris. The main objective of the national accounts research unit was to design a Simplified System of National Accounts for the use of Member countries of the OEEC. This System evolved into the Standardised System of National Accounts that was discussed at a Conference on National Accounts held at the OEEC in Paris in September 1951. Stone was mainly responsible for drafting the ensuing 1952 OEEC publication that describes the System. The foreword to the 1952 publication describes national accounts as "a comparatively new device to present a comprehensive and consistent picture of a nation's economy in quantitative terms". Already, under Stone's influence, the emphasis had shifted away from trying to define and measure a few macro-economic aggregates, such as national income or expenditure, towards building an integrated system of macro-economic accounts designed for economic analysis and policy making. According to the Introduction to the 1952 Standardised System of National Accounts, the main objective was "to draw up a set of standard tables which provide the basic information to be extracted from a system of

national accounts in a form which experience shows to be highly useful for economic description and analysis including the comparison of the structure and development of two or more economic systems". Notice the distinction drawn between the set of standard tables and the underlying system of national accounts consisting of concepts, definitions, classifications and accounting rules.

Meanwhile the UN Statistical Commission had decided that a standard international system was also needed at a world level. An expert group, chaired, of course, by Stone, was established by the United Nations in 1952 which led to the publication of the 1953 report "A System of National Accounts and Supporting Tables", to be known later as the 1953 SNA. The System was basically similar to the 1952 OEEC System.

During the mid and late 1950's Stone continued his association with the OEEC whose Director of Economics and Statistics by then was Milton Gilbert who had been one of the leading authorities responsible for the development of the United States National Income Accounts. Stone was asked to prepare a report on the measurement of prices and volumes in a national accounts context. This led to the OEEC 1956 publication on "Quantity and Price Indexes in National Accounts" in which Richard Stone is named explicitly as the author. In the Introduction to this report, Milton Gilbert explains that OEEC's "Standardised System of National Accounts" was only the first in "a projected series of publications relating to problems which arise in social accounting. The report on "Quantity and Price Indexes" carries the systematic examination of conceptual problems, which in the "Standardised System" remained confined to problems connected with national accounts expressed in money terms, one stage further. It deals with the problems of making comparisons between the accounts of successive periods in such a way that they are free, as far as possible, from the effect of changing prices. Such comparisons can be achieved through the construction of price and quantity index numbers on a systematic and co-ordinated plan. The report, like the earlier "Standardised System" itself, was discussed at a conference of national accounts experts held at the OEEC prior to publication in 1956.

A major weakness of the vast main stream economic literature on index number theory is that it is almost entirely focussed on comparisons between two situations, time periods or locations, for a single aggregate, or basket of goods and services, taken in isolation from other flows. Stone's report, however, contains a significant chapter in which he addresses the problem of trying to construct complete sets of accounts that balance in real terms. In Chapter VII on "The Treatment of Non-Commodity Flows" he concludes that "in general, it is impossible to find a unique set of deflated values of the non-commodity transactions in an accounting system such that the accounts continue to balance in real terms." He points out that for a particular non-commodity flow, the choice of deflator may be obvious for one particular type of analysis but not within an integrated system of accounts in which the same flow appears in more than one account.

For example, he points out that "it may be satisfactory for many purposes to deflate personal saving by means of a price index of consumers' goods and services ...In a social accounting system, however, saving will also appear as a incoming flow into the capital transactions account and from the investors' point of view it may seem more appropriate to deflate saving by means of the price index for domestic asset formation..." Stone also discusses the problem of trying to measure real national income and gives a clear statement of the problem of measuring changes in real income for an open economy when the terms of trade, are changing. He anticipates much of the subsequent literature on the measurement of trading gains.

In June 1961 was published the third in the series of methodological studies on national accounting planned by the OEEC, namely the report by Richard Stone on "Input-Output and National Accounts". In the introduction the report is described as being "concerned with the problems of classifying and analysing the product flows which emerge if the product account in a system of national accounting is subdivided by industry." The distinctive feature of the report is not what it has to say about input-output analysis as such but its explanation of the way in which input-output can be integrated into the wider system of national accounting as a whole. Stone starts his explanation with a very simple highly aggregated social accounting matrix which is then subdivided in such a way as to incorporate an input-output table within the more general transactions matrix. Stone shows that while input-output tables and analysis evolved independently as a means of describing and analysing the productive process of a complete economic system they can also be regarded as the result of subdividing the production account in a system of national accounts so that many different forms of production are displayed. Chapter III of the 1961 report on "Input-Output and National Accounts" contains a detailed discussion of valuation problems using the concepts of producers' and purchasers' values to supplement the traditional distinction between market prices and factor costs used in national accounts. Producers' and purchasers' values have, of course, come to play a prominent role in both the 1968 and 1993 SNAs.

In the late fifties, minor revisions of the 1952 OEEC and 1953 UN systems were undertaken with a view to eliminating differences between them. However, in the early 1960s the need for a more fundamental revision and extension of the joint system was perceived and in 1963 Stone was once again approached by the United Nations to see if he would help, a request which he readily accepted. The influence of Stone's earlier OEEC reports on Quantity and Price Indexes and on Input-Output on the opening chapters of the 1968 SNA is obvious. The first four chapters of the 1968 SNA which lay out the grand scheme of the revised system are in many respects a further elaboration of the ideas already developed in the earlier reports. Chapter III on the "System as a basis for input-output analysis" and Chapter IV on the "System as a basis for quantity and price comparisons" are clearly extensions and refinements of the results presented by Stone in the earlier reports. The

drafting of the 1968 SNA was shared between Stone and Abraham Aidenoff of the United Nations Statistical Office, with Stone writing the first four chapters and Aidenoff almost all the other chapters. The final draft was completed in 1968 in Cambridge.

The overview of the System as elaborated in Chapter II of the 1968 SNA represents a classic exposition of an integrated system of national accounts using the matrix presentation so favoured by Stone. The basic architecture and structure of the 1968 System survives through the recently completed 1993 revision.

National accounting, and indeed economics in general, is not the kind of science in which one magical observation, deduction or inference may sometimes merit the award of a Nobel Prize and ensure a place in history. Stone's contribution consists of a sustained input over a period of many years from 1941 onwards which has shaped the evolution of national accounts towards the complex sophisticated system as it exists in the 1993 version of the SNA. Others also made contributions, of course. In addition to the work in England under the inspiration of Stone, major progress in national accounting was being made during and after the Second World War in several other countries, notably the United States, the Scandinavian countries and the Netherlands. It would be individuou to single out a few individuals by name, but it can be seen in retrospect that many important developments were proceeding in parallel, and to some extent independently, in these countries. There is, however, one distinctive feature of Stone's work which deserves specific mention, namely his use of transactions matrices as both an analytical and expository tool.

Stone's approach to national accounting can be seen in two of his earlier papers written for academic journals while he was still actively involved in the elaboration of the international systems of national accounts in the years following the Second World War. The first paper written in 1948 is the article on "Social Accounting, Aggregation and Invariance" published in French in *Economic Applique*, Vol. II, No. 1, 1949, while the second is the paper on "Simple Transaction Models, Information and Computing" delivered at a Conference in 1951 and published in *The Review of Economic Studies*, Vol XIX, No. 49, 1951-52. In these papers Stone adopts a micro approach to national accounting by focussing on the transactions between individual economic agents. Transactions are the building blocks from which the entries in systems of accounts relating to economic flows in the economy are constructed. A transaction is essentially an interaction between two different economic agents in which the ownership of some good or asset is transferred, or some service provided, typically but not necessarily as the counterpart to the payment of money or incurrence of a financial liability. Transactions take such a wide variety of different forms that is difficult to define or describe them in a succinct manner. Nevertheless, the values of the goods, service or assets involved in transactions provide the basic elements from which the entries in accounts are derived, directly or indirectly. As many transactions take place on markets deliberately organised to facilitate the large scale conduct of

transactions, accounts based on transactions are clearly important for analysis of market economies.

The objective of economic accounting is to organise, classify and aggregate the countless transactions taking place within an economy in a way which is informative and useful for purposes of economic analysis, policy making and decision taking. Stone seems to have been among the first to approach national accounting from this viewpoint. It is a very far cry from approaching the subject of national accounting at the level of the economy as a whole and engaging in almost semi-philosophical discussions about how certain macro-economic aggregates including national income itself should be defined, measured and valued. Stone's approach may be illustrated by the following quotation from section II of the RES article:

"It is usual to classify transactions with respect to three criteria. The first is the institutional character of the payer and receiver.... The second is the form of economic activity with which the transaction is associated in the affairs of the payer and receiver. ... The third criterion is the type of transaction ... ."

(These ideas were already to be found in the 1947 Appendix to the United Nations report.)

Stone points out that by grouping transactors according to the first criterion, the different sectors of the economy are defined. The second criterion enables three major types of economic activity, namely production, consumption and accumulation to be measured for each of the sectors. The third criterion involving classification by type of good, service or asset involved in the transactions opens up endless possibilities. Stone then goes on to construct transaction matrices in which the various cells correspond to the aggregate values of sets of transactions defined with reference to the kind of criteria outlined above. In the 1948 article referred to earlier Stone had pointed out that matrices are a very natural way of recording transactions in which the rows and columns can be used to refer to individual transactors, or groups of transactors, i.e. sectors, the entries in a given cell showing the values of the transactions between the two parties, or sectors, concerned. The use of matrices to represent interactions between economies agents and the interdependence of different parts of the economy has a long history in economics, but Stone seems to have been among the first to realise the potential advantages to be gained by systematically exploiting this methodology for national accounting purposes. Another Nobel prize winner, W. Leontieff had, of course, also exploited one particular kind of transaction matrix in the late 1930s to construct input-output tables.

In the 1948 paper Stone presents the basic Keynesian identities:

$$C + S = C + I = Y$$

in the form of a three by three matrix in which each row and column represents an account: thus,

	(i)	(ii)	(iii)
(i)	—	C	I
(ii)	Y	—	—
(iii)	—	S	—

The first row/column refers to production, the second to consumption and the third to accumulation. This simple matrix was to reappear in many subsequent publications by Stone and others. For example, it was exploited in a somewhat different context by Stone in the chapter on the treatment of non-commodity flows in the 1961 publication on *Quantity and Price Indexes in National Accounts* to demonstrate the arbitrary nature of attempts to balance an accounting system in real terms.

This illustrates the fact that a matrix not only provides an alternative framework to traditional accounts for the assembly and presentation of data but also constitutes an analytical tool of considerable potential. Stone was, of course, well aware of the latter aspect and much of the 1951 paper and later papers is devoted to the use of transaction matrices for economic modelling and analysis. The combined accounting and analytical advantages of transaction matrices are particularly evident in input-output tables which were integrated into mainstream national accounting as part of the 1968 revision of the SNA. Transaction matrices naturally evolved into the more sophisticated social accounting matrices, or SAMs as they are usually called. It is not proposed to pursue the subject of SAMs here as they are likely to be the subject of other papers. It is sufficient to note that embryonic SAMs can be observed in the very earliest papers of Stone on the subject of national accounting in those critical years of the 1940s and 1950s. Chapter II of the 1968 SNA, which appears to have been mainly drafted by Stone, can be regarded as the culmination of this work on transaction matrices. The advantages of the matrix presentation of accounts are clearly explained in paras. 1.23 to 1.27 of Chapter I, the Introduction of the 1968 SNA.

## Conclusion

This paper has been concerned only with Richard Stone's contribution to national accounting. Even though Stone was awarded the Nobel Prize for work on national accounting, such work formed only part of his activities and it is necessary to remember that he also made major contributions to economic analysis, especially demand and growth analysis, that are not covered at all in this paper. All his work was firmly grounded on economic principles and he developed national accounts, not as an end in itself, but as a means to obtaining greater insights into

the working of an economic system through improved data systems designed specifically for purposes of economic modelling and analysis. It is perhaps all too easy to forget that national accounts determine the way in which economists, analysts and policy makers actually perceive the working of the economy. We still observe the economic system through spectacles prescribed by Stone, even though the prescription may need to be revised from time to time.



# COMMODITY AND SECTOR CLASSIFICATIONS IN LINKED SYSTEMS OF NATIONAL ACCOUNTS

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## 1. Introduction

Professor Stone conceived "A System of National Accounts," which was authorized by the United Nations in 1968. Traditional accounting is by means of so called T-tables, one for each account. Professor Stone's device of matrix accounting is ingenious. Instead of a T-table, an account is a pair of a row and a column (with the same index). With T-tables it is cumbersome to locate the debit and the credit entries of a single transaction; with a matrix of accounts it is automatic. Matrix accounting employs the consistency of a system of accounts: a transaction has the same debit and credit values. T-tables need not be consistent. A seller and a buyer may report a transaction differently. In matrix accounting one must decide on a common value. This problem emerges in a well known form: matrix accounts must be balanced. Professor Stone has not only recognized the consistency requirements of a matrix system of accounts, but also offered a scientific resolution, see his article, "Balancing the National Accounts" (1984) and the references given there.

Professor Stone's contributions are relatively timeless. It is only now that his system of national accounts has been revised. The Revised System of National Accounts (see United Nations, 1992) will be published by the United Nations in December 1993. Even more striking is the substance of the revision. The first paragraph of Annex II ("Changes from the 1968 SNA") speaks for itself:

"The revised System of National Accounts, (revised SNA), retains the basic theoretical framework of its predecessor A System of National Accounts (1968 SNA). However, in line with the mandate of the United Nations Statistical Commission, it contains clarifications and justifications of the concepts presented, it is harmonized with other related statistical systems and it introduces a number of features that reflect new analytical and policy concerns of countries and international organizations."

The System of National Accounts is so stable because of its flexibility. Classification problems can be accommodated by introducing separate accounts. The prime example of this flexibility is Professor Stone's resolution of classification problems called for by input-output analysis. Professor Leontief's transactions table of the sectors of the American economy and his inversion constitute the first application of general equilibrium analysis. The power of his analysis has a price: it is rigid. The concept of a sector consolidates a commodity and an activity. In practice it is difficult to classify enterprises in this sectoral framework. The U.S. Bureau of Economic Analysis juggles with the so called transfer method in order to produce a transactions table. Professor Stone simply enters separate accounts for separate items, such as commodities and activities. A clean collection and organization of statistics is facilitated and manipulations are relegated to where they belong: the economic analysis.

The revised System of National Accounts (United Nations, 1992, chapter II, pp. 11 and 12) proposes that commodities and activities are classified according to the Central Product Classification and the International Standard Industrial Classification of All Economic Activities, respectively. Classification problems persist. Modern establishments engage in a multitude of activities. Moreover, the specification of the latter hinges on primary output which cannot always be identified. In this paper I wish to point out that Professor Stone's system is so flexible that a standard classification is not required. I will investigate some traditional economic problems, the determination of productivity, competitiveness and comparative advantages, and show how they can be analyzed in the framework of a System of National Accounts with different establishments classification across countries.

## 2. The Measurement of Sectoral Productivity Rates

Productivity is the ratio of output to input. For a national economy, output comprises commodities and input comprises capital and labor. We need prices to measure output and input. The appropriate numerical values will be determined in the next section. As regards notation commodity prices are listed in a row vector,  $p$ , and the prices of capital and labor are denoted  $r$  and  $w$ , respectively. Then productivity is  $py/(rM + wN)$  where  $y$  is the net output commodity vector of the economy and  $M$  and  $N$  are capital and labor inputs. If the commodity prices coincide with production costs, then productivity equals one by the equality of the national product ( $py$ ) and income ( $rM + wN$ ). The formula becomes more interesting when it is used to account for the growth of productivity. The weights are held constant and factor productivity growth becomes the growth rate of the numerator,  $pd y/(py)$ , minus the growth rate of the denominator,  $(rdM + wdN)/(rM + wN)$ . In short, total factor productivity growth equals

$$\rho = (pd y - rdM - wdN)/(py)$$

where we invoked the national income identity.

A sectoral decomposition of total factor productivity growth using the System of National Accounts is as follows. Let the use and make tables be  $U$  and  $V$ . The commodity inputs and outputs of sector  $j$  are in column  $j$  and row  $j$  of  $U$  and  $V$ , respectively.  $(V^T - U)$  is the net output vector of sector  $j$ . Let the sectoral employment row vectors be  $K$  and  $L$ , respectively. Then  $y = (V^T - U)e$ ,  $M = Ke$  and  $N = Le$ , where  $e$  is the summation vector (all entries equal to one). Substitution yields

$$\begin{aligned} \rho &= [pd(V^T - U) - rdK - wdL]e/(py) \\ &= \sum_j [pd(V^T - U)e_j - rdK_j - wdL_j]/(py). \end{aligned}$$

The numerator is a sum of sectoral terms and each term denotes the growth of real value added per factor input. (The weights are still  $p$ ,  $r$  and  $w$ .) Note that this decomposition of total factor productivity growth does not require that the number of sectors is equal to the number of commodities.

Intuitively, a great sectoral contribution to total factor productivity growth signals greater strength of the sector, a greater likelihood that a comparative advantage resides in this sector. Comparative advantages can be determined by a model of free trade between at least two economies. For a number of reasons such a model requires that there is a unique classification of commodities, common to both economies. First and foremost, total net exports are zero for each commodity and this fact can be used to balance the accounts and to specify a model of trade with sensible feasibility constraints only if net exports can be summed on a commodity by commodity basis. The United Nations Statistical Commission recommends the Central Product Classification (CPC). The aggregation level can be selected by choice of digit level (1 to 5).

A sector is a segment of the economy where factor and commodity inputs are transformed into outputs. The statistical unit is the establishment. Ideally a unit engages in only one productive activity at a single location. A number of complications seems to plague the System of National Accounts. First, reporting units may be large and, therefore, engage in more activities. The System of National Accounts distinguishes primary and secondary activities and recommends separation of the latter. Second, productive activities may include more than a single product. The System of National Account notes that in practice, by-products are treated in the same way as secondary products, the products of secondary activities. Third, how to group statistical units. The System of National Accounts recommends to identify a principal activity on the basis of value added and to group establishments that have the same principal activity in industries according to the International Standard Industrial Classification. (United Nations, 1992, chapter XI, p. 4 and chapter II, p. 11, 1992). It acknowledges that this procedure does not eliminate secondary activities, but outlines in great detail how the use and make tables can be converted into product-by-product input-output tables (chapter XV, pp. 33-45).

In many cases there is no need to relate the sectoral classification to the product classification. An example is the above decomposition of total factor productive growth. The decomposition is by direct application on the use and make tables, without invoking the usual input-output coefficients table. Not only is there no need to reconcile sectoral classification with the CPC, but it is not even necessary to have a unique classification of sectors. International comparisons and trade studies are perfectly feasible when reporting units accommodate country specific sectors. The need to classify statistical units by primary activity and the practice to separate secondary activities stem from the imposition of the International Standard Industrial Classification. If productive activities are not only specified by their inputs and outputs, but also by location, why group them according to primary activities by ISIC? It is in the spirit of input-output analysis where commodities, activities and industries are conveniently identified by means of the concept of a sector, but there are no analytical requirements on the international comparability of industries.

### 3. The Location of Comparative Advantages

The extension of productivity analysis to the location of comparative advantages may illustrate my point.  $U$  and  $V$  are the use and make tables of the home country.  $K$  and  $L$  the sectoral factor employment row vectors with totals  $M$  and  $N$ . Introduce a foreign country, with accounts given by  $U$ ,  $V$ ,  $K$  and  $L$  (and totals  $M$  and  $N$ ). The commodity classification is the same, but the sectoral classification may be different.  $U$  and  $U$  have the same rows dimensions, but the columns dimensions differ. For  $V$  and  $V$  it is the other way round.  $K$  and  $K$  have different dimensions as have  $L$  and  $L$ . The net output vectors  $y = (V^T - U)e$  and  $y = (V^T - U)e$  reside in the common commodity space ( $e$  and  $e$  have all entries equal to one but are of different dimensions). Net output consists of domestic final demand,  $f$ , and net exports,  $g$ :  $y = f + g$  and  $y = f + g$ . In a two country model,  $g + g = 0$ , since the net exports of one country are the net imports of the other. If  $p$  is the row vector of terms of trade, then  $pg$  is the trade surplus of the home country or the deficit of the foreign. To locate the comparative advantages, let us determine the reallocation of activity prompted by competitive markets, including free trade. I make the conservative assumption that the economic agents want to stick to the observed domestic final demand proportions. If this assumption is dropped, further reallocations would take place. In other words, we will condition the comparative advantages on the observed patterns of domestic final demand. I also make the conservative assumption that no substitution takes place within sectors. (I consider them ideal statistical units in the sense of the System of National Accounts (United Nations, 1992, chapter II, p. 11). It is consistent with the country specific classification of activities. If the assumption is not fulfilled, further reallocation effects are to be expected.)

Invoking the relationship between general equilibrium and Pareto optimality, the allocation of activity under free trade can be determined

by the maximization of the domestic final demand level subject to a foreign final demand level, the material balance for the commodities and the factor input constraints:

max  $c$  subject to

$$(V^T - U)s + (V^T - U)s \geq fc + fc$$

$$Ks \leq M, Ks \leq M, Ls \leq N, Ls \leq N, s \geq 0, s \geq 0.$$

The commodity accounts are pooled and the factor input accounts are separate, assuming mobility of the former and immobility of the latter. These specifications can be altered in accordance with the facts. In general, mobile inputs have pooled balances and immobile inputs have separate balances. Now let us consider the distribution of final demand. The bigger the foreign level of final demand,  $c$ , the smaller the domestic level of final demand,  $c$ .  $s$  and  $s$  determine the allocations of activity under free trade. Net exports are the difference between net output and domestic final demand:  $(V^T - U)s - fc$  for the home country and  $(V^T - U)s - fc$  for the foreign country. In the solution, the material balance will be binding and the net exports vectors are opposite. Its value is the deficit. The deficit of the foreign economy is a monotonic function of parameter  $c$ , its consumption level. Equation with the observed deficit fixes the value of this parameter. The consequent net exports vector determines the pattern of free trade and locates the comparative advantages on a commodity basis. The underlying activity vectors,  $s$  and  $s$ , identify the competitive sectors. If a sectoral component exceeds unity, that sector would expand under competitive conditions.

The relationship with factor productivities is established by the shadow prices to the constraint of the maximization program. Active sectors break even and inactive sectors are unprofitable. Consequently the ratios of value added and factor costs are one and smaller than one, respectively. For the national economies, factor productivities are  $r$  per unit of capital and  $w$  per worker and their rates of change  $\dot{r}$  and  $\dot{w}$ . Total factor productivity growth is obtained by weighting by the factor input stocks and the result coincides with the traditional total factor productivity growth expression,  $\rho$ , by differentiation of the main theorem of linear programming. It is the values of these shadow prices that ought to be used in the total factor productivity growth measure.

Hallmarks of economic analysis, measurement of productivity, allocation of comparative advantages, identification of competitive sectors can be based on a System of National Accounts without a standard industrial classification. International sectoral comparisons can be made in terms of productivity, but do not hinge on a common classification scheme. Consider, for example, the question if agriculture is more efficient at home than abroad. Typically, agriculture is classified as the first sector. One might compare  $s_1$  and  $s_1$  in the solutions to the above program. One might also evaluate the value added/factor costs ratios of the sectors. But strictly speaking the issue of efficient agriculture boils down to the question which sector produces those commodities and there

is no reason to limit the candidate sectors to the first ones of the respective economies. It is conceivable that the products will be produced as secondary output of some other sectors. The very industrial organization or products, as determined by the make table, may in one country be different and possibly more efficient than in another.

Once it is fully recognized that activities are location specific, the identification of sectors across countries becomes redundant. A more formal approach is given by a simple rewrite of the constraints of the above model. The material balance reads

$$\left( \begin{bmatrix} V \\ V \end{bmatrix}^T - (U \ U) \right) \begin{pmatrix} s \\ s \end{pmatrix} \leq f_c + f_c$$

and the factor constraints are

$$\begin{pmatrix} K & 0 \\ 0 & K \\ L & 0 \\ 0 & L \end{pmatrix} \begin{pmatrix} s \\ s \end{pmatrix} \leq \begin{pmatrix} M \\ M \\ N \\ N \end{pmatrix}$$

The tables can be conceived as a system of world accounts in which activities remain reported separately when they take place at different locations. For example, the world use table,  $(U \ U)$ , has a row for each commodity and a column for each national sector. The sectors are simply stacked next to each other and there is no need to have equal numbers of them in the different countries, let alone a standard classification.

#### 4. Conclusion

A standard industrial classification is of course a useful device to organize enterprise data in a system of national accounts. But, unlike the classification of commodities, there is no economic analytical requirement for uniformity across national economies. Moreover, since the industrial classification is independent of the commodity classification anyway, it may be refined to accommodate enterprise data which otherwise are difficult to classify. In other words, the national sectoral classification may reflect the industrial organization of its economic activities. The classification of commodities must be as disaggregated as possible, uniform across national accounts.

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# RECENT DEVELOPMENTS IN ITALIAN NATIONAL ACCOUNTS: THE INFLUENCE OF RICHARD STONE

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Quantitative (\*) Stone's analysis has strongly influenced the setting of the Italian National Accounts System. I would like to go through the facts, the thoughts and the analysis we have made since 1980 to arrive at our current system, which synthesizes more than ten years of our activity.

The events that have characterized the Italian economy in the last twenty years (the crisis of industrial relations, the energy and raw material crisis, the cost inflation and the unemployment dynamics) have introduced structural changes in the production system and in the social behaviour. In particular: a fragmentation of the supply side of the economy, i.e. the production activity spread in the provinces and in the households, and an acceleration of the growth process of the service sector. The changes in the institutional framework have favoured the creation of a labor market segmented in heterogeneous realities, for example: the "at-home" jobs, the work of illegal immigrants, the double jobs and the development of activities concealed to the revenue authorities and to the statistical surveys.

Then the rapid and uneven development of the economic system made the official statistical information increasingly unsuitable to represent it, in particular to represent an emerging reality with its new aspects.

The unavoidable lag between the moment in which the structural and behavioural change take place and the moment in which the surveys are

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(\*) The views expressed in this paper are those of its author and do not necessarily reflect those of the OECD or its Member countries.

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conducted and the methodologies are developed, is typical not only of the Italian case, but also of the experiences of other European countries and in general of countries belonging to the industrialized area.

In that historic moment the system of Italian National Accounts System found remarkable limitations and constraints: a progressive destruction of myths made it less reliable than before.

There was much evidence, in research projects carried out by economists, sociologists and statisticians, of an underground economy not documented but parallel to the official economy and of the existence, in some parts of the territory, of a labor market divided into various segments and not reducible to a unique and homogeneous reality.

At the beginning of the Eighties, the Italian State, conscious of the importance of statistics in the public decision process, convened an International Commission of Statistics the presidency of which was given to Sir Claus Moser, charged to analyze in collaboration with Istat, the information capacity of Italian official statistics and to provide suggestions how to improve them.

A new consciousness regarding these problems had been already developed in 1976 by Eurostat, the statistical office of the European Commission, which issued a document titled: "The Current Situation and Development Perspectives of Base Statistics Necessary to Elaborate Accounts and Tables in EEC Countries".

All this was reflected in the choices of national accountants who paid in the last ten years close attention to the creation of new instruments and informational basis, providing a more accurate observation of the reality and an enlargement of the National Accounts schemes, without, however, modifying them in their core.

Main efforts were made:

- a) finding better statistical sources;
- b) enlarging the observation field with sample surveys centered around the investigation of the production and cost structure of small and very small enterprises and with multipurpose surveys directed to highlight specific phenomena relative to the agents' subjects behaviors;
- c) specifying questions in the questionnaires;
- d) increasing the data quality controls. Beside this, techniques and methodologies aimed to allow different and multipurpose analysis of structural data have been developed. The national accountants have dedicated many resources to improve the use and the reading of statistical sources. For this purpose Istat is now involved in constructing an integrated statistical system suitable to represent the social, economic and financial phenomena through an accurate and careful analysis of their different aspects, functional to the objective of the National Accounts (1).

This, on the other hand, will allow the realization of a more transparent link among aggregates and micro-data, having ensured the

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(1) The construction of this data base will allow the optimization of the information capacity of the primary sources and will be also useful for the 1992 benchmark.



migration from one to another, using formal models and estimation techniques.

The research projects aimed to complete the account schemes and to pursue a higher degree of integration among different statistical sources also were concerned with the area of constant prices estimates. In this respect an effort was made to insert the available information on prices – either derived from direct surveys or implicitly derived from the utilization of indicators of value and quantity in an integrated price Input-Output scheme. An estimation of value added at constant prices with the twice deflation method was possible by using the price Input-Output scheme.

The outcome was the construction of a scheme suitable to measure in the most exhaustive possible way the production activity and then to reduce that part of the economic system which evaded the statistical observation. This also implied the planning of well tested techniques of accounting and the construction of an equilibrium system at a disaggregated level.

A new method of work was then developed which was more conscious, more rigorous, more scientific and made easier by the use and the diffusion of electronics in the data management process. This last point is not of small importance, in that it has contributed not only to the permanent use of computerized systems, but also to the set-up of those coherence constraints useful for the quality control of estimates.

The adoption of this new method of work and the harmonization of units classification of the economic and the financial accounts allowed the completion and the enlargement of them on an annual and a quarterly base, at a national and regional level. It made possible:

- 1) a quantification of the underground economy;
- 2) a quantification of the transactions relative to units belonging to sets and subsets with the highest degree of homogeneity with respect to the function;
- 3) a development of satellite accounts and then a development of the analysis representing the distributive process;
- 4) an increase in the degree of congruence between economic and financial accounts.

The underground economy, whose extension is a function of a set of aspects and behaviours of the economic system – i.e. the size of enterprises, the revenue regulation, the social control, forms of behaviour historically consolidated, new forms of behaviour connected to emerging social and economic phenomena characterizes the Italian economy as well as other industrial economies.

However, the methods used in Italy for its quantification are different from those used in other countries. With respect to them, the "Italian method" is original in that the "underground economy" phenomenon has been dealt with ex-ante and overall terms and therefore the methodological solution adopted to quantify the phenomenon is included in the system.

In the estimation methods of the production by the supply side, the criterion consisting of expansion of the per capita values for labor units,

is prevailing <sup>(2)</sup>. This criterion is used after having estimated the overall labor underlying the product and after having corrected the percapita values for occasional undervaluation. It clearly shows the fundamental role played by employment in all national accounts estimates.

The Italian accountants designed a methodology suitable to identify and analyse the various segments of regular and irregular economy in the labor market by choosing a definition of "employee" close to the economic reality where, apart from the occasional and marginal labor, a new type of worker emerges: a worker with many job positions allocated across different branches. They also chose a disaggregation level by economic activity and by territory which maximizes the final results.

A definition of employee in accordance with the current economic reality requires to break down the identity of worker-work activity. Rather, the total number of different job positions are taken into accounts.

The quantity of work represented by the set of many different job positions is equal to the number of workers ("heads") only up to the point in which each position is unique for the worker; beyond this limit the quantity of work has to be increased by all job positions classified as "secondary activities". The volume of work performed in the production process must in fact include this group of activities.

This means that neither the number of "the persons who work" nor the number of "job positions" may permit a satisfactory measure of the volume of work which contributes to the production process, therefore a new concept needs to be introduced by National Accounting authorities: the "Labor Unit". It is characterized by an equal involvement in the productive process by each economic activity, obtained by reducing the series "job position" to approximately homogeneous units. More explicitly, the work units are estimated in the confines of each branch, establishing a correspondence, through a reduction coefficient, between the discontinuous and part-time (principal and secondary) jobs and the full-time jobs.

This coefficient is equal to the ratio between the hours actually worked by a person employed part-time in any job position and the hours actually worked by a person who reports himself "employed" in the survey on industry and service labor forces. Regarding the agriculture, the same ratio is calculated by using the actual work days as an indicator.

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(2) Other estimation methods for products are:

a) methods consisting in direct estimates of revenues and costs from balance sheets. These data, adjusted for underreporting and incompleteness, are reclassified in the branches in which the homogeneous productivity units are classified. They are relative to the large enterprises, public enterprises and Public Institutions;

b) methods consisting in the aggregation of estimates of produced quantity and prices. They are used to estimate the agriculture, buildings, and energy production.

c) methods consisting in estimating the product by collecting information on Uses. These are criteria generally adopted to estimate the activity of "invisible" units or units of a very small dimension that are interested in remaining concealed and therefore do not record their production. They are in the personal services, like education, recreation, social security, ecc.

More synthetically, it can be said that the national accounts employment is expressed by a series called "Labor Units", which reduces an intermediate series called "job positions" to approximately homogeneous units. The per capita values used for the estimate of the production is subject to control and the phenomenon of "underreporting" by entrepreneurs is bypassed by making adjustments.

An "underreporting" of the income produced is hypothesised if the hourly wage of someone self-employed is lower than that of a dependent worker who works, at parity of work hours, in the same activity.

If the hypothesis is verified, the original data relative to the production and income value are corrected by using a procedure based on the "Franz method". The procedure of calculus made for economic activity and classes of employee follows these steps:

a) the per capita income of dependent workers (managers, white collars, blue collars) is recalculated on the basis of the same numbers of hours of those who are self-employed;

b) the net income for each enterprise gives the possibility to calculate the net per capita income of the self-employed <sup>(3)</sup>;

c) if the per capita income of the self-employed is lower than those of workers with long-term contracts, the latter income is attributed to each self-employed.

In the estimation of the product the national accountants consider the use in the production process of irregular work and the related underreporting as principal features of the "underground" economy.

For many branches of economic activity, in particular those in which the production realized by small and very small units is substantial, direct and specific sample surveys are made, in order to let emerge the aspects of underground and irregular economy.

The phenomena observed through sample surveys "ad hoc" are the expenses afforded by the households for the vacations, the outside beverage and food consumption and the dwellings repairs.

The surveys' results on the vacation and the outside food and beverage consumption, integrated with the expenses afforded by foreigners in hotels and extra-hotel accomodation, allow the estimation of the product created by the branch "Hotels and Catering". The results of the survey on ordinary and extraordinary dwellings' repair highlighted a part of the product realized with the second job and were used for the calculus of the product of the branch "building".

The total product of this branch was then calculated by also estimating the illegal buildings. This estimate was made by comparing the information drawn by municipality registers and census data.

The set of methodologies has allowed to uncover the underground activity for the statistical information but, due to the unavoidable

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(3) This is estimated by subtracting from the Gross Domestic Product the incomes of dependent workers, the bank commissions, the depreciation of fixed capital, the rents and the insurance premia.

approximation in estimations, the accounts' coherence, a fundamental prerequisite of a national accounts model, can only be approximated.

In order to ensure the accounts' coherence, the system must be balanced at a disaggregated level. "The balancing is necessary", says Stone, "...because the economic information available is to some degree incomplete, inconsistent, and unreliable. It is impossible to construct a set of national accounts by direct measurement without the appearance of unidentified terms and errors, variously termed statistical discrepancies.... [Balancing entries] have to be introduced before the identities which connect the true values of the entries in the accounts are satisfied" (R. Stone, 1988).

The development of a balancing methodology for Italian National Accounts was strongly influenced by Stone's work. Based on the scheme originally proposed by Stone, Champernowne and Meade (1942), adapted to suit the Italian reality, it consists of a procedure which integrates a purely subjective method of correcting errors with a more sophisticated approach, which allows to evidence disequilibrium in the accounts by distributing residuals among aggregates. The residuals' redistribution is made not only by taking into accounts the aggregates' weights, but by exploiting any available information on the estimation methodologies, i.e. coverage degree, use of proxies, indicators, etc.

Starting from the I-O standard scheme, let us define  $X$  an accounting matrix and  $\hat{X}$  the same accounting matrix, unbalanced, that is the matrix of entries' estimates in  $X$ .

The authors suggest the balancing of the system:

$$\check{X} = \hat{X} - VG'(G\hat{X})^{-1} (G\hat{X} - h) \quad [1]$$

where  $\hat{X}$  is a vectorization of initial estimates supposed unbiased,  $\check{X}$  is the same vector where the accounting system is balanced,  $G$  is the constraints' matrix,  $V$  is a matrix of weights which shows how the residuals of the unbalanced system are to be distributed among the aggregates,  $G\hat{X}$  are the residuals of the unbalanced system of equations  $\hat{X}$  and  $h$  is a vector of known terms such that  $G\check{X} = h$ . The vector is formed by the known terms of the equation system whose valuation, by definition, is not included in the residuals to be distributed among aggregates.

The peculiarity of Stone-Champernowne-Meade methodology consists mainly in the use of the  $V$  matrix in [1], since its correct specification allows a reasoned redistribution of the residuals, not a proportional one, as happens with other methodologies. By using in an appropriate way this property in the balancing process, the procedure carried out by Italian national accountants takes into proper accounts the various methods followed in estimating the aggregates in  $\hat{X}$ . This gives rise to a more incisive revaluation process for entries whose estimates are perceived to be less reliable than others.

In particular, since the the  $V$  matrix is expressed in the same unit of account as  $\hat{X}$ , it can be considered as the covariance matrix of  $\hat{X}$  such that the covariance matrix of  $\hat{X}$  can be expressed as:

$$\Sigma_{xx} = V - VG' (G V G')^{-1} GV \quad [2]$$

In National Accounts applications, the determination of the V matrix components is carried out by assigning to each set or subset of estimates in  $\hat{X}$  a value between 0 and 1 reflecting in a partly subjective way, as we already saw, the reliability degree attached to initial estimates and multiplying the  $\hat{X}$  values by the complements to 1 of those valuations.

It must be noted that if n is the number of elements in  $\hat{X}$ , V is a matrix of order  $n^2$  of the variance-covariance of  $\hat{X}$ . In practice V is a diagonal matrix since it is assumed that all the estimates of elements in  $\hat{X}$  are independent.

This last hypothesis, that at a first glance appears quite limiting, is not in the practice if one considers that the N. A. estimates are drawn from different sources and no objective methods exist for the estimation of the covariances in  $\hat{X}$ . It is evident that, being V a matrix of absolute values of the same size of  $\hat{X}$ , the distribution of residuals is carried out on the basis of the variances of the elements of each equation of the system and not in a direct way, on the basis of the weights' system used to generate the V matrix.

However, from a thorough analysis of [1] and [2] it is clear that the approach proposed can introduce only small advantages with respect to analogous methodologies if the constraints' system is wrong or the V matrix is misspecified. In fact in the first case the  $\hat{X}$  estimator will be biased and in the second case it will be inefficient. Moreover, the use of V matrix must be very careful because a misspecification of such matrix can lead to final results economically not significant. The balancing methodology is very sensible to the use of a variances system not coherent with the aggregates' estimation methodologies, mainly due to the simultaneity of the balancing process, in a highly disaggregated system. In the first phase of construction of the variances, it is not rare to obtain a change of the sign of some entries, mainly those relatively small with a high variance, due to the inconsistency between the residuals and the variance matrix.

This result is even worst when an equation contains a high percentage of constrained values with respect to the unconstrained values.

The sensitivity of the system is enhanced when the estimates are precise and coherent.

The first interesting application of the Stone-Champernowne-Meade methodology was successfully made in the construction of the National Accounts time series for 1971-1987. The accounting structure used is described in the scheme 1 and 2. This scheme concerns only the goods and services flows and reproduces the Input-Output scheme only for the totals of the various aggregates (given by the sum of national production and imports).

In order to understand the accounting system, it is necessary to take into account that the I-O scheme considers the equilibrium between

flows going in the production process of the branch (inputs) and flows going out of the branch towards intermediate and final uses (outputs). This, in the scheme 1 is expressed by the equilibrium condition among aggregates in each row and column. Each elements of figure 1 is a matrix of values whose content is represented in the scheme 2.

The fundamental equation of the system is the first of the scheme 1:

$$Ic + If = C + VA + D + M + MA + T \quad [3]$$

It imposes the equality between the value of production for intermediate consumption ( $Ic$ ) and final consumption ( $If$ ) (households' consumption, General Government's and private NPIS' consumption, Gross Fixed Capital Formation, change in inventories, goods' and services' exports) and the sum of production costs of enterprises ( $C$ ), the value added at factor costs ( $VA$ ), the value of net indirect taxes, (taxes on production net of production contributions), the imports ex-custom ( $M$ ) (CIF imports and taxes on imports), the commercial margins ( $MA$ ) and the products' transfer ( $T$ ) <sup>(4)</sup>.

The equation has been applied to the 42 production branches corresponding exactly to the NACE-CLIO except for those belonging to the sector "Public Administration" (NACE-CLIO 81,85 and 89), which are grouped in one branch.

The Figure 1 includes 5 additional equations whose function is that of "closing" the accounts system since they express simply the constraints among entries' components in [3] and the value of their sum. These equations are however very important in those cases in which there is a reliable estimate of the total value of some aggregates with respect to their distribution in branch or in components, so that it becomes necessary to elaborate a different weights system in the process of balancing of the entries.

The last equation of the system represents, finally, the [3] at an aggregate level. It takes into account that for the entire economy the sum of costs beared by enterprises in the production process is equal to costs for intermediate consumption of the same enterprises and the sum of transfers in goods and of trade and transport margins across branch is zero. In simbols we have:

$$If = VA + D + M \quad [4]$$

A balancing scheme completely similar to this, with the same obvious difference of disaggregation of the General Governement Account in the three branches considered in NACE-CLIO, is used for the balancing of the estimates of aggregates' in the yearly National Accounts.

(4) In an Input-Output scheme it is necessary to take into accounts products transfer in order to link the effective production and the distributed production. The value of transfer is equal to the production of joint commodities.

**Figure 1 – Accounts scheme**

	1	2	3	4	5	6	7
1	–	C	I	–	–	–	–
2	DSC	–	–	–	–	–	–
3	–	–	–	–	–	–	SI
4	R	–	–	–	–	–	–
5	RM	–	–	–	–	–	–
6	MAR	–	–	–	–	–	–
7	–	–	–	SR	SRM	SMAR	–

**Figure 2 – Contents of the matrices**

- C** = total intermediate flows (42 × 42)
- I** = final uses (42 × 6)
- final households consumption
  - General Government Consumption
  - Private Non Profit Institutions consumption
  - gross fixed capital formation
  - change in stocks
  - exports
- R** = taxes and V.A. (5 × 42)
- value added at factor cost
  - Indirect taxes on production
  - production and exports contributions
  - products' transfers
  - VAT on products and imports
- RM** = Imports and taxen linked to imports
- CIF imports
  - indirect taxes on imports
- MAR** = margin ( × 42)
- trade margins
  - transmort margins
- DSG** = diagonal matrix of total cost (42 × 42)
- SI** = vector of totals of final demand components
- SR** = vector of totals of R matrix components
- SRM** = vector of totals of RM matrix components
- SRM** = vector of totals of matrix MAR components

The enlargement of ESA scheme was also realized for the construction of institutional sectors' accounts. In particular the relationships between the household sector and the enterprise sector, the enterprise sectors and the General Government sector were made more transparent by creating subsets of institutional sectors as homogenous as possible with respect to their function.

At the operating level this required first to modify the contents of sectors and subsectors and secondly to design a methodology

harmonized to the methodology used for the homogeneous product units. It allowed to obtain an estimate for all aggregates in sectors' accounts aggregates without <sup>(5)</sup>.

The new definition of sectors "non-finance corporation and quasi-corporation" and "Households" was conceived in order to determine the production activity of enterprises of any size and at the same time to isolate a set of aggregates which highlights the behavior of the households seen only as consumers.

The analysis of the households as consumers will be deepened by linking the functional income distribution to the personal income distribution.

An additional improvement of the field of analysis was realized by compiling satellite accounts for the Social Security System and the Health System. They permitted to analyse the redistribution aspect and enlarge the knowledge of social phenomena affecting the economic framework.

The observation of interrelationships among socio-demographic factor and economic variables should help in analyzing and solving many problems such as the unemployment, the faults of the Social Security System, of the Health System and of the Educational system.

This approach however should be developed in order to include the quantification of externalities caused by pollution, the housework, the voluntary work and to tackle the main issues of welfare.

Stone suggests to recur to the Input-Output analysis. The renewal process which invested the Italian National accounts in the Eighties is proceeding now toward a macroeconomic representation based on a deep knowledge of Input-Output interdependences and on its integration with the sectors' accounts. This means to build a social matrix which allows the analysis of structural relationships of the economy and the income distribution among sectors.

This task is not beyond our statistical system's reach but adequate resources and political willingness to use information are necessary.

I would like to finish this lecture with His own words and make them mine: " ....I hope that economics and econometrics will become more empirical because, while theory is essential, its purpose is to help us to

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(5) The sector non-financial corporate and quasi-corporate enterprises (SaS) consists of corporations, public and private enterprises with more than 20 employees, divided into: "Non-financial corporate and quasi-corporate public enterprises and Non-financial and quasi corporate private enterprises". This classification will make possible to build the satellite accounts of the "Enlarged Public Sector" and the "Total Public Sector";

The households sector covers households as consumers and households as entrepreneurs. The latter are formed by individual enterprises and partnerships with less than 20 employees. The consumption and production function are evidenced in separate accounts ("household as consumers", including also NPI's, and "individual enterprises"). In taking this decision it was considered also that in the 'nineties the difficulty to separate the decisions of households as consumers from those of households as entrepreneurs should be overcome by the obligation to compile balance sheets for Fiscal and Social Security purposes.



interpret and understand the world we live in. Spinning theories is good fun, especially when they are expressed mathematically; testing them quantitatively is a swot, but it is the only way of finding out whether they have any validity. I know that theorising is considered a nobler pursuit than number crunching and is therefore held up as the highest achievement to all who aspire to fame. As a consequence, thousands of theoretical papers are published every year. I doubt whether thousands of worthwhile theories are produced every year".

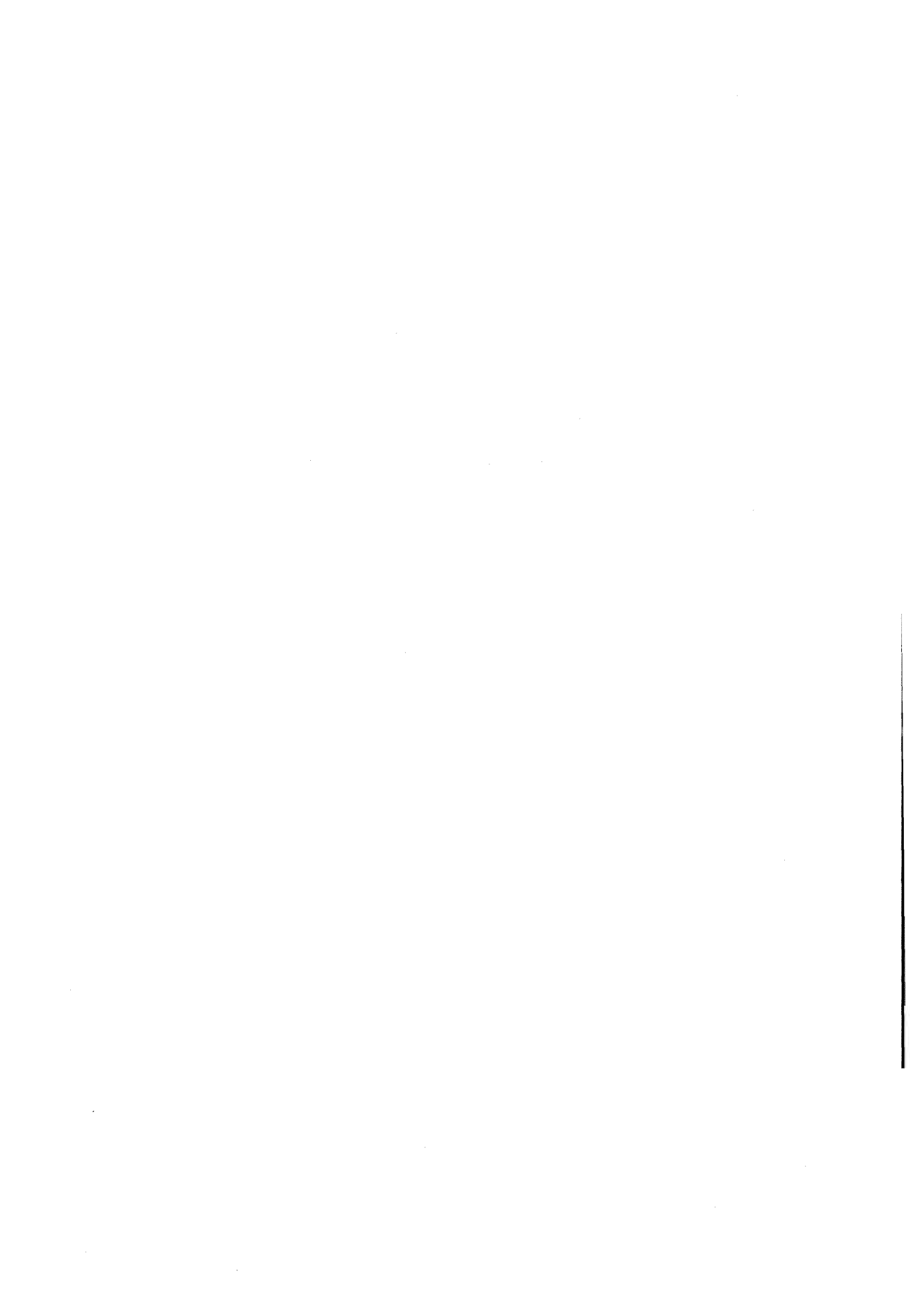
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**II SESSION**  
**ACCOUNTING FRAMEWORKS AND**  
**ECONOMIC ANALYSIS**

Chairman: Angus Deaton (*Princeton University – U.S.A.*)



# RATIONAL EXPECTATIONS IN DISAGGREGATED MODELS: AN EMPIRICAL ANALYSIS OF OPEC'S BEHAVIOUR

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## 1. Introduction

In this paper we consider the problem of information heterogeneity in the context of a linear Disaggregated Rational Expectations (DRE) model which relates decision variables of an individual agent (or a group of agents) to the individual-specific variables, and to the agent's expectations of economy-wide variables. Examples of DRE models include the isolated "island" models of Phelps (1970) and Lucas (1973), game-theoretic models of price and output decisions of oligopolistic firms, and the sectoral models of money wage determination where wages in a given sector are set in the light of expectations of wage settlements in other sectors (as in Lee and Pesaran (1993)).

The analysis of DRE models does not present any new technical problems when expectations are formed with respect to the same information set. Under homogeneous information the problem of disparities in expectations across agents disappears and the DRE model reduces to the familiar simultaneous-RE model already discussed in the literature (1). However, when the focus of the analysis is on aggregate behaviour one needs to consider the aggregation problem which arises when individual-specific variables enter different equations of the DRE model with different coefficients. The analysis of the aggregation

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(1) The solution, identification and estimation of multivariate RE models are discussed in Revankar (1980), Wallis (1980), Pesaran (1981, 1987), Broze and Szafarz (1987), and Binder and Pesaran (1995a).

problem in the context of DRE models is outside the scope of the present paper <sup>(2)</sup>.

The situation is very different when information is heterogeneous across agents. In this case the decision-making will be subject to "behavioural" or strategic uncertainty, and a determinate solution for the DRE model in general will not be available <sup>(3)</sup>. The solution of the model involves individuals' forecasts of what others are forecasting, what the average opinion expects the average opinion to be, and so on. In this paper we follow the approach in Pesaran (1987), and derive a general solution for the DRE model by the "infinite regress" method, and explore the conditions under which the infinite regress method yields a determinate solution. We then show that when these conditions are met the infinite regress solution can also be obtained directly by a method similar to the martingale-difference method used in obtaining the solution of standard linear RE models [see, for example, Broze et al. (1985) and Pesaran (1987, ch. 7)]. This solution method considerably simplifies the econometric analysis of DRE models and enables one to derive sufficient conditions under which consistent estimation of the parameters of the DRE model can be obtained by the Instrumental Variable (IV) method or by the Generalized Method of Moments (GMM).

Disaggregated Rational Expectations models arise naturally in game situations where behavioural uncertainty is prevalent and individuals' actions depend significantly on their perception of other individuals' actions and beliefs. As an example, we formulate and estimate an output allocation model for OPEC member countries. The analysis of OPEC's output decisions has attracted a great deal of attention in the literature. See, for example, the papers by Fischer et al. (1975), Griffin and Teece (1982), Gately (1984), Griffin (1985) and Salehi-Isfahani (1986). But none of these studies explicitly allow for the heterogeneous nature of the information structure that underlies OPEC's behaviour. In this paper we employ the Nash bargaining framework and derive a DRE model for the empirical analysis of output policies of individual OPEC member countries paying particular attention to the problem of information heterogeneity. Our analysis abstracts from the dynamic considerations that arise because of the ultimately exhaustible nature of the oil reserves

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(2) The problem of aggregation in standard linear models has been addressed in some detail in the literature. See, for example, Theil (1954), Grunfeld and Griliches (1960) and Boot and de Wit (1960) for an early analysis, and Pesaran et al. (1989), Barker and Pesaran (1990) and Lee et al. (1990a,b) for a more recent treatment of the subject.

(3) The concept of "behavioural" uncertainty is defined in Pesaran (1987, Ch2) and refers to situations where the subjective probability distribution of uncertain events faced by individuals is affected by their own actions or the perception they may have of the action of others in the market place. This type of uncertainty is particularly prevalent in oligopolistic market conditions, or in labour markets where wage contracts are struck among a few powerful unions and employers. Keynes' example of a "beauty contest" where each participant is required to anticipate "what average opinion expects the average opinion to be" represents a good instance of how behavioural uncertainty may come to dominate individuals' decisions.

and considers, for empirical purposes, a relatively simple formulation of "outside options" open to the individual OPEC member countries.

## 2. Disaggregated RE Models – Solution and Estimation

### 2.1. Examples of Disaggregated RE Models

Disaggregated RE models have been discussed in the literature primarily in the context of isolated "island" models of Phelps (1970) and Lucas (1973). Here we consider other examples and motivate the way that we propose to approach the problem of information heterogeneity. The first example deals with the case where individual-specific decision variables depend on *current* expectations of the macro variables, and the second example relates current decision variables to the *future* expectations of the macro-variables.

#### **Example 1** A Disaggregated Model of Wage Determination <sup>(4)</sup>

Consider the disaggregated model:

$$y_{it} = \alpha_i + \beta_i E_{it}(\Pi_t) + \delta_i' x_{it} + \varepsilon_{it} \quad i = 1, 2, \dots, m \quad (1)$$

$$\Pi_t = \theta_1 y_t + \theta_2' z_t + v_t, \quad (2)$$

$$y_t = \sum_{i=1}^m w_i y_{it} \quad (3)$$

Equation (1) gives the rate of change of money wages ( $y_{it}$ ) in the  $i$ -th sector as a function of sector-specific variables ( $x_{it}$ ), and the sector's expectations of the overall rate of inflation in the economy ( $E_{it}(\Pi_t)$ ). Other macro-variables could also be included in (1) but are left out to keep the exposition simple. Equation (2) represents a standard mark-up equation linking the rate of inflation ( $\Pi_t$ ) to the average rate of change of money wages ( $y_t$ ), and other macro-variables ( $z_t$ ) such as the rate of change in material and fuel prices and cyclical variables. In equation (3),  $w_i$  is the  $i$ -th sector's share of wages in the total wage bill in the base year;  $\varepsilon_{it}$  and  $v_t$  represent white-noise processes.  $E_{it}(\Pi_t) = E(\Pi_t | \Omega_{it})$  is the expected value of  $\Pi_t$  formed on the basis of  $\Omega_{it}$ , the information set of agent  $i$  at time  $t$ . The above DRE model allows for sectoral interactions in the wage-setting process through the inflation expectations variable and has a number of important potential applications. The model, for example, provides a framework for the analysis of the empirical relevance of wage formation theories that emphasize the importance of inter-sectoral wage comparisons, as negotiators look to settlements outside their industry to form a reference wage against which they decide on their own wage claims.

(4) For a discussion of the theoretical underpinnings of this model see Lee and Pesaran (1993).

Under the RE hypothesis equations (1)-(3) reduce to:

$$y_{it} = \lambda_i E_{it}(y_t) + h_{it}, \quad i = 1, 2, \dots, m \quad (4)$$

$$y_t = \sum_{i=1}^m w_i y_{it} \quad (5)$$

where

$$h_{it} = \alpha_i + \phi_i' E_{it}(z_t) + \delta_i' x_{it} + \varepsilon_{it} \quad (6)$$

$$\lambda_i = \theta_1 \beta_i \quad \text{and} \quad \phi_i = \theta_2 \beta_i \quad (7)$$

The solution of the DRE model (4) crucially depends on the nature of the information set,  $\Omega_{it}$ . It is analytically useful to decompose  $\Omega_{it}$  into a public information set,  $\psi_{t-1}$ , available to all agents, and a sector-specific private information set,  $\Phi_{it}$  namely  $\Omega_{it} = \Phi_{it} \cup \psi_{t-1}$ . In the case where information is homogeneous (or symmetric) across sectors (agents) there is no private information ( $\Phi_{it} = 0$ ) and the problem of disparities in expectations across sectors (agents) disappears. In this case the DRE model (4) reduces to a simultaneous RE model already discussed in the literature by, for example, Revankar (1980), Wallis (1980), Pesaran (1981, 1987), Broze and Szafarz (1987), and Binder and Pesaran (1995a), and the econometric analysis of DRE models does not present any new technical difficulties. The situation is, however, different when information across agents is not homogeneous.

Two types of information disparities across agents may be distinguished: asymmetric and heterogeneous information structures. Under the former it is assumed that economic agents can be divided into two groups; with one group being more informed than the other. <sup>(5)</sup> Under the latter (the heterogeneous information case) it is simply assumed that information differs across agents. There are no presumptions that any one agent has more information than any other one. Our aim is to develop operational methods for the empirical analysis of disaggregated models such as (4), both under asymmetric and heterogeneous information structures. The present paper focuses on the heterogeneous information case. The case of asymmetric information is dealt with in Binder and Pesaran (1995b).

## Example 2 A Simple Model of Land (Gold) Speculation

The model in the previous example relates the individual decision variables to the current expectations of the economy-wide variables. Here we consider a simple model of land speculation discussed in Futia (1981), where expectations of future land prices enter the decision

(5) This definition is more general than the one usually adopted in the literature, where it is assumed that one group is informed while the other is uninformed [see, for example, Futia (1981, p. 188)].



equations of individual speculators. In this model the demand for land at time  $t$  by speculator  $i$  ( $q_{it}^d$ ) is given by

$$q_{it}^d = \{k_i V(P_{t+1} | \Omega_{it})\}^{-1} \{E(P_{t+1} | \Omega_{it}) - (1+r)P_t - c\} \quad (8)$$

where  $P_t$  is the price of land at time  $t$ ,  $k_i$  is the Arrow-Pratt index of absolute risk-aversion,  $r$  is the rate of interest (assumed fixed), and  $c$  is the transaction and maintenance costs per parcel of land [for a derivation of (8) in the context of an inventory model, see for example, Pesaran (1987, p. 113)]. Suppose now that the supply of land for speculative purposes, which we denote by  $s_t$ , is generated according to a general linear stochastic process. Then under the assumption that innovations in  $\{s_t\}$  are homoscedastic, we have  $V(P_{t+1} | \Omega_{it}) = \sigma_i^2$  (a fixed constant), and the market clearing condition for this problem yields:

$$P_t = \beta \sum_{i=1}^m \omega_i E(P_{t+1} | \Omega_{it}) + x_t \quad (9)$$

where

$$\beta = 1/(1+r), \quad \lambda = \sum_{i=1}^m \lambda_i, \quad \omega_i = \lambda_i/\lambda, \quad \lambda_i = (k_i \sigma_i^2)^{-1},$$

and

$$x_t = -\beta(c + s_t/\lambda).$$

The above equation generalizes the familiar Cagan's money demand equation to the case where price expectations in the market are formed with respect to disperse information. This is an important generalization and allows a more satisfactory analysis of speculative behaviour in capital and financial markets. Futia (1981) has already considered the solution of a special case of (9), when speculators can be divided into informed and uninformed categories.

## 2.2. Solution of RE Models Under Heterogeneous Information

In general, the solution of DRE models under heterogeneous information is indeterminate and involves individuals' forecast of what others are forecasting, what the average opinion expects the average opinion to be, and so on. The solution will be subject to the "infinite regress" problem discussed in the literature by Phelps (1983), Di Tata (1983), Townsend (1983), and Pesaran (1987, ch. 4), among others. For example, in the case of the money wage growth model set out above, in order to determine the rate of growth of money wages in any one sector, agents need to form expectations of the growth of money wages in the other sectors. Here we briefly describe the infinite regress method and suggest a simple and a plausible method of dealing with the indeterminacy of the infinite regress solution. Clearly a satisfactory resolution of the infinite regress problem is needed before DRE models under heterogeneous information can be implemented empirically.

### 2.2.1. The Infinite Regress Solution

First aggregate (4) across sectors to obtain

$$y_t = \lambda F^1(y_t) + h_t, \quad (10)$$

where

$$h_t = \sum_{i=1}^m w_i h_{it}, \quad \lambda = \sum_{i=1}^m w_i \lambda_i, \quad (11)$$

$$\tilde{w}_i = w_i \lambda_i / \lambda, \quad \left( \sum_{i=1}^m \tilde{w}_i = 1 \right), \quad (12)$$

and

$$F^1(y_t) = \sum_{i=1}^m \tilde{w}_i E_{it}(y_t) \quad (13)$$

which is the first-order "market expectations" of  $y_t$ . Now using

$$E_{it}(y_t) = \lambda E_{it}[F^1(y_t)] + E_{it}(h_t),$$

it follows that

$$F^1(y_t) = \lambda F^2(y_t) + F^1(h_t), \quad (14)$$

where  $E_{it}[F^1(y_t)]$  is the  $i$ -th agent's expectations of the (first-order) market expectations,

$$F^2(y_t) = \sum_{i=1}^m \tilde{w}_i E_{it}[F^1(y_t)] \quad (15)$$

is the second-order "market expectations" of  $y_t$ , and

$$F^1(h_t) = \sum_{i=1}^m \tilde{w}_i E_{it}(h_t). \quad (16)$$

Substituting (14) into (10) yields

$$y_t = \lambda^2 F^2(y_t) + h_t + \lambda F^1(h_t).$$

Continuing this process *ad infinitum* and assuming that  $\lim_{r \rightarrow \infty} \lambda^r F^r(y_t) = 0$  gives the "infinite regress" solution:

$$y_t = h_t + \sum_{r=1}^{\infty} \lambda^r F^r(h_t), \quad (17)$$

where

$$F^r(h_t) = \sum_{i=1}^m \tilde{w}_i E_{it}[F^{r-1}(h_t)], \quad r = 2, 3, \dots$$

and  $F^1(h_t)$  is given by (16).

### 2.2.2. A Determinate Solution

The above solution depends on the unobservable average expectations  $F^1(h_t)$ ,  $F^2(h_t)$ , ... and in general will not be determinate. Further assumptions are therefore needed if the analysis of DRE models under heterogeneous information is to be empirically operational. One possibility would be to assume that  $h_{it}$  is in  $\Omega_{it}$ , and that in forming expectations of  $h_{jt}$  agent  $i$  ( $i \neq j$ ) conditions his/her expectations only on the public information set,  $\psi_{t-1}$ . This assumption can be stated formally as:

**Assumption A:** It is "common knowledge" that

$$E_{it}(h_{jt}) = \begin{cases} h_{jt} & i = j \\ E(h_{jt} | \psi_{t-1}), & i \neq j. \end{cases} \quad (18)$$

This seems a plausible assumption, considering that in the present model agent  $i$  does not observe  $\Omega_{jt}$  ( $i \neq j$ ) and with  $m > 2$  has no way of inferring it from the publicly available information set,  $\psi_{t-1}$ . We now have

**Proposition 1** Under the "common knowledge" Assumption A and assuming that  $|\lambda| < 1$ , the infinite regress solution (17) yields the unique solution

$$y_{it} = h_{it} + \left( \frac{\lambda_i}{1 - \lambda} \right) E(h_{it} | \psi_{t-1}) + \left( \frac{\lambda_i w_i}{1 - \lambda_i w_i} \right) \varepsilon_{it}^h \quad i = 1, 2, \dots, m \quad (19)$$

where

$$\varepsilon_{it}^h = h_{it} - E(h_{it} | \psi_{t-1}), \quad (20)$$

and

$$E(\varepsilon_{it}^h | \psi_{t-1}) = 0. \quad (21)$$

The proof of this and other propositions in the paper are collected in the Mathematical Appendix.

This same solution can also be obtained if we assume that the individual-specific expectations,  $E_{it}(y_t)$  in (4) can be decomposed into a "common" and an individual-specific component. More specifically we have

**Proposition 2** Suppose that the individual-specific expectations  $E_{it}(y_t)$  can be decomposed into a "common" and an individual-specific component, namely

$$E_{it}(y_t) = E(y_t | \psi_{t-1}) + w_i [y_{it} - E(y_t | \psi_{t-1})]. \quad (22)$$

Then the DRE model (4) has the same infinite regress solution as in (19).

The above solution strategy can also be adapted to the analysis of multivariate RE models under heterogeneous information with current and/or future expectations [see, Binder and Pesaran (1995b)].

### 2.3. Estimation and Hypothesis Testing

Assumption A, or equivalently the decomposition 22, seems a plausible starting point for dealing with the infinite regress problem and is particularly convenient for empirical analysis of DRE models under heterogeneous information. To use the above disaggregated solution for estimation purposes we first note from (19) that <sup>(6)</sup>

$$y_t = h_t + \left( \frac{1}{1 - \lambda} \right) E(h_t | \psi_{t-1}) + \sum_{i=1}^m \left( \frac{\lambda_i w_i^2}{1 - \lambda_i w_i} \right) \varepsilon_{it}^h.$$

Taking conditional expectations with respect to the public information set we have

$$E(h_t | \psi_{t-1}) = (1 - \lambda) E(y_t | \psi_{t-1}),$$

which may also be written as

$$E(h_t | \psi_{t-1}) = (1 - \lambda)y_t - (1 - \lambda)\eta_t, \quad (23)$$

where  $\eta_t = y_t - E(y_t | \psi_{t-1})$  is a martingale difference process with respect to the public information set and satisfies the orthogonality property  $E(\eta_t | \psi_{t-1}) = 0$ . Substituting from (6) and (23) in (19) and using the decomposition

$$y_t = w y_{it} + y_{-i,t},$$

where

$$y_{-i,t} = \sum_{\substack{j=1 \\ j \neq i}}^m w_j y_{jt} \quad (24)$$

we have

$$y_{it} = \left( \frac{\lambda_i}{1 - w_i \lambda_i} \right) y_{-i,t} + \left( \frac{\alpha_i}{1 - w_i \lambda_i} \right) + \left( \frac{\phi_i'}{1 - w_i \lambda_i} \right) E_{it}(\mathbf{z}_t) + \left( \frac{1}{1 - w_i \lambda_i} \right) \delta_i' \mathbf{x}_{it} + v_{it}$$

$$i = 1, 2, \dots, m, \quad t = 1, 2, \dots, n. \quad (25)$$

where

$$v_{it} = \left( \frac{1}{1 - w_i \lambda_i} \right) \left\{ \varepsilon_{it} - \lambda_i \eta_t + \left( \frac{\lambda_i w_i}{1 - w_i \lambda_i} \right) \varepsilon_{it}^h \right\}. \quad (26)$$

In situations where the process generating the macro-variables  $\mathbf{z}_t$  is "common knowledge",  $E_{it}(\mathbf{z}_t)$  can be replaced by  $E(\mathbf{z}_t | \psi_{t-1})$ , although in principle sector-specific expectations,  $E_{it}(\mathbf{z}_t)$ , can be substituted in (25) directly, when they are available. For a consistent estimation of the parameters of (25), and hence the structural parameters of the DRE model (4) we now make the following further assumptions:

(6) See also result (35) in the Appendix.

*Assumption B:* The micro disturbances  $\varepsilon_{it}$  in (6) have zero means, are serially uncorrelated and are distributed independently of the variables entering the DRE model.

*Assumption C:* The correlation matrices of the instruments (taken from the publicly available information set) and the individual specific variables in (25) have full ranks both in finite samples and asymptotically.

**Proposition 3** *Under Assumptions A-C, the parameters of the DRE model (4) can be consistently estimated using (25) by the IV method or the GMM using variables in the publicly available information set as instruments.*

### 3. An Empirical Nash Bargaining Model of OPEC's Behaviour

The output and pricing policies of OPEC viewed as a unified entity have been the subject of numerous studies, some of which are discussed and reviewed in Griffin and Teece (1982) and in Gately (1984). In contrast, the output policies of individual OPEC member countries and the strategic interactions that exist amongst them have not attracted a great deal of attention in the empirical literature on OPEC. (7) An important exception is the empirical study by Griffin (1985) which does consider the output equations of individual OPEC member countries but does not allow for the heterogeneous nature of the information structure that underlies OPEC's decision environment and hence uses inappropriate econometric methods in his analysis. Griffin's specification is also based on a static formulation of the oligopoly model and does not adequately deal with the interactions that exist amongst OPEC member countries. In this section we develop and estimate a generalized Nash bargaining model of OPEC where the expected profit of each OPEC member from oil production is compared to an assumed "threat point" or a "status quo" profit level. The generalized Nash maximand for this problem is given by (8)

$$L_t = \prod_{i=1}^m (p_t^o q_{it} - c_i q_{it} - \bar{\pi}_{it})^{\gamma_{it}} \quad (27)$$

where  $\gamma_{it}$  is the "bargaining power" parameter for country  $i$ .  
 $q_{it}$  = oil output of country  $i$ ,

(7) Game-theoretic models of OPEC have been considered in the literature by Salant (1976), Hnyiliczka and Pindyck (1976), Loury (1986), and Carraro and Siniscalco (1987). None of these studies, however, provide an empirical analysis of the bargaining model of OPEC.

(8) The standard symmetric two-person bargaining solution is due to Nash (1950, 1953), and as is noted by Harsanyi (1956) was also obtained earlier by Zeuthen (1930) based on a concession principle. For a lucid account of the two-person cooperative games see Luce and Raiffa (1957, Ch. 6). Generalization of the Nash-Zeuthen solution concept to  $n$ -person asymmetric games has been discussed, for example, in Roth (1979) and Binmore and Dasgupta (1987). Also see Shapley (1967), for an early development of the basic theory.

$p_t^e$  = expected real price of oil,  
 $c_i$  = marginal cost of oil production in country  $i$  (assumed fixed),  
 and  
 $\bar{\pi}_{it}$  = "threat point" or reservation profit of country  $i$ .

This is a rather simple formulation and does not allow for the exhaustible nature of oil reserves and the dynamic complications involved in wealth maximization models developed in the literature for a single oil producing country (9). The above formulation also assumes that all members of OPEC have the same oil price expectations. The demand for OPEC oil,  $q_t$ , which in equilibrium is assumed to be equal to OPEC supply, is determined as a residual after supply of non-OPEC oil,  $S_n$ , is subtracted from the world demand for oil,  $D_w$ :

$$\begin{aligned}
 q_t &= \sum_{i=1}^m q_{it} = D_w(p_t^e, y_t) - S_n(p_t^e, z_t) + \varepsilon_t, \\
 &= H(p_t^e, y_t, z_t) + \varepsilon_t
 \end{aligned} \tag{28}$$

where  $y_t$  is the level of world economic activity,  $z_t$  is a vector of exogenous supply variables, and  $\varepsilon_t$  is a disturbance term standing for the unobserved effects of shocks to world demand and non-OPEC supplies. The Nash bargaining solution to the above problem is given by maximizing  $L_t$  with respect to  $q_{it}$  subject to the oil demand constraint (28). For given values of the "threat points" it is easily seen that

$$q_{it} = \left( \frac{\bar{\pi}_{it}}{p_t^e - c_i} \right) + a_{it} \left\{ E_{it}(q_t) - \sum_{j=1}^m \left( \frac{\bar{\pi}_{jt}}{p_t^e - c_j} \right) \right\}, \tag{29}$$

$i = 1, 2, \dots, m$

where  $a_{it} = \gamma_{it} / \sum_{j=1}^m \gamma_{jt}$  is the "relative" bargaining power parameter of country  $i$  with  $\sum_{i=1}^m a_{it} = 1$ . The above equations determine the allocation of output over the OPEC member countries. The output of OPEC as a whole is determined as a residual from (28), and the oil price expectations,  $p_t^e$ , satisfy the equilibrium condition:

$$p_t^e = \left( 1 - \frac{1}{e_t} \right)^{-1} \left( \sum_{i=1}^m \frac{a_{it} q_{it}}{\chi_{it}} \right)^{-1} \left( \sum_{i=1}^m \frac{a_{it} c_i q_{it}}{\chi_{it}} \right), \tag{30}$$

where

$$\chi_{it} = (p_t^e - c_i) q_{it} - \bar{\pi}_{it}$$

and  $e_t$  is the price elasticity of demand for OPEC oil given by  $e_t = - (p_t^e / q_t) (\partial H_t / \partial p_t^e)$ . The system of equations (28), (29) and (30) may, in principle, be solved for  $p_t^e$  and  $(q_{1t}, q_{2t}, \dots, q_{mt})$  in terms of  $y_t, z_t, \bar{\pi}_{it}$  and the shocks  $\varepsilon_t$  and this solution could be made part of a full information

(9) Building on the pioneering work of Hotelling (1931), wealth maximizing models with OPEC viewed as a monolithic entity have been discussed in the literature by, for example, Dasgupta and Heal (1979), Devarajan and Fisher (1982) and Pindyck (1978).

estimation procedure of the unknown parameters of the model. Here our aim is much more modest and centres on the estimation of the allocation equations (29) for given oil price expectations and a given specification of the "threat point" profit levels,  $\bar{\pi}_{it}$ .

There are a large number of ways that  $\bar{\pi}_{it}$  can be modelled. One possibility would be to assume a competitive market solution and write

$$\bar{\pi}_{it} = q_{it}^c (p_t^c - c_i), \quad q_{it}^c \leq \bar{q}_{it}$$

where  $p_t^c$  and  $q_{it}^c$  represent the price and output combinations that are expected to prevail in competitive conditions (i.e., under the assumption of an OPEC's collapse), with  $\bar{q}_{it}$  representing the maximum sustainable output capacity of the  $i$ -th country. This formulation, however, requires time series data on output capacities that are not available on a regular basis, and also involves the determination of  $p_t^c$  in a counterfactual exercise which is beyond the scope of the present paper (10). An alternative formulation of  $\bar{\pi}_{it}$  which has a weaker basis in theory but is much easier to apply in practice is to take  $\bar{\pi}_{it}$  to be proportional to the profit earned by the  $i$ -th country in the recent past. This formulation is justified if OPEC countries take seriously the target revenue hypothesis as characterizing a threatening behaviour (11). In the present application we take

$$\bar{\pi}_{it} = \theta_i q_{i,t-1} (p_{t-1} - c_i) \quad (31)$$

where  $\theta_i$  ( $\theta_i > 0$ ) is a free parameter measuring the extent to which country  $i$  is committed to maintaining the level of its past revenue. We refer to  $\theta_i$  as the revenue commitment parameter.

Under (31) and using Proposition 2, the estimatable equations corresponding to (29) may be written as:

$$q_{it} = \theta_i \phi_{it} q_{i,t-1} + \left( \frac{\alpha_{it}}{1 - \alpha_{it}} \right) (q_{-i,t} - \Phi_{-i,t}) + v_{it} \quad (32)$$

where  $E(v_{it} | \psi_{t-1}) = 0$ .

$$q_{-i,t} = \sum_{\substack{j=1 \\ j \neq i}}^m q_{jt}$$

$$\phi_{it} = (p_{t-1} - c_i) / (p_t^e - c_i),$$

$$\Phi_{-i,t} = \sum_{\substack{j=1 \\ j \neq i}}^m \theta_j \phi_{jt} q_{j,t-1},$$

and  $\psi_{t-1}$  represents the publicly available information at time  $t$ , which we take to include at least observations on lagged prices and lagged outputs.

(10) Data on output capacity of individual OPEC countries is available only for isolated years and what is available is of doubtful quality. See Fesharaki et al. (1989).

(11) For a discussion of target revenue models of OPEC and their empirical relevance see, for example, Teece (1982).

### 3.1. Empirical Results

With the exception of the output equation for Ecuador, all the estimations are carried out over the period 1974(3)-1987(4) <sup>(12)</sup>. The choice of the estimation period is based on the widely held view that it was only in the aftermath of the first oil shock in 1973/74 that OPEC started to assert significant influence on output decisions <sup>(13)</sup>. In the pre-1973 period by and large the oil companies decided how much oil to produce and how to allocate production across the oil producing nations <sup>(14)</sup>. For this earlier period an oligopolistic model of the oil companies seems to be more appropriate for the determination of output shares across OPEC.

As Table 1 shows there are important differences across countries in OPEC both in terms of their reserves and absorptive capacities that may have significant effects on their relative bargaining powers,  $a_{it}$  <sup>(15)</sup>. In addition, important political developments such as the revolutionary upheavals in Iran may have also had a significant impact on  $a_{it}$ 's. In the empirical analysis we assume that

$$a_{it} = a_i + b_i RD_t, \quad \sum_{i=1}^m a_i = 1, \quad \sum_{i=1}^m b_i = 0, \quad (33)$$

where  $RD_t$  is a dummy variable taking the value of zero before 1978, the start of the revolutionary upheavals in Iran, and unity thereafter. We also consider the inclusion of an oil reserve variable (i.e., the share of  $i$ -th country reserves in OPEC's total reserves) in the determination of  $a_{it}$ , but decided not to pursue this line of research further for two reasons: first, the data on oil reserves are not available on a quarterly basis and the annual data that are available are not of a high quality <sup>(16)</sup>. Second, as can be seen from Table 2, there are only minor variations in the distribution of reserves across OPEC during our sample period and the inclusion of a reserve variable in (33) seems unlikely to have a significant impact on the empirical results.

In estimating the output equations (32), the value of  $c_i$  was set equal to 1\$/b for the Gulf countries, 2\$/b for the North African countries and

(12) Due to a major earthquake, oil production in Ecuador was seriously disrupted in 1987 and the output equation for Ecuador was estimated over the period 1974(3)-1986(4).

(13) The output series are seasonally unadjusted and are compiled from the various issues of the *Petroleum Economist*. The real oil price,  $p_t$  is computed as  $p_t = PAL_t/PX_t$ , where  $PAL_t$  is the price of Arabian Light Crude and  $PX_t$  is the average quarterly index of export prices of industrial countries. For the sources and other details see the Data Appendix in Pesaran (1990).

(14) See, for example, Rustow and Mugno (1976, Ch. 1).

(15) Here we are using population as a proxy for absorptive capacity.

(16) For example, over the period 1985 to 1988 both Iran and Iraq revised the estimates of their proven reserves by almost a factor of two from 47.9 and 44.1 bln barrels at end of 1985 to 92.2 and 100 bln barrels at the end of 1988, respectively. See *BP Statistical Review of World Energy*, June 1986 and July 1989.



**Table 1 – Selected Data for OPEC Member Countries: 1986**

OPEC MEMBER COUNTRIES	Population 1985 (Mln)	International Reserves <sup>(1)</sup> End 1985 (\$ Bln)	Proven Oil Reserves <sup>(2)</sup> (Bln Barrels) (1,000 b/d)	Oil Production <sup>(3)</sup> (1,000 b/d)	Reserve Production Ratio (years)	Quota <sup>(4)</sup> (1st Qtr. 1988)
<b>GULF COUNTRIES</b>						
Saudi Arabia	11.0	22.9	166.6	5,317	85.8	4,343
Iran	43.4	—	48.8	1,885	70.9	2,369
Iraq	14.1	—	47.1	1,725	74.8	2,369
Kuwait	1.7	5.1	91.9	1,423	176.9	996
U.A.E.	1.3	2.9	32.9	1,545	58.3	948
Qatar	0.3	0.4	3.2	350	25.0	299
<b>Total Gulf</b>	<b>71.8</b>	<b>31.3</b>	<b>390.5</b>	<b>12,245</b>	<b>87.4</b>	<b>11,324</b>
<b>OTHERS</b>						
Libya	3.6	5.5	21.3	1,070	54.5	996
Nigeria	92.0	1.5	16.0	1,455	30.1	1,301
Algeria	20.5	2.8	8.8	990	24.4	667
Gabon	1.1	0.2	0.6	160	10.3	159
Venezuela	16.8	9.7	25.0	1,845	37.1	1,571
Ecuador	9.3	0.6	1.7	275	16.9	221
Indonesia	159.3	4.6	8.3	1,400	16.2	1,190
<b>Group Total OPEC</b>	<b>302.6</b>	<b>24.9</b>	<b>81.7</b>	<b>7,195</b>	<b>31.1</b>	<b>6,105</b>
	<b>374.4</b>	<b>56.2</b>	<b>472.2</b>	<b>19,440</b>	<b>66.5</b>	<b>17,429</b>

Sources: <sup>(1)</sup> IMF International Financial Statistics.

<sup>(2)</sup> BP Statistical Review of World Energy, June 1987. Proven reserves are reserves proved by drilling and recoverable with prevailing technology and prices, b/d stands for barrels per day. Both Iran and Iraq have revised the estimates of their proven reserves from 47.9 and 44.1 bin barrels at the end of 1985 to 92.9 and 100 bin barrels at the end of 1988, respectively.

<sup>(3)</sup> BP Statistical Review of World Energy, June 1987.

<sup>(4)</sup> Petroleum Economist, London. There was no agreement over Iraq's production quota. Here we have assumed parity with Iran's production quota.

Indonesia, and 3\$/b for the rest of the OPEC <sup>(17)</sup>. Our attempt to estimate  $c_i$ 's with other parameters of the model did not prove successful. There does not seem to be enough variations in marginal extraction costs relative to the observed variations in real prices to allow a precise estimation of  $c_i$ 's from the data. Finally, to construct oil price expectations we considered a number of univariate time series models and found that an AR(2) specification provided a satisfactory approximation to the price process over our estimation period <sup>(18)</sup>. We also tried other variables such as lagged values of OECD oil consumption, OPEC and non-OPEC oil supplies in the price equation but did not find their effect to be statistically significant.

During the sample period under consideration all countries in OPEC experienced wide variations in their production-shares. See Figures 1-6. Changes in Iran's production share, under the influence of revolution and war, have been most dramatic, ranging from 21.5% in 1973(4) to 2.8%

(17) I am grateful to Robert Mabro for suggesting the use of these estimates.

(18) Notice that due to the non-linear way that price expectations enter the output equations, the application of the errors-in-variables approach whereby  $p_t$  is replaced by its actual value, will not lead to consistent estimates even if the resultant equations are estimated by the IV method. See, for example, Hausman et al. (1988).

**Table 2 – Distribution of Population, Oil Production and Reserves Among OPEC Countries**

OPEC MEMBER COUNTRIES	Population %	Proved Reserves (%)					Production (%)						
		End 1977	End 1979	End 1981	End 1985	End 1986	1973	1978	1979	1980	1981	1985	1986
<b>GULF COUNTRIES</b>													
Saudi Arabia	2.7	37.8	38.1	36.5	35.9	35.3	24.5	26.9	30.8	36.7	43.7	20.3	27.4
Iran	11.6	13.5	13.5	12.4	10.2	10.3	19.1	18.1	9.7	5.7	5.8	14.0	9.7
Iraq	4.0	7.3	7.2	9.2	9.4	10.0	6.4	8.6	11.2	10.0	4.1	9.0	8.9
Kuwait	0.4	15.1	15.2	14.4	19.1	19.5	9.8	6.5	7.4	5.9	5.0	6.0	7.3
U.A.E.	0.2	7.1	6.9	7.3	7.0	7.0	4.9	6.3	6.1	6.3	6.7	7.2	7.9
Qatar	0.1	0.9	0.9	0.8	0.7	0.7	1.8	1.7	1.6	1.7	1.8	1.9	1.8
<b>Group Share</b>	<b>19.0</b>	<b>81.7</b>	<b>81.8</b>	<b>80.6</b>	<b>82.3</b>	<b>82.8</b>	<b>66.5</b>	<b>68.1</b>	<b>66.8</b>	<b>66.3</b>	<b>67.1</b>	<b>58.4</b>	<b>63.0</b>
<b>OTHERS</b>													
Libya	0.9	5.5	5.5	4.8	4.5	4.5	7.0	7.0	6.8	6.6	5.0	6.6	5.5
Nigeria	23.5	4.2	4.1	3.8	3.5	3.9	6.6	6.2	7.9	7.6	6.4	9.5	7.5
Algeria	5.9	1.4	2.0	2.1	1.9	1.9	3.5	4.3	4.1	4.0	3.6	4.3	5.1
Gabon	0.2	0.5	0.1	0.1	0.2	0.1	0.5	0.6	0.6	0.6	0.7	1.1	0.8
Venezuela	4.2	4.1	4.2	4.8	5.4	5.3	10.9	7.4	7.7	8.0	9.3	10.8	9.5
Ecuador	2.5	0.3	0.3	0.3	0.4	0.3	0.7	0.7	0.7	0.8	0.9	1.8	1.4
Indonesia	43.7	2.3	2.2	2.1	1.8	1.7	4.3	5.7	5.3	5.8	7.1	7.5	7.2
<b>Group Share</b>	<b>80.9</b>	<b>18.3</b>	<b>18.4</b>	<b>18.0</b>	<b>17.7</b>	<b>17.7</b>	<b>33.5</b>	<b>31.9</b>	<b>33.1</b>	<b>33.4</b>	<b>33.0</b>	<b>41.6</b>	<b>37.0</b>
OPEC as % of Communist Block	–	80.8	77.9	75.1	75.9	76.4	68.5	62.9	62.0	56.6	51.7	38.6	42.4
OPEC as % of World	–	68.3	66.9	65.6	67.1	62.7	53.3	48.4	48.0	43.3	38.6	27.6	30.6

Sources: Oil and Gas Journal, Petroleum Economist, and BP Statistical Review of World Energy.

in 1980(4). Similar variations can also be observed in Iraq's production share, ranging from 2.27% in 1980(4) to 13.39% in 1987(4). There is also a clear negative relationship between production shares of Saudi Arabia and Iran suggesting an important degree of accommodation by Saudi Arabia with respect to the exogeneously determined variations in Iran's oil output. (See Figure 1). These effects, namely revolution, war and Saudi Arabia's production response to the political situation in Iran are difficult to model. In what follows we have relied on war and revolution dummy variables to deal with some of these influences.

Table 3 gives the estimates of the structural parameters ( $a_p$ ,  $b_p$ ,  $\theta_i$ ) obtained by the non-linear instrumental variable (NLIV) method using  $q_{i,t-1}$ ,  $q_{i,t-2}$ ,  $q_{-i,t-1}$ ,  $q_{-i,t-2}$ ,  $\phi_{it}$  and the intercept term as instruments<sup>(19)</sup>. These estimates impose the cross-equation parametric restrictions implicit in (32) and are computed iteratively. (The estimation process converged after 5 iterations, starting from  $\theta_i = 0.7$  for all  $i$ .) The results are generally satisfactory and, except for the estimates obtained for Iran

(19) In the case of the output equations for Saudi Arabia, Iran, Iraq and Indonesia the list of instruments also included the variables,  $RD_{t-1q-i,t-1}$  and  $RD_{t-2q-i,t-2}$ . The computations were carried out using Microfit 3.0 (see Pesaran and Pesaran (1991)).

**Table 3 – Estimates of the Structural Parameters of the Output Equations of OPEC Member Countries (1)**

OPEC MEMBER COUNTRIES	STRUCTURAL PARAMETERS				DIAGNOSTIC STATISTICS (2)				
	$\hat{a}_i$	$\hat{b}_i$	$\hat{\theta}_i$	Intercept	$\hat{\sigma}$	$\bar{R}^2$	$\chi^2_{SC}$	$\chi^2_{FF}$	$\chi^2_H$
<b>GULF COUNTRIES</b>									
Saudi Arabia (3)	0.2189 (0.0345)	0.0603 (0.0256)	0.9059 (0.0443)	-1501.4 (597.33)	676.81	0.9158	9.97 [0.96]	0.58 [0.55]	1.81 [0.82]
Iran (4)	0.3349 (0.0488)	-0.1519 (0.0370)	0.3822 (0.1270)	433.1 (325.89)	554.04	0.9040	4.42 [0.65]	4.03 [0.96]	0.04 [0.16]
Iraq (5)	0.0787 (0.0200)	0.0399 (0.0137)	0.8339 (0.0585)	-329.7 (149.24)	227.50	0.9281	2.89 [0.42]	1.48 [0.78]	0.00 [0.05]
Kuwait	0.0849 (0.0185)	-	0.6265 (0.0853)	-	224.71	0.8440	2.03 [0.27]	0.53 [0.53]	6.48 [0.99]
U.A.E.	0.0547 (0.0104)	0.0164 (0.0068)	0.7240 (0.0581)	-	142.03	0.8083	1.05 [0.10]	4.91 [0.97]	0.00 [0.04]
Qatar	0.0162 (0.0037)	-	0.5654 (0.0984)	65.63 (36.49)	54.47	0.6296	1.92 [0.25]	4.16 [0.96]	0.10 [0.25]
<b>OTHERS</b>									
Libya	0.0639 (0.0149)	-	0.6972 (0.0748)	-	197.86	0.8302	5.88 [0.79]	0.00 [0.05]	0.03 [0.14]
Nigeria	0.0746 (0.0153)	-	0.5755 (0.0891)	231.50 (134.46)	220.13	0.7154	1.62 [0.19]	1.51 [0.78]	4.15 [0.96]
Algeria	0.0327 (0.0069)	-	0.7371 (0.0578)	-	76.71	0.8715	1.29 [0.14]	4.93 [0.97]	1.88 [0.83]
Gabon	0.0058 (0.0011)	-	0.5884 (0.0956)	36.65 (12.95)	13.47	0.8267	6.74 [0.85]	5.49 [0.98]	1.84 [0.83]
Venezuela	0.0606 (0.0104)	-	0.6142 (0.0723)	400.79 (143.56)	165.74	0.7361	2.78 [0.40]	3.86 [0.95]	0.27 [0.40]
Ecuador (6)	0.0014 (0.0025)	-	0.7235 (0.1163)	50.57 (40.89)	19.41	0.7335	3.36 [0.50]	2.89 [0.91]	16.93 (1.00)
Indonesia	0.0368 (0.0051)	0.0167 (0.0051)	0.7880 (0.0333)	-	92.58	0.6488	3.91 [0.58]	7.92 [1.00]	8.14 [1.00]

(1) The estimates summarized in this table are computed applying the Nonlinear IV estimation method to the output equations (32) using seasonally unadjusted quarterly data over the period 1974(3)-1987(4). The estimates impose the cross-equation parametric restrictions in (32). It is assumed that the bargaining power parameters vary according to  $\alpha_i = a_i + b_i RD_i$ , where  $RD_i$  is a dummy variable taking the value of zero before 1978(3), the start of the revolutionary upheavals in Iran, and unity thereafter. The figures in round brackets are the asymptotic standard errors and those in square brackets are probability values. The symbol "-" represents that the statistically insignificant parameter estimates was set equal to zero.

(2)  $\hat{\sigma}$  is the estimated standard error  $\bar{R}^2$  in the adjusted  $\bar{R}^2$ ,  $\chi^2_{SC}$ ,  $\chi^2_{FF}$  and  $\chi^2_H$  are diagnostic statistics asymptotically distributed as chi-squared variates (with degrees of freedom in parentheses) for tests of residual serial correlation, functional form misspecification and heteroskedasticity, respectively. These diagnostic statistics are based on the nonlinear IV residuals. The details of the computations and algorithms are similar to the linear case discussed in Pesaran and Pesaran (1991).

(3) The output equation for Saudi Arabia includes a Gulf War dummy variable and the unexpected change in oil prices. Oil price expectations were constructed according to a second-order autoregressive process estimated over the period 1974(3)-1987(4).

(4) Includes a Revolution dummy variable which takes the value of unity in 1979(1) and zero elsewhere. Notice that this is a once and for all dummy variable and differs from  $RD_i$ , which shifts the bargaining parameter  $\alpha_i$ .

(5) Includes a Gulf War dummy variable which takes the value of unity in 1980(4) and zero elsewhere.

(6) Estimated over the 1974(3)-1986(4) period. As a result of a major earthquake Ecuador experienced a serious disruption to her oil production in 1987.

and Saudi Arabia, give plausible estimates for the structural parameters of the model. The bargaining power parameters are statistically significant, have the correct signs and satisfy the adding up restrictions  $\sum_{i=1}^m a_{it} = 1$ , with a reasonable degree of approximation, both before and

after the Iranian Revolution (see Table 4). <sup>(20)</sup> Notice that the estimation procedure does not impose the adding-up restrictions and the fact that the estimates seem to satisfy these restrictions provide further empirical support for the estimated equations. The estimate of the bargaining

**Table 4 – Estimates of Bargaining Powers, Quota and Production Shares of OPEC Member Countries**

OPEC MEMBER COUNTRIES	Bargaining Powers <sup>(1)</sup>		Quota Shares <sup>(2)</sup> (1st Qtr. 1988)	Production Shares <sup>(3)</sup>
	Before Iranian Revolution	After Iranian Revolution		
<b>GULF COUNTRIES</b>				
Saudi Arabia	0.2189 (0.0345)	0.2792 (0.0510)	0.2492	0.2959
Iran	0.3349 (0.0488)	0.1830 (0.0559)	0.1359	0.1334
Iraq	0.0787 (0.0200)	0.1186 (0.0309)	0.1359	0.0792
Kuwait	0.0849 (0.0185)	0.0849 (0.0185)	0.0571	0.0657
U.A.E.	0.0547 (0.0104)	0.0711 (0.0143)	0.0544	0.0667
Qatar	0.0162 (0.0037)	0.0162 (0.0037)	0.0172	0.0174
<b>Group Total</b>	<b>0.7883</b>	<b>0.7530</b>	<b>0.6497</b>	<b>0.6583</b>
<b>OTHERS</b>				
Libya	0.0639 (0.0149)	0.0639 (0.0149)	0.0571	0.0605
Nigeria	0.0746 (0.0153)	0.0746 (0.0153)	0.0746	0.0728
Algeria	0.0327 (0.0069)	0.0327 (0.0069)	0.0383	0.0363
Gabon	0.0058 (0.0011)	0.0058 (0.0011)	0.0091	0.0080
Venezuela	0.0606 (0.0104)	0.0606 (0.0104)	0.0901	0.0899
Ecuador <sup>(4)</sup>	0.0014 (0.0025)	0.0014 (0.0025)	0.0127	0.0099
Indonesia	0.0368 (0.0051)	0.0535 (0.0085)	0.0683	0.0644
<b>Group Total</b>	<b>0.2759</b>	<b>0.2926</b>	<b>0.3502</b>	<b>0.3417</b>
<b>OPEC</b>	<b>1.0641</b>	<b>1.0455</b>	<b>1.00</b>	<b>1.00</b>

<sup>(1)</sup> The estimates are based on the results reported in Table 3. Bracketed figures are asymptotic standard errors.

<sup>(2)</sup> *Petroleum Economist*, London. See also footnote 4 to Table 1.

<sup>(3)</sup> Production shares are computed as the average of the quarterly shares over the period 1974(3)-1987(4).

<sup>(4)</sup> The results for Ecuador are based on the observations over the period 1974(3)-1986(4).

power parameter obtained for the pre-revolutionary Iran is, however, too high and suggests possible model misspecification. Perhaps this is to be expected considering that Iran has been subject to a major revolution and a prolonged war the effects of which are difficult to model formally.

(20) A formal statistical test of the adding up restrictions require joint estimation of the output equations and is not attempted here.

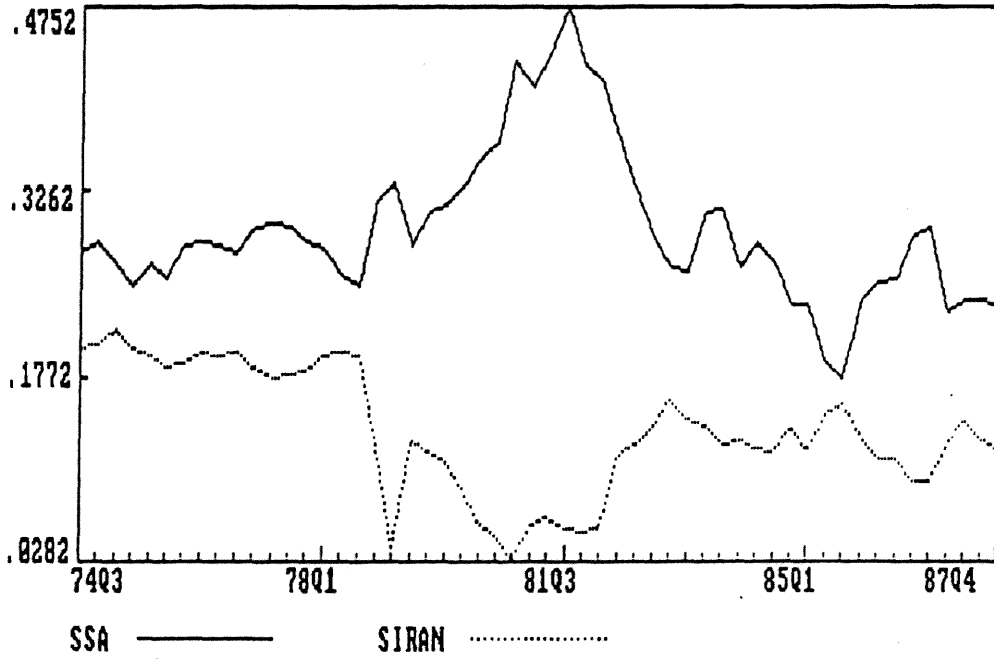


Figure 1 – Production shares of Saudi Arabia and Iran

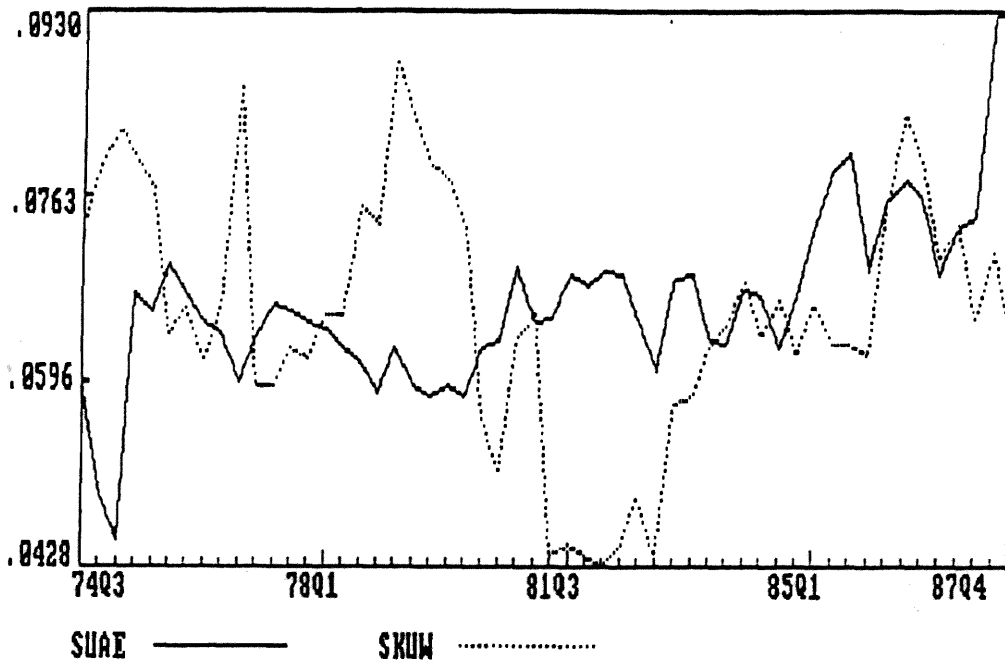


Figure 2 – Production shares of U.A.E. and Kuwait

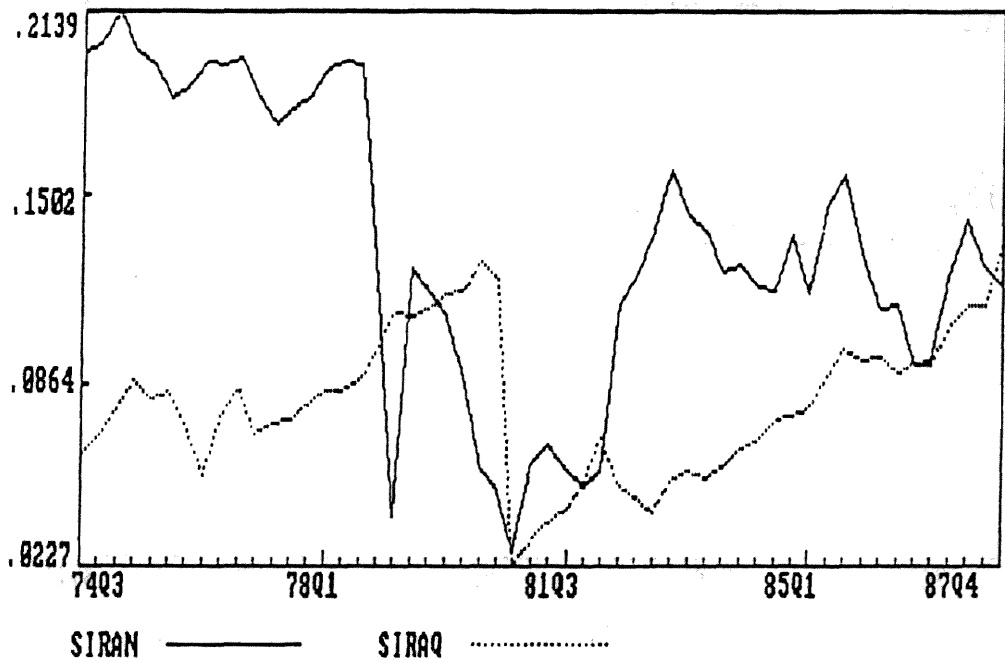


Figure 3 – Production shares of Iran and Iraq

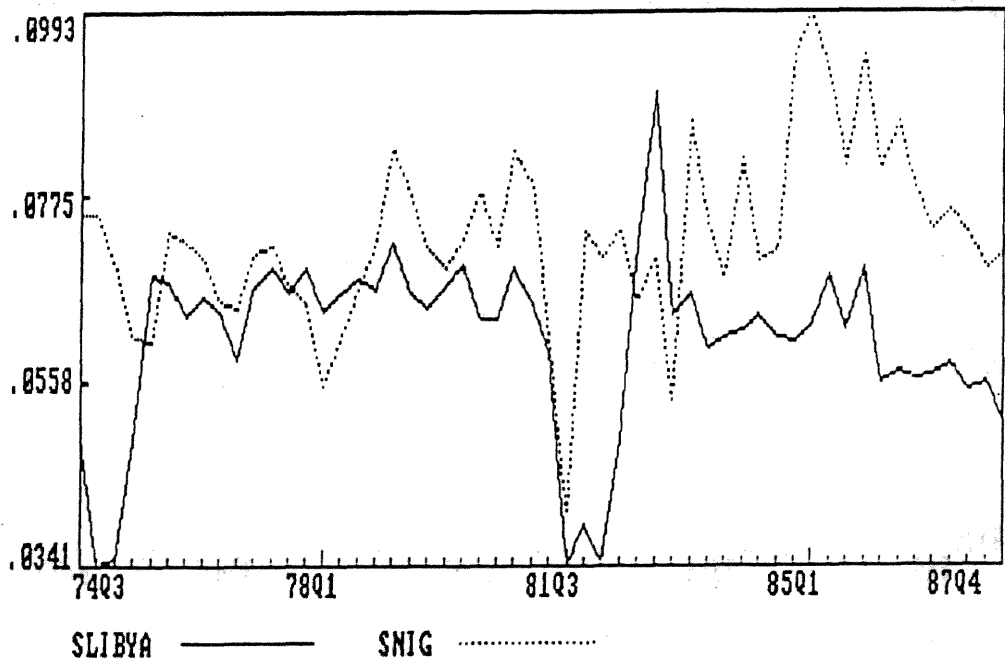


Figure 4 – Production shares of Libya and Nigeria

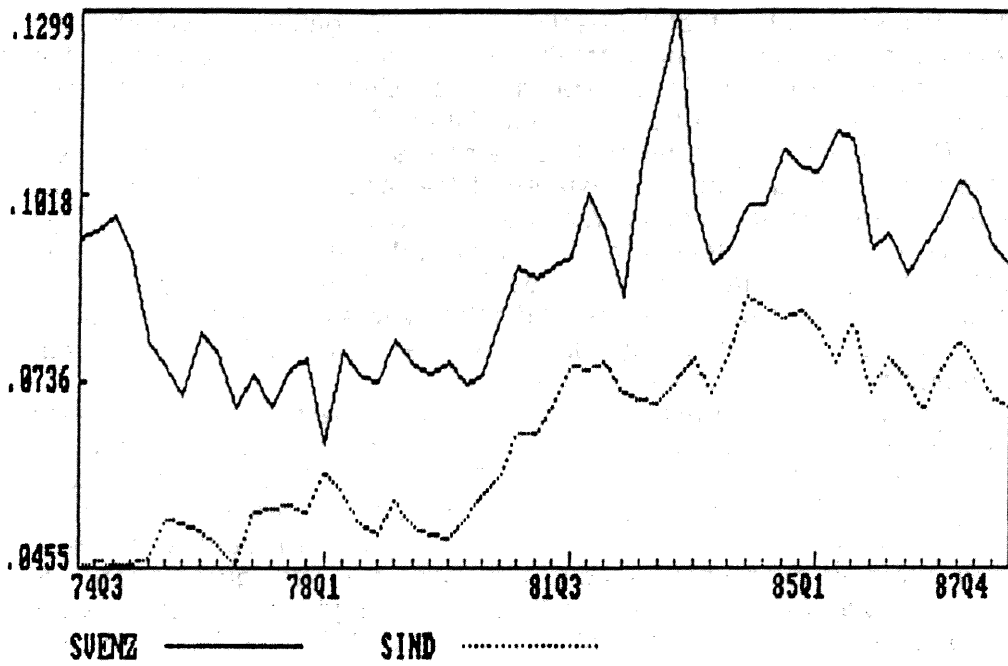


Figure 5 - Production shares Venezuela and Indonesia

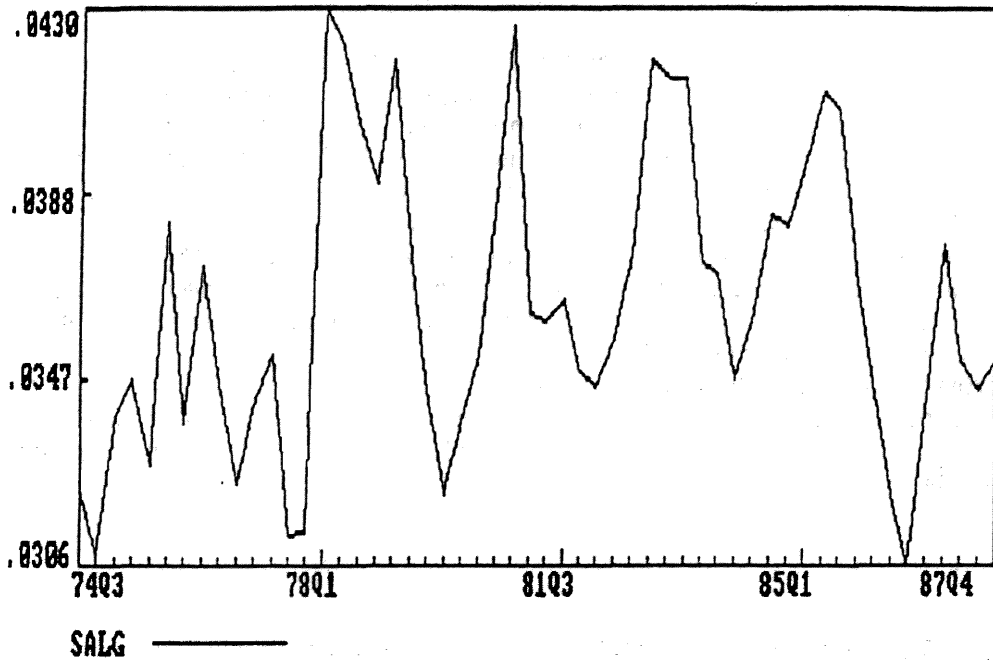


Figure 6 - Production share of Algeria

It is also worth noting that the decline in Iran's bargaining power after the revolution has been almost totally compensated by increases in the bargaining power of Saudi Arabia, Iraq, United Arab Emirate and Indonesia, all on the oil-trade route to Japan <sup>(21)</sup>.

The revenue commitment parameters,  $\theta_i$ , are also precisely determined and are mostly clustered in the range 0.60-0.70 with Saudi Arabia, Iran, Iraq and Indonesia being the exceptions. The relatively high estimate obtained for Iraq fits well with this country's known determination to maintain revenues. The high estimate obtained for Saudi Arabia is, however, rather difficult to explain. This may be due to the special role played by Saudi Arabia inside OPEC which our formulation fails to capture adequately <sup>(22)</sup>.

The estimates of the bargaining powers (before and after the Iranian Revolution), the production shares and the quota shares (agreed in the 1st quarter of 1988) are compared in Table 4 and Figure 7. The results suggest that the bargaining power of the Gulf countries exceed their quota shares and is more in line with their share of reserves than with their share of production. Among the Gulf countries, quota shares of Saudi Arabia, Iran and Kuwait are below their bargaining power while the reverse is true of Iraq. Among the other OPEC countries it is only Venezuela that has been given a quota share well in excess of its bargaining power.

#### 4. Concluding Remarks

In this paper we have emphasized the importance of information heterogeneity in the analysis of disaggregated rational expectations models, and have shown that under relatively plausible assumptions these models can be estimated consistently. The focus of our attention in this paper has been on models with current expectations of the endogenous variables, but the analysis can be extended to DRE models with future expectations. (See Binder and Pesran (1995b)). The application to OPEC's output allocation problem is based on rather strong assumptions and abstracts from the intertemporal nature of output decisions. Nevertheless, the results are encouraging and generally provide plausible estimates of the relative bargaining powers of OPEC member countries. The next stage in the analysis would be to see how robust the results are to other methods of modelling the threat point profit levels.

(21) Over 95 percent of Japan's oil imports in 1978 originated from Middle East and South East Asia. See, *BP Statistical Review of the World Oil Industry*, 1978.

(22) In view of the residual supplier role of Saudi Arabia in OPEC we did include an unexpected price variable in the Saudi Arabia output equation, which turned out to be statistically significant, but failed to have a significant impact on the estimates of  $\theta_i$ .



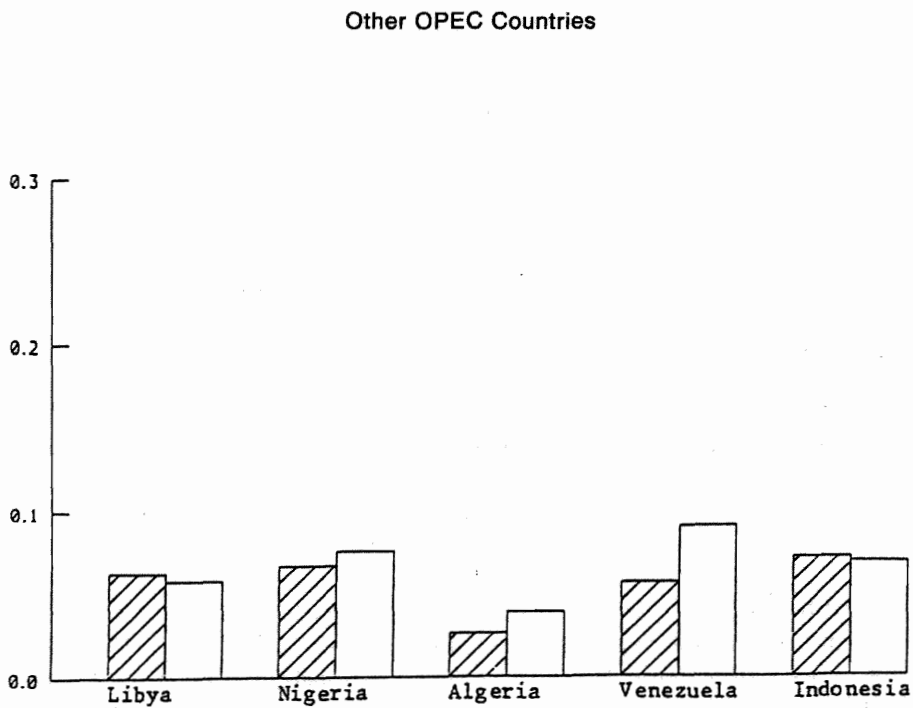
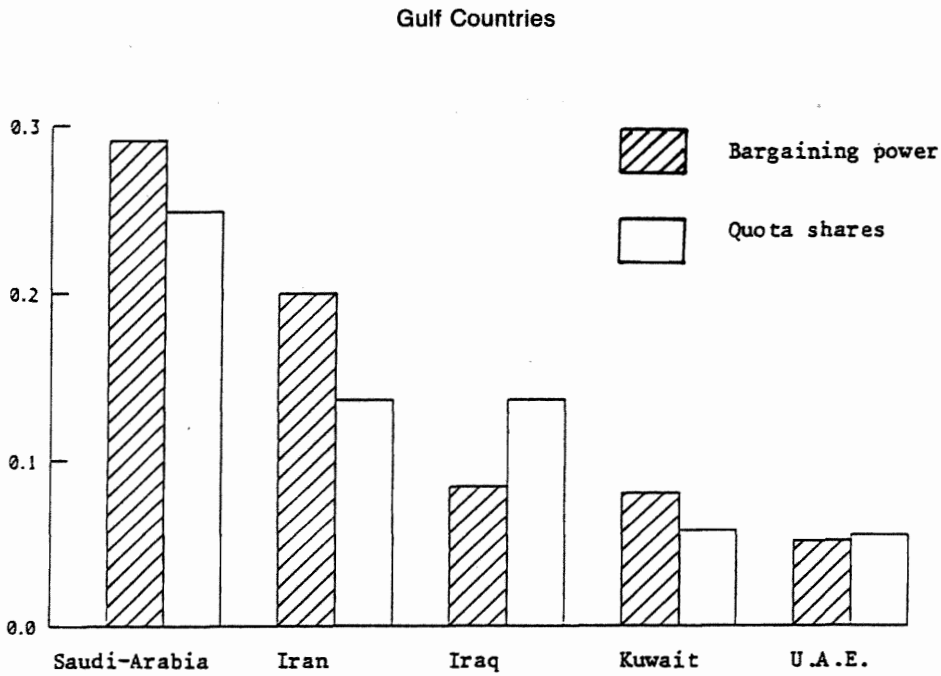


Figure 7 - Bargaining power estimates and quota shares of OPEC Countries

## 5. Mathematical appendix

*Proof of Proposition 1: Under Assumption A*

$$\begin{aligned} E_{it}(h_t) &= w_i[h_{it} - E(h_{it} | \psi_{t-1})] + E(h_t | \psi_{t-1}), \\ &= w_i \varepsilon_{it}^h + E(h_t | \psi_{t-1}) \end{aligned} \quad (34)$$

Using this result in (16) we have <sup>(23)</sup>

$$F^1(h_t) = \sum_{i=1}^m w_i \tilde{w}_i \varepsilon_{it}^h + E(h_t | \psi_{t-1}).$$

To obtain  $F^2(h_t)$  we first note that under (18)

$$\begin{aligned} E(\varepsilon_{it}^h | \Omega_{jt}) &= \varepsilon_{it}^h, \quad i = j \\ &= 0, \quad i \neq j, \end{aligned}$$

and

$$\begin{aligned} F^2(h_t) &= \sum_{i=1}^m \tilde{w}_i E\{F^1(h_t) | \Omega_{it}\}, \\ &= \sum_{i=1}^m w_i \tilde{w}_i^2 \varepsilon_{it}^h + E(h_t | \psi_{t-1}). \end{aligned}$$

Similarly, by repeating the above steps we have

$$F^r(h_t) = \sum_{i=1}^m w_i \tilde{w}_i^r \varepsilon_{it}^h + E(h_t | \psi_{t-1}),$$

for  $r = 1, 2, \dots$ . Substituting this result in (17) now yields (assuming  $|\lambda| < 1$ )

$$y_t = h_t + \left( \frac{\lambda}{1 - \lambda} \right) E(h_t | \psi_{t-1}) + \sum_{i=1}^m \left( \frac{\lambda_i w_i^2}{1 - \lambda_i w_i} \right) \varepsilon_{it}^h. \quad (35)$$

To obtain the disaggregated solution (19) we first note from (35) that

$$E_{it}(y_t) = E_{it}(h_t) + \left( \frac{\lambda}{1 - \lambda} \right) E(h_t | \psi_{t-1}) + \left( \frac{\lambda_i w_i^2}{1 - \lambda_i w_i} \right) \varepsilon_{it}^h.$$

Substituting for  $E_{it}(y_t)$  from (34) and using the resultant expression in (4) now gives the disaggregated solutions (19) in the text. The fact that  $E(\varepsilon_{it}^h | \psi_{t-1}) = 0$  follows immediately from the martingale difference property of  $\varepsilon_{it}^h$  defined in (20).

(23) Note from (12) that  $\sum_{i=1}^m \tilde{w}_i = 1$ .

*Proof of Proposition 2:* Using (22) in (4) we have

$$y_{it} = \lambda_i E(y_t | \psi_{t-1}) + \lambda_i w_i [y_{it} - E(y_{it} | \psi_{t-1})] + h_{it}. \tag{36}$$

Taking conditional expectations with respect to  $\psi_{t-1}$  and subtracting the resultant expression from (36) we obtain

$$y_{it} - E(y_{it} | \psi_{t-1}) = \left( \frac{1}{1 - \lambda_i w_i} \right) \varepsilon_{it}^h, \tag{37}$$

where  $\varepsilon_{it}^h$  is defined by (20). Substituting (37) in (36) and aggregating across  $i$  using weights  $w_i$  now yields

$$y_t = \lambda E(y_t | \psi_{t-1}) + \sum_{i=1}^m \left( \frac{\lambda_i w_i^2}{1 - \lambda_i w_i} \right) \varepsilon_{it}^h + h_t.$$

Taking conditional expectations with respect to  $\psi_{t-1}$  and noting that  $E(\varepsilon_{it}^h | \psi_{t-1}) = 0$ , we also have ( $|\lambda| < 1$ )

$$E(y_t | \psi_{t-1}) = \left( \frac{1}{1 - \lambda} \right) E(h_t | \psi_{t-1}). \tag{38}$$

Hence

$$y_t = h_t + \left( \frac{\lambda}{1 - \lambda} \right) E(h_t | \psi_{t-1}) + \sum_{i=1}^m \left( \frac{\lambda_i w_i^2}{1 - \lambda_i w_i} \right) \varepsilon_{it}^h,$$

which is the same as the aggregate solution (35) obtained using the infinite regress solution (17) under Assumption A. The disaggregated solution (19) can also be obtained using the decomposition (22). Using (37) and (38) in (22) we have

$$E_{it}(y_t) = \left( \frac{1}{1 - \lambda} \right) E(h_t | \psi_{t-1}) + \left( \frac{w_i}{1 - \lambda_i w_i} \right) \varepsilon_{it}^h$$

which upon substitution in (4) yields (19).

*Proof of Proposition 3:* Two cases may be distinguished: (i) when the generating process of the macro-variables  $\mathbf{z}_t$  is "common knowledge", and hence  $E_{it}(\mathbf{z}_t) = E(\mathbf{z}_t | \psi_{t-1})$  for all  $i$ , and (ii) when expectations of  $\mathbf{z}_t$  vary across agents. In either case  $E_{it}(\mathbf{z}_t)$  in (25) may be replaced by  $\mathbf{z}_t + \varsigma_{it}$  where the expectations errors,  $\varsigma_{it}$  are given by  $\varsigma_{it} = \mathbf{z}_t - E(\mathbf{z}_t | \psi_{t-1})$  under (i) and by  $\varsigma_{it} = \mathbf{z}_t - E(\mathbf{z}_t | \Omega_{it})$  under (ii). In both cases  $E(\varsigma_{it} | \psi_{t-1}) = 0$ . With this in mind (25) may be written in matrix notations as

$$\mathbf{y}_i = \mathbf{G}_i \gamma_i + \mathbf{u}_i \quad i = 1, 2, \dots, m \tag{39}$$

where  $\gamma_i = \{1/(1 - \lambda_i w_i)\} (\lambda_i a_i \phi_i', \delta_i)'$ ,  $\mathbf{G}_i = (\mathbf{y}_{-i}, \mathbf{1}, \mathbf{Z}, \mathbf{X}_i)$ ,  $\mathbf{y}_{-i}$  is the  $n \times 1$  vector of observations with elements  $y_{-i,t}$  defined by (24),  $\mathbf{Z}$  is the  $n \times k_z$  matrix of observations on the macro-variables  $\mathbf{z}_t$  and  $\mathbf{X}_i$  is the  $n \times k_i$  matrix of observations on the micro-variables  $\mathbf{x}_{it}$  and  $\mathbf{u}_i$  is the  $n \times 1$  vector of disturbances with elements:

$$u_{it} = \left( \frac{1}{1 - \lambda_i w_i} \right) \left\{ \varepsilon_{it} - \lambda_i \eta_t + \left( \frac{\lambda_i w_i}{1 - \lambda_i w_i} \right) \varepsilon_{it}^h - \phi_i' \zeta_{it} \right\}. \quad (40)$$

Under Assumptions A and B all the elements of this composite disturbance term are orthogonal to the public information set,  $\psi_{t-1}$ .

Efficient estimation of the parameters of the disaggregated system of equations in (39) requires a complete specification of the temporal and cross-sectoral correlations of the disturbances  $u_{it}$ , and in view of the presence of  $\eta_t = y_t - E(y_t | \psi_{t-1})$  in (40) does not seem to be a simple undertaking and will not be pursued here. Instead we follow a limited information approach and derive consistent estimators of the parameters of (39), considering the relations for each sector one at a time.

Consider the problem of estimating the parameters of the equation for the  $i$ -th sector (agent). Since  $E(u_{it} | \psi_{t-1}) = 0$ , in principle there exists a set of instruments obtainable from the public information set  $\psi_{t-1}$  which can be used for a consistent estimation of the parameters,  $\gamma_i$ . Denote the  $n \times s_i$  matrix of observations on these instruments by  $F_i$ ,  $s_i \geq k_i + k_z + 2$ . Then the generalized instrumental variable estimator of  $\gamma_i$  in (39) is given by

$$\hat{\gamma}_i = (\mathbf{G}_i' \mathbf{P}_i \mathbf{G}_i)^{-1} \mathbf{G}_i' \mathbf{P}_i \mathbf{y}_i$$

where  $\mathbf{P}_i = \mathbf{F}_i (\mathbf{F}_i' \mathbf{F}_i)^{-1} \mathbf{F}_i'$ . It is now easily seen that under Assumption C these estimators are consistent, and that asymptotically

$$\sqrt{n}(\hat{\gamma}_i - \gamma_i) \stackrel{a}{\approx} N(0, \mathbf{V}_i),$$

where

$$\mathbf{V}_i = \left[ \underset{n \rightarrow \infty}{\text{plim}} \left( \frac{\mathbf{G}_i' \mathbf{P}_i \mathbf{G}_i}{n} \right) \right]^{-1} \underset{n \rightarrow \infty}{\text{plim}} \left( \frac{\mathbf{G}_i' \mathbf{P}_i \sum_i \mathbf{P}_i \mathbf{G}_i}{n} \right) \left[ \underset{n \rightarrow \infty}{\text{plim}} \left( \frac{\mathbf{G}_i' \mathbf{P}_i \mathbf{G}_i}{n} \right) \right]^{-1},$$

and  $\sum_i = E(\mathbf{u}_i \mathbf{u}_i')$ .

In practical applications we can estimate  $\mathbf{V}_i$  by the heteroskedasticity and autocorrelation consistent covariance matrix proposed by Newey and West (1987). The orthogonality conditions  $E(u_{it} | \psi_{t-1}) = 0$  can also be utilized in the development of GMM estimators for  $\gamma_i$ , along the lines suggested by Hansen (1982), Hansen and Singleton (1982) and Dumbly et al. (1983), among others.

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# A REGIONAL AND STRUCTURAL MODEL FOR EUROPE

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## 1. Introduction

*.. improvements seem to come largely through the analysis of observations, through the relating of theories and facts by appropriate methods of estimation and inference.*

*Richard Stone, "Model building and the social accounts: a survey",  
Income and Wealth, Series IV, 1955, p. 27.*

The development of coherent systems of accounts for social phenomena (eg national and regional economies, population, education and the environment) was a feature of Richard Stone's contribution to the social sciences. Such disaggregated accounts form the statistical basis necessary to implement models of social behaviour. One of the great strengths of his work is that he never seems to lose sight of the purposes and uses of the accounts. He invariably discusses and shows how they can be used to model the underlying behaviour they represent.

In 1961 and 1962, when Richard Stone and Alan Brown published details of their proposed new disaggregated economic model in several papers (Stone, 1961b, 1962; Stone and Brown 1962a,b) including the first volume of *A Programme for Growth* (1962b), one important feature was the linking of a demand system and an input-output table. (This was not the first such empirical synthesis: Leif Johansen's *A Multi-sectoral Study of Economic Growth* (MSG) published in 1960 linked an input-output table for Norway in 1950 with a Cobb-Douglas production function and a demand system; the book had been completed in 1959 while he was a Visiting Scholar in the Department of Applied Economics, University of Cambridge.) Stone and Brown provided a bridge between the time-series econometric analysis of consumer demand and the cross-section analysis of industrial demand represented by the input-output table. The value of the input-output table (especially when incorporated into the wider coverage of a Social Accounting Matrix) in providing a disaggregated framework for the National Accounts was

earlier recommended in the UN's *System of National Accounts* (1968) prepared under Stone's leadership.

This paper briefly describes an ambitious project, in the tradition of Stone and Brown's venture thirty years ago, also bringing together different modelling traditions. The challenge is to model energy-environment-economy (E3) interactions for the regions of the European Community in a common framework combining elements of the general equilibrium modelling tradition and that of disaggregated time-series models estimated by conventional econometric techniques.

Such models, designed to project E3 variables and analyse E3 policies, are one important use of disaggregated accounts. The paper emphasises the national accounting aspects of the model being developed.

## 2. A general Energy-Environment-Economy Model for Europe

This is to be a general model for Europe capable of addressing issues that link developments and policies in the areas of energy, the environment and the economy. The essential purpose of the model is to provide a framework for evaluating different policies, particularly those aimed at achieving sustainable energy use over the long term. One of first tasks of the model, which will determine the design and the priorities in construction, will be the evaluation of policies reducing anthropogenic emissions of greenhouse gases in Europe by 60% over the next 50 years.

The European economy is becoming more and more integrated; at the same time, the relationship between economic activity, energy use and the environment is of pressing concern for European policy and political debate. These circumstances justify the development of a new modelling tool to inform policy-makers at all levels of government in Europe and to guide strategic decisions of business. The guiding principles for the model are that it should be:

- elaborated at a *European* rather than at a national level, with the national economies being treated as regions of Europe;
- dealing with energy, the environment, population and the economy in *one modelling framework*;
- designed from the outset to address issues of central importance for *economic, energy and environmental policy* at the European level;
- capable of analysing *long-term structural change* in energy demand and supply and in the economy;
- focused on the contribution of *research and development*, and associated technological innovation, on the dynamics of growth and change.

An important purpose of the model is to help the European Commission (EC) and member governments of the European Union (EU)

in the preparation and implementation of energy, environmental and economic policies for the EC over the medium and long term. This is a rather general purpose, and so some examples of the policies which the model will cover are in order.

- (i) The CEC and EC member governments are aiming to reduce emissions of Greenhouse Gases (GG): the model should be able to show the effects of a Community-wide carbon/energy tax on GG emissions, the economy and energy use. This assessment should include the effects of such a tax on international competitiveness.
- (ii) The model should also be able to assess the effects on long-term growth and international competitiveness of policies designed to raise R&D, particularly spending in the area of improving energy efficiency.
- (iii) Another example of the use of the model would be the assessment of the effects of further fiscal harmonisation between member states in duties on hydrocarbons.

A further purpose of the model is to provide the meso-economic framework for sectoral and environmental analysis across the European economy. For this purpose, as well as the requirement to help with economic and energy policy, the model is to be capable of making annual forecasts and analysing the consequences of budgetary action, eg the introduction of a carbon/energy tax, over the medium term. This means that the model must be capable of short-term disequilibrium analysis, where resources are not in full employment, as well as long-term equilibrium, where the labour market is adjusting towards long-term balance.

Although it makes sense to model the European economy as a single entity, there is no avoiding the fact that there exist severe regional divergences in economic structure. The most extreme of these are between the agricultural southern Mediterranean fringe (Portugal, Greece) and the service-oriented centre, and between the former GDR and western Germany. These differences have implications for migration, regional transfers of funds and economic development. Any model which ignored them would be very limited in usefulness for policy. An additional consideration is that application of the subsidiarity principle within the EU implies that many functions of government will continue to operate at the level of the national state or at a more local level, hence national economic development is likely to continue to be a central concern.

The character of a Single Market with important regional/national divergences can be met in modelling terms by the general model dividing Europe into regions, consisting of each of the 12 member states with Germany divided into east and west and Italy divided into north and south. This will allow the European economy to be analysed as:

- (a) the industrialised centre and north;
- (b) the agricultural south and west;
- (c) the new market economy in the east.

These areas have quite different requirements for energy policy: the industrialised centre tends to lead in terms of energy efficiency; the agricultural south is in the process of rapid development and urbanisation with the expectation that fuel use will rise rapidly; and the new market economies have the problems of transformation of their energy markets with a change to much higher relative prices of energy and accelerated replacement of their capital stock.

This consideration of structure also means that the model must also distinguish many industries. Input-output tables have been constructed by EUROSTAT at the level of 59 sectors for several EC countries for 1985. EUROSTAT also publishes time-series data at a 25 industry level, although with energy and water shown as one group. If energy and water is disaggregated into 6 sectors and two new sectors for environmental control and waste recycling added then the model will contain 32 industries. The input-output tables can be aggregated into these 32 sectors and a database can be constructed at this level of aggregation for the 14 European regions.

### **3. Approaches to long-run E3 modelling**

Research on E3 models and policies is proceeding at a rapid pace. This brief review concentrates on E3 modelling; for a survey of the economics of CO<sub>2</sub> abatement policies see (Mors, 1991); for a survey of the modelling and estimation of the economic costs of abatement, see (Boero, Clarke and Winters, 1991); for a review of the role of input-output in E3 modelling, see (Pearson, 1989); and for a survey of recent developments in the theory of economic growth, see (Solow, 1991).

The models for E3 analysis which have been developed and published fall into four groups:

- static applied general equilibrium models;
- dynamic applied general equilibrium models;
- multisectoral (input-output) dynamic econometric models;
- operational multi-model systems.

In addition developments in two other areas of economics are important:

- regional input-output modelling;
- the application of modern growth theory.

#### **3.1. Static Applied General Equilibrium Models (GEM)**

These models stem from economic theory going back to Walras and developed by Arrow and Debreu (1954); they have been applied by calibrating them on input-output tables. Recent developments are elegantly surveyed by Whalley (1991). These models have much in common with Johansen's MSG model (1960) and developed as a means of measuring the effects of tax policy (Shoven and Whalley, 1984).

The static GEM approach has some features worth adopting in a general model (viz calibration, a comprehensive approach to welfare, and the inclusion of relationships and simulated behaviour in areas where no data exist), but suffers some drawbacks. The main weakness of the approach is in its treatment of the dynamic aspect of economic behaviour. Often the models are based on just one snapshot set of cross-section information, such as an input-output table, so that they cannot hope to capture the process of economic development and change. A further assumption usually made is that the economy in the calibration year is in equilibrium and operating at maximum efficiency and welfare. This view of the economy is then used as a standard or reference point to compare with the position after a change, eg after the imposition of a carbon tax. The method is well illustrated in an initial paper reporting results of the GREEN model (Burniaux et al, 1991a): the paper reports that OECD welfare is reduced by the imposition of a carbon tax, but the model assumes that the base line does not include distortions and that there are no effects of the tax in stimulating energy-saving R&D which in turn stimulates long-term growth.

### 3.2. Dynamic Applied General Equilibrium Models (GEM)

The static GEM can be developed into a dynamic model by breaking the solution period into segments and optimising over each segment. The OECD GREEN model is the most comprehensive and advanced global model of this type (Burniaux et al, 1991b), solving over 5-year segments, and it has been applied to a range of questions concerning the implementation of carbon and energy taxes in different world areas, together with different schemes for distributing the revenues.

One of the leading US modelling programmes on E3 policies builds on Jorgenson's Dynamic General Equilibrium Model (Jorgenson and Wilcoxon, 1989, 1990, 1992). This combines the GEM approach with annual time-series econometric estimation of important sets of relationships (eg production functions). One more recent development in this direction is the G-CUBED multi-country model (McKibbin and Wilcoxon, 1992; McKibbin, 1992), a model combine dynamic optimisation with (proposed) econometric estimation of consumption and production functions, although at present the model is estimated on input-output tables and trade data for 1987. Another such model which has been developed to look at fiscal policy in France is reported in (Assouline and Epaulard, 1990; Schubert and Letournel, 1991). However, the models remain ones of equilibrium under the assumption of full employment and constant returns to scale, so that the problem remains of calibration to economies clearly out of equilibrium.

### 3.3. Multisectoral (Input-Output) Dynamic Econometric Models

These models also use input-output tables, but combine them with time-series analysis used in macroeconomic models. The Cambridge

Econometrics UK model (MDM) (Barker and Peterson, 1987; Barker and Lewney, 1991; Barker, 1995) is the latest in a continuous line of development via the Cambridge Growth Project, tracing back to the Stone and Brown model (1961). It is the leading E3 model for the UK, having recently been developed to assess the feasibility of the achievement of a 60% reduction in UK CO<sub>2</sub> emissions below 1990 levels by 2040. The model has been used (Barker, Baylis and Madsen, 1993) to analyse in detail the effect of the CEC carbon/energy tax on the UK economy for 4 scenarios (with VAT or income tax offset; and on EU-basis with energy-intensive industries exempt or on an OECD-basis with energy-intensive industries covered). Models in the same tradition for national economies have been developed by the INFORUM group of modellers (see Almon, 1991) and by the Norwegian Central Bureau of Statistics in its MODAG model (Cappelen, 1991) which operates alongside an MSC model in the Johansen tradition.

These are however usually national models and E3ME is to be a regional model. In addition, the long-term equilibrium properties of these models have not been extensively investigated and reported. (The long-term properties of macroeconomic models in general and the UK NIESR's model in particular, have recently been discussed by Britton (1992); the long-term properties of the disaggregated models such as MDM can in principle be developed in the same way.)

### 3.4. Operational Multi-Model Systems

This term is used to describe the systems of models at present being used to analyse the effects of carbon taxes and related policies at the European level. It includes the HERMES-MIDAS system of models discussed by Bradley (1991) at a meeting in Amsterdam, September 1991 and the DRI system of macroeconomic, industrial and energy models used in the study on the effects of a carbon/energy tax commissioned by the CEC (DRI, 1991 and 1992).

In principle, linked models such as the HERMES-MIDAS system could be estimated and solved consistently for all the economies involved. However, in practice, this often proves difficult, if not impossible, and considerable resources have to go into linking. In the new model any linkage problems will not be obvious to the user because there will be only one model. However, there are costs to such a unified approach: the general nature of the model means that country-specific detail may be lost. The model will include a facility to introduce condensed forms of country models to help to reduce this loss; in addition, emphasis will be placed on validation of equations and results by country experts.

Even if the consistency problem in linkage can be solved by successive iterative solutions of the component models, as reported in the DRI study (1991, 1992), there remains a more basic problem with the multi-model approach if it attempts to combine macroeconomic models with detailed industry or energy models. This problem is that the system cannot adequately tackle the simulation of "bottom-up" policies.

Normally these systems are first solved at the macroeconomic level, then the results for the macroeconomic variables are disaggregated by the industry model. However if the policy is directed at the level of industrial variables, it is very difficult (without substantial intervention by the model operator) to ensure that the implicit results for macroeconomic variables from the industry model are consistent with the explicit results from the macro model. As an example, it is probably very difficult to use a macro-industry two-model system to simulate the effect of exempting selective energy-intensive industries from the carbon/energy tax.

### 3.5. Regional Input-Output Modelling

Stone (1961) provides a useful analysis of the development of regional models. The possibilities of regional applications were recognised early in input-output analysis (Isard, 1951). Isard's model envisaged a full set of input-output tables for all the regions and a full set of data on inter-regional trade, ie trade was to be disaggregated for each region with every other region. Trade between the regions was in fixed proportions to output. This was soon followed by two variants, both limiting the data requirements and altering the restrictive assumption of proportionality. The first by Leontief (1953), elaborated further by Leontief and Strout (1963), assumed that each industry had the same cost structure in each region, that commodities were either entirely locally produced and consumed or traded, and that the traded commodities were traded in fixed proportions to total output. The second variant (Moses, 1955) allowed cost structures to vary, but fixed the trading patterns. The choice between these models depends on the data available and the likely importance of distance and differences in technology between regions. If distance is less important and the regions can be expected to share common technologies, then Leontief's model seems more appropriate.

More recent surveys and research (Nijkamp et al, 1986; Hewings and Jensen, 1986; Harrigan and McGregor, 1988) demonstrate the large scale of research activity in the last 30 years in the theory and application of regional input-output models. Two developments are worth noting here. The first is the extension of the Social Accounting Matrix framework (the basis of the accounting system used for MDM) to regional accounts (Round, 1988). The second is the integration of regional input-output tables with national General Equilibrium Models, such as the ORANI model of Australia (Dixon et al, 1982) and with time-series econometric models (Joun and Conway, 1983). The regionalised E3ME is an example of this second type of development.

### 3.6. The Application of Modern Growth Theory

Neo-classical growth theory has been recognised as seriously deficient (Romer, 1987) and has been largely ignored as irrelevant in applied long-term modelling. However, new models developed by Romer (see Solow, 1991 for a review) and Scott (1989) look much more

promising, and fit in well with a new interest by applied economists in the effects of innovation on trade and growth (Greenhalgh, Taylor and Wilson, 1990). Bowen et al (1991) illustrates the quantitative importance of innovation in economic growth and shows how measures of traditional R&D activities help to explain productivity growth and international competitiveness. They present estimates of innovation's contribution to GDP growth (1913-84) ranging from 24% in Japan to above 60% in France and the UK. The rates of return to R&D spending appear to be in the range of 10-20% and therefore considered as a worthwhile investment. An innovatory feature of E3ME is that some of the insights from the new theories will determine the specification and development of the long-term model.

#### **4. Characteristics of the E3ME approach**

##### **4.1. Integrated Modelling of the EC's Regional Markets**

The approach allows full interaction between the regional economies and the EC economy in the analysis of E3 issues. It will permit a full simulation of the effects of policies and changes in behaviour across a range of sectors (industries, consumers, government and transport) on the macroeconomy. For example, the link between investment growth in Germany and output in other regions of the EU, will be explicitly identified within the model rather than as the result of a linking exercise.

##### **4.2. Analysis of Specific Energy/Environment Infrastructure Projects**

It is much easier to incorporate assumptions on new projects, where there is partial and incomplete information, into a fully specified economic model than into a reduced form model. For example, estimates may be available for costs and impacts of infrastructure projects such as new gas pipelines from the east. These will have strong regional effects. If investment is fully specified in the model, such exogenous increases can be introduced explicitly into the forecast.

For certain location-based activities, such as transport and distribution, the location of the infrastructure in the form of transport links and warehousing will determine the regional supply. The location of large new investment projects, eg tidal barrages or the Channel Tunnel, can be introduced directly into the regional investment projections.

##### **4.3. Regional Accounts**

The treatment allows the calculation of full accounting balances for each member state for energy (thermal units) and commodities (monetary units in constant prices) supply and demand, exactly corresponding to the balances for the whole of the European Community. These balances cover inter-regional trade and an allocation of any imbalances across the regions. The balances provide an important



consistency check on any forecasts of regional energy and commodity output and the components of regional final demand.

#### 4.4. Forecasting Capability

The connection of the model-building with ERECO (1994) on sectoral projections for Europe will mean that substantial benefits from the project could accrue to a wide range of companies and businesses interested in Europe. It will provide a consistent framework for constructing medium-term and long-term forecasts and scenarios for the macroeconomies and industries of Europe over a period of potentially huge changes in the structure of energy supply and demand.

### 5. Preliminary specification fo E3ME

The characteristics of the model which underly the specification are that:

- it will reach its long-term results through a general equilibrium solution, but it can also be solved out of equilibrium and therefore valuable for meso-economic medium-term forecasting and that;
- it will draw together the consistency of the general equilibrium approach and the impact of R&D spending and innovation in the determination of long-term growth.

#### 5.1. The Development of a General E3 Model for Europe

The construction of a general E3 model has appeared until now to be too ambitious in relation to the available data and the resources required. The new approach is to build up a general model based on data from EUROSTAT/OECD/IEA in the form of input-output tables for 1985, national accounts, energy balances and energy prices and taxes. The model will have a clear economic and accounting structure, it will use incomplete and partial data, and it will apply techniques drawn from general equilibrium modelling and the co-integration literature. The forecasts and projections for the recent past will be calibrated so as to reproduce the available data for energy and output.

A considerable reduction of data requirements can be made by adopting Leontief's approach (Leontief, 1953; Leontief and Strout, 1963) in which each region trades with all the rest as a group rather than with each other region separately, so that the main requirement for trade data is on each region's exports and imports in total. (The data requirements involved in a full inter-regional model are of a higher order of magnitude - see Polenske, 1980). An important guiding principle in the modelling will be that the classifications and definitions adopted are as far as possible those of EUROSTAT. This means that a relatively detailed industry classification will be adopted for the commodity and industry

variables (such as gross output, GDP, employment, regional exports and imports), although the variables may be aggregated for presentation or comparison with the results from national models.

## 5.2. The Model

The following brief account of the first version of E3ME emphasises the accounting balances to be adopted. The functional forms for the main sets of equations are not fixed and indeed will be the subject of research, but we expect to draw on the experience of the HERMES models and to aim for robustness in equation properties and parsimony in their specification. An account of the specification and estimation of equation sets for employment, wage rates and industrial prices is given in Barker and Gardiner (1995).

The variables to be considered are described below. In general any reduced-form or condensed-form European equations will follow their regional (member country) counterparts in these explanatory variables. The first version of the model will comprise: the accounting balances for commodities and energy carriers; environmental emission flows; and 6 sets of time-series econometric equations (energy demands, commodity exports, total consumers' expenditure, disaggregated consumers' expenditure, industrial fixed investment and industrial employment). Energy supplies and population stocks and flows will be treated as exogenous. The second version will include a treatment of incomes, prices and values.

### *Energy Balances and Imbalances*

For each region, in thermal units (eg tonnes oil-equivalent, toe):

$$J + JM0 = JY0 + JC0 + JG0 + JS0 + JX0 \quad (1)$$

where each variable is a vector as follows:

- J outputs by energy carriers (coal, oil, gas, electricity)
- JM0 imports
- JY0 consumption of energy by industries, including the energy industries
- JC0 consumers' expenditure classified by energy carrier
- JG0 government final current expenditure classified by energy carrier
- JS0 stockbuilding
- JX0 exports

and in current prices:

$$VJ + VJM0 = VJY0 + VJC0 + VJG0 + VJS0 + VJX0 \quad (2)$$

where the variables of equation (1) are now in monetary units in current prices.

### *Commodity Balances and Imbalances*

For each region, in constant prices:

$$Q + QM0 = QY0 + QC0 + QG0 + QK0 + QS0 + QX0 \quad (3)$$

where each variable is a vector as follows:

- Q outputs by commodities (goods and services)
- QM0 imports
- QY0 absorptions of commodities by industries
- QC0 consumers' expenditure classified by commodity
- QG0 government final current expenditure classified by commodity
- QK0 investment in fixed assets classified by commodity
- QS0 stockbuilding by source commodity
- QX0 exports

and in current prices:

$$VQ + VQM0 = VQY0 + VQC0 + VQG0 + VQK0 + VQS0 + VQX0 \quad (4)$$

where the variables of equation (3) are now in current prices.

### *The Inter-Regional Export-Import Balance*

There is an accounting balance for net regional trade and European trade with the rest of the world. This can be found from equation (3). If the regional commodity balances in equation (3) are summed across the regions, and the European totals for each variable are subtracted from each side, the fundamental inter-regional export-import identity can be found. Since regional gross output and each component of domestic demand (ie excluding exports) adds up to the corresponding European variable, in constant prices and for each commodity, these variables cancel out in the identity, leaving European exports and imports and inter-regional exports and imports.

In other words, taking all European regions together, regional imports plus imports of the rest of the world (ie European exports) are equal to regional exports plus exports of the rest of the world (ie European imports). This balance is enforced in the projections of the model as an adding-up constraint on regional exports.

### *Energy Consumption*

Energy consumption by region will be derived from energy demand equations which will include the effects of real incomes or levels of activity, relative fuel prices and temperature. They will be estimated from time-series data, and if necessary long-term price and income own and cross elasticities will be imposed in the equations from national studies.

Fuel use by the electricity supply industries (ESI) will be determined by country/region specific sub-models. Unlike other models of energy use, the plant build programmes produced by the electricity sub-models will not be determined according to optimisation rules based on available technologies and their costs. The capital and fuel costs of alternative plant are taken into account, but are not the only factors used in deciding the quantity of each plant type built. The sub-models have the flexibility to allow other considerations, such as the desire for fuel security or government subsidisation, to overwrite pure cost considerations and for some aspects of plant build and generation to be determined exogenously.

### *Environmental Emissions*

The first version of the model will concentrate on emissions into the atmosphere. Data on the regional emissions of principal greenhouse gases and sulphur dioxide will be collected by category of the main fuel users. These will initially be related to energy consumption by these categories.

### *International Trade*

An important part of the modelling concerns international trade. The basic assumption here is that, for most commodities, there is a European "pool" into which each region supplies part of its production and from which each region satisfies part of its demands. This might be compared to national electricity supplies and demands: each power plant supplies to the national grid and each user draws power from the grid and it is not possible or necessary to link a particular supply to a particular demand. The demand for a region's exports of a commodity will be related to domestic demand for the commodity in all the European regions weighted by their economic distance from the region in question, to activity in the main European export markets as measured by GDP or industrial production and to relative prices, including the effects of exchange rate changes. The economic distance variable is normalised with a weight of 1 being given to activity in the home region; the weights for the other regions are inversely proportional to the economic distances of the other regions from the exporting region.

Regional imports will be related to demand and relative prices by commodity and region.

In addition, measures of innovation (eg patenting activity or spending on R&D) will be introduced into the trade equations to pick up an important long-term dynamic effect on economic development. There will be a reduced form equation for total regional exports and imports to allow for effects of relative prices, interest rates and incomes.

### *Intermediate Demand*

For each region, QY0 is defined as a vector comprising the row sums of the industrial absorption matrix for each region,

$$Y = \text{matp}(\text{QYC}, \text{diag}(Q)) \quad (5)$$

where

*QYC* a matrix of input-output coefficients,

*matp* a matrix multiplier operator

*diag* an operator converting from a vector to a matrix.

For the first version of the model, the EUROSTAT national input-output tables for 1985 will be used for each of the regions. The coefficients are calculated as inputs of commodities from whatever source, including imports, per unit of gross industrial output. In the case of the two parts of Italy and Germany the national input-output coefficients are therefore applicable to the regional economies on the assumption that the regional technology and mix are the same as those for the whole economy for each industry; this is clearly not so, and a priority in the research will be to allow for different structures, especially for energy use.

#### *Consumers' Expenditure*

Total consumer spending by region will be derived from consumption functions estimated from time-series data as in HERMES and transferred to the new model. These equations relate consumption to regional personal disposable income and interest rates.

In dividing this spending, the approach makes the most of the disaggregated data on consumers' expenditure available by region from the EUROSTAT. Again a set of equations is to be estimated from time-series data relating the spending per capita to the national spending using the Rotterdam system or a simplified Almost Ideal Demand System. The substitution between categories as a result of changes in relative prices is achieved at the national level.

#### *Government Expenditure*

This will be treated as exogenous.

#### *Fixed Investment*

New investments are one of the ways that companies adjust to the new challenges introduced by energy and environmental policies, so the quality of this data and the way it is modelled are of great importance to the performance of the whole model. Regional investment by investing industry will be determined in the model as intertemporal choices depending on expected output and investment prices, where the current prices of investment goods incorporates expectations about all future prices of capital services and all future discount rates (see Jorgenson 1989). There are problems with this approach since the measurement of capital is controversial (Scott, 1989), there are usually no satisfactory data on economic scrapping, and the econometric identification of the

effects of the user cost of capital on investment is elusive. Furthermore, investment is central to the determination of long-term growth and a coherent and convincing theory of growth should underlie the long-term behaviour of the investment and employment equations (see below).

### *Stockbuilding*

Stock levels are assumed to be proportional to output levels in the long term. Stockbuilding will be calculated as the changes in these stock levels.

### *Gross Output*

Gross commodity output will be calculated as a residual supply and net output is calculated by deducting intermediate demand from gross output.

### *Employment*

Employment will be treated as a demand for labour derived from the regional demand for goods and services. Regional employment equations will be estimated relating industrial employment in each industry to its output in the region, together with relative wage rates and measures of innovation. There will be a reduced-form equation for total national employment to allow for effects of real wage rates, interest rates and output.

### *Incomes and Expenditures*

Personal disposable incomes will be derived from income from employment, government transfers and taxes. Expenditure will be related to incomes. Output generates employment and income, with wage rates determined by a regionalised version of the model developed by Lee and Pesaran (1993).

## **6. Software**

The commercial modelling packages AREMOS, GAMS, MODLER, MREG and TROLL amongst others have been considered for the model, but most have limits of the size of models which preclude their use. Cambridge Econometrics has developed its own package IDIOM for the management and solution of large-scale models. The original program was described in (Barker, Peterson and Winters 1984) and its design and use is discussed by Peterson (1987). Economic models with many thousands of equations are solved as a matter of course using IDIOM on a laptop or desktop personal computer. The most important characteristic of the package for this purpose is that it is designed for very large-scale energy-environment-economy modelling. In contrast to

MODLER, TROLL or AREMOS, IDIOM deals with classes or categories of variables, organised according to the United Nations' System of National Accounts (SNA) rather than with single variables. For example, IDIOM can handle the input-output matrix of coefficients as one symbol. The flexibility of the software comes from the fact that the user can define up to 104 different classes or categories. Examples of the classes which would be included in E3ME are as follows:

- the regional dimension of 14 regions;
- industries;
- environmental emissions;
- energy carriers;
- energy users;
- final domestic demands (government current consumption, private consumers' expenditure, stockbuilding, social capital formation, private fixed investment);
- world areas;
- policies;
- objectives;
- years;
- 5-year periods.

## 7. Data

The basic data sets to be used in the first version of the model are as follows:

- input-output tables for 1985;
- national accounts with 25-sector detail;
- investment data;
- energy balances and energy prices;
- electricity stations data;
- air emissions.

The data from EUROSTAT has the advantage of covering the whole EU on a consistent basis (as far as possible), but the data are usually published later than the corresponding data from national sources. Recent data and short-term forecasts, at least for the larger EU economies, would be used to calibrate the model for the period between the last year of EUROSTAT data and the first year of a long-term forecast, usually taken as the second year into the future. Hopefully there will be an improvement in the timeliness of EUROSTAT data over the next few years, so that this process of calibration will become less important.

The input-output tables for 1985 have been published for Spain, Italy and Denmark. Tables for other EU economies for 1985 are expected to

appear before the end of 1992. However the data are on different bases for different countries, some treatment allowing for the differences will have to be included in the model.

In addition, a new database and set of equations has been developed to project economic and population variables in over 200 regions in Europe, published as *European Regional Prospects* (ERECO, 1994). This annual ERECO publication puts particular emphasis on detailed regional change in urban areas. The database of this regional model will provide information for the new model on population and the regional division of Italy and Germany. The requirements for E3ME tie in very well with EUROSTAT's five-yearly programme on input-output analysis and services.

## 8. Conclusions

The construction of a new E3 model for Europe combining solutions for both long-term general equilibrium and short-term dynamic disequilibrium is ambitious but feasible given the comprehensive and consistent data available for the EU12, the use of co-integration techniques to identify long-term behaviour and the adoption of existing software, specially written for the task. The insights from several distinct bodies of literature (input-output analysis, macroeconometric, multisectoral, general equilibrium and regional modelling) can be brought to bear on the problems. However, in order for the project to be a practical proposition and for the model to be operational, it has to be data-driven, flexible and limited in scope.

### – *Data-driven*

The classes adopted will be determined by the form in which the data is prepared. The emphasis will be on the use of regularly supplied official data on a comparable basis across the member states. The level of disaggregation will be largely determined by the data, although there may be special theoretical or practical reasons for further special disaggregation (eg division of Italy into north and south).

### – *Flexible*

The system should allow for different functional forms, so that a start can be made on a complete solution fairly soon, if necessary by adopting less demanding specifications of equations, and to allow expansion of the classes eg for new members of the Community.

### – *Limited*

It is not a short-term, quarterly forecasting model but rather a model for simulation and analysis, explaining the consequences of changes in policy.



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# SYSTEM FOR INTEGRATED ENVIRONMENTAL AND ECONOMIC ACCOUNTING (SEEA) OF THE UNITED NATIONS

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## **Introduction**

It is a great honour for me to participate in the International Conference in Memory of Sir Richard Stone. I did not have the opportunity to cooperate with him and to learn from him in direct personal contact. Our relationship was restricted to some talks during international conferences. Nevertheless, his personality deeply impressed me and his spirit of work has influenced to a great extent my own attempts in the field of national accounts. His ideal to extend national accounting to achieve a comprehensive description of social and economic performance was a very important key-note for me in developing concepts of integrated environmental and economic accounts. I am grateful that I could learn from him and that I now have an opportunity to discuss his work with his former students and colleagues.

Environmental accounting was not one of the main topics of Sir Richard Stone's work. Nevertheless, he gave valuable advice with regard to this field. In his Nobel Memorial Lecture (8 December 1984) he said: "The three pillars on which an analysis of society ought to rest are studies of economic, socio-demographic and environmental phenomena. Naturally enough, accounting ideas are most developed in the economic context (...) but they are equally applicable in the other two fields." In 1972, he published the paper "The Evaluation of Pollution: Balancing Gains and Losses" in *Minerva*, vol. X (1972), no. 3, pp. 412-25. This paper is a very important contribution to the question of how to value the economic use of the natural environment. He described in his paper input-output tables with separate pollution elimination activities and discussed the problem of valuing pollution. In his conclusions, he proposed that the extent of pollution elimination could be determined neither by consumers nor by producers but by a public decision based on social welfare functions. With regard to social welfare functions he

applied Voltaire's aphorism about God: "If they did not exist it would be necessary to invent them and, I may add, to act in their light (p. 425)". His approach is similar to the concept of prevention costs connected with setting standards which are derived from macro-economic standards rather than individual utilities.

Another very important contribution of Sir Richard Stone to integrating environmental and economic accounting are his considerations on "Supplementing the National Accounts for Purposes of Welfare Measurement", a paper prepared for the United Nations and distributed by the United Nations Economic and Social Council for the eighteenth session of the Statistical Commission in Geneva, 7-18 October 1974 (E/CN.3/459/Add.1, 9 August 1974). Inter alia, he proposed in his paper the development of natural asset accounts. "It is important to cover non-reproducible assets in the national accounts as they are important elements of wealth and the reserves for production and are the subject of major questions concerning man's environment (para. 132)." He also stressed the importance of a functional breakdown of transactions which would also include an explicit record of expenditures connected with environmental protection activities. "The outlays on treatment (of pollution C.S.) are already included in the accounts; the major difference is that an endeavour is made to enumerate the sources and output of pollutants and to detail the outlays on treatment (para. 36)". Furthermore, he proposed that environmental statistics containing physical data on the economic-environmental interrelationship should be linked to national accounts. "Thus, while there are many problems of taxonomy and data collection to be solved, it appears to be possible to cover a number of aspects of pollution in a national accounting framework; in view of the importance of the subject, it is desirable to do so. It is most convenient to deal with this and other aspects of pollution together, in a coherent body of statistics of the environment which is coupled with the national accounts (para. 38)."

Twenty years later, we are now able to follow the recommendations of Sir Richard Stone. The revised SNA contains natural asset accounts and a section on integrated environmental and economic accounting in a satellite system of national accounts (1992a). A further elaboration of integrated environmental and economic accounting is given in the System for Integrated Environmental and Economic Accounting (SEEA) which will be published in an SNA Handbook this autumn (United Nations, 1992b). The SEEA contains, as Sir Richard Stone proposed, an environment-related disaggregation of traditional national accounts, linked physical data and values of the economic use of the natural environment following the concepts of market valuation as well as prevention costs according to environmental standards.

The following paper gives an overview of the present "interim" version of the SEEA. This version has the task to make existing methodologies widely available in order to facilitate a broad consensus on a commonly acceptable integrated framework. At the same time, the feasibility of the concepts and methods proposed has to be tested by

implementing the SEEA in countries at different stages of development. The results of the theoretical discussion and the empirical work will be used to prepare the "final" version of the SEEA. As I hope, the results of our discussion during this conference could also be used to further improve the concepts of the SEEA.

## 1. Approaches of Environmental and Economic Accounting Systems

As actual experience and the conceptual discussion have shown, there is a large variety of approaches in the design of statistical systems describing the interrelationship between the natural environment and the economy (see United Nations, Economic Commission for Europe, 1991). Two extreme positions are the following:

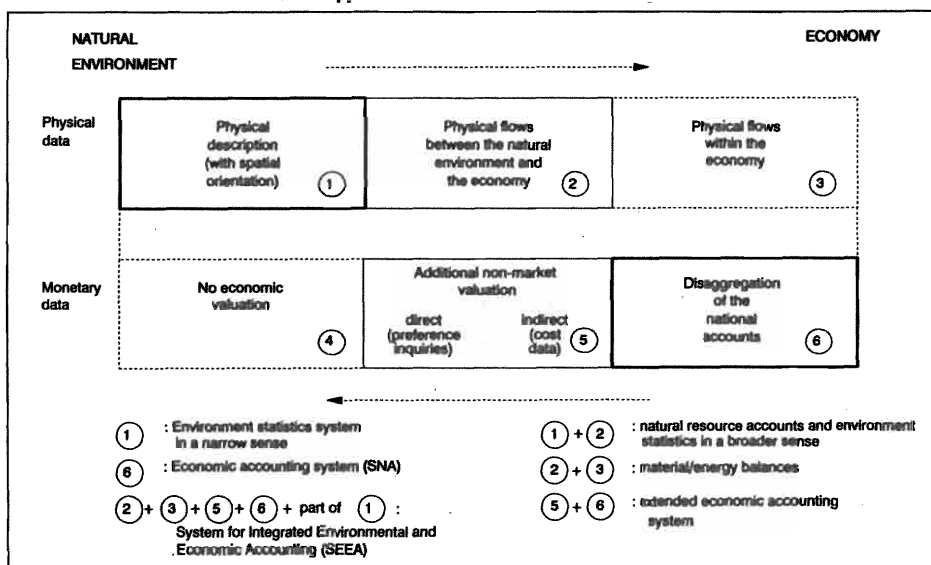
- The description is solely focussed on the natural environment. The environmental-economic linkages are described with special regard to economic impacts on the environment. An important part of such a statistical framework is dedicated to a *spatial description* of the natural environment, using for instance maps of the respective region. The information is normally presented in physical units.
- A second extreme type of statistical frameworks for environmental-economic accounting focuses only on the economy and takes into account the environmental-economic linkages only as far as they are connected with *actual economic transactions* (e.g. environmental protection expenditures, actual damage costs). These data systems are normally extensions of the traditional national accounts. They disaggregate the flows and assets of the accounting system with regard to environmental aspects. The results are mainly restricted to monetary data which reflect actually observable market data.

In Table 1 these two concepts are indicated under the number ① and ②, respectively.

Approaches which are located *between* these two extremes could be classified with regard to the use of statistical units: Some statistical data systems take into account either physical or monetary data, some present a combination of them. Systems which mainly use a *uniform* type of statistical units are the following:

- The physical description of the natural environment could be extended to include information on the physical flows between the environment and the economy (use of natural resources, flow of residual products). The present systems of *natural resource accounting* and environment statistics comprise mainly these data (in Table 1: ① and ② (cf. e.g. United Nations, 1984, 1988, 1991). This description in physical terms could be extended to information on transformation processes within the economy. The approach of *material/energy balances* comprises a physical description of the use of natural resources, their transformation by production and consumption activities and the flow

Table 1 – Approaches of environmental accounting systems



of residuals back to the natural environment (in Table 1: ② and ③ (cf. United Nations, 1976). Natural resource accounting and material/energy balances have statistical areas of overlapping, especially the flows between the economy and the environment (in Table 1: ②).

The description of economic activities in *monetary* terms could be *extended* to a valuation of the economic use of the natural environment. Comprehensive measurement of costs and benefits of economic activities and their environmental impacts could be the result of such calculations (in Table 1: ⑤ + ⑥, cf. e.g. Bartelmus, Stahmer, van Tongeren, 1991).

These statistical systems do not seem to be sufficient for a complete monitoring framework of the environmental-economic linkages. On the one hand, the suitable statistical unit for giving a detailed picture of the natural environment is the physical unit and, from an ecological point of view, the restriction to monetary valuation indicates a complete dominance of the natural environment by the economy. On the other hand, the interrelationship between the environment and the economy could not be sufficiently described for economists if it could not be translated into money values, the common economic language. This approach not only facilitates the access of economists to environmental problems but also creates a common scale which allows the compilation of results on a highly condensed level. Therefore, the System for Integrated Environmental and Economic Accounting comprises *both monetary and physical data*.

A complete system of Integrated Environmental and Economic Accounts would have to contain the traditional System of National



Accounts (see United Nations, 1992a) as a data system for describing economic activities, a System of Environmental Accounts and all monetary and physical flows which could describe the interrelationship between the environment and the economy (in Table 1: ①, ②, ③, ⑤ and ⑥). This *ideal concept* cannot be implemented at the present time. The main reason is a missing comprehensive data system for describing the natural environment. Ambitious approaches have been employed in several countries, but no overall description of the natural environment has been implemented up to now. This lack of success cannot only be explained by inadequate financial support. It is true that more financial resources would probably have brought about more success in developing environmental statistics and comprehensive statistical systems in this field. However, the main reason for the absence of comprehensive environmental accounting seems to be the extraordinary difficulty encountered in describing the natural environment with its climatic, biological, physical and chemical changes during a reporting period in a model which describes this complex interrelationship adequately. At present it seems possible to describe sufficiently the state of the natural environment at a certain moment. This could be done by mapping or by tables monitoring the situation at a given time. But it has been – at least up to now – nearly impossible to portray the natural dynamics between two points of time. An example of such complex interrelations is the difficulty in developing weather models. It is relatively easy to draw weathermaps, but it is much more difficult to explain the reasons for the observed weather situation and to describe the changes. A complete integration of existing environmental and economic data systems therefore seems to be impossible at this moment.

It seems necessary to concentrate efforts in this field first of all on improving environmental statistics and to develop consistent systems for describing the natural environment as a second step. The *Framework for the Development of Environment Statistics* (FDES) of the United Nations and the work of the different regional organizations of the United Nations (e.g. the Economic Commission for Europe) in the field of environmental statistics seem to be a promising starting point (United Nations, 1984, 1988, 1991, and United Nations, Economic Commission for Europe, 1988). The French work in the field of Natural Patrimony Accounting (see INSEE, 1986) could play a prominent role in further conceptual improvements in this field.

Difficulties in describing the natural environment in a comprehensive and sufficiently detailed manner should not prevent the attempts to describe the interrelationship between the natural environment and the economy as completely as possible. Concepts of *natural resource accounting* which focus on describing the natural environment from the point of view of economic use and the experience which has been gained in this field in several developing and developed countries could be used to establish a consistent data system. In this context, the conceptual considerations in the context of *material/energy balances* may also prove to be helpful.

The relatively comprehensive *System for Integrated Environmental and Economic Accounting* (SEEA) comprises four parts:

1. Parts of the established economic accounting system (System of National Accounts (SNA) of the United Nations, see United Nations, 1992a) which are of special relevance to environmental aspects and which will have to be partly disaggregated to identify monetary flows and assets which are related to the use of the natural environment (Table 1: parts of ⑥).
2. Additional non-market valuation of the economic use of the environment in monetary terms (in Table 1: ⑤).
3. Physical data on the flows from the natural environment to the economy, on their transformation within the economy and on the flows of the residuals of the economic activities back to the natural environment (in Table 1: ② and ③).
4. Description of the natural environment as far as it is necessary to analyze the impacts of economic use. This part would not lay claim to comprehensiveness. It would rather have a supplementary character (in Table 1: parts of ①).

This concept does not pretend to provide an overall accounting system which comprises a complete description of the natural environment, the economy and its interrelations. It only focuses on describing the *interrelationship* between the *environment* and the *economy*. Economic activities as well as events within the natural environment are only taken into account as far as they are necessary to understand the relations between the economy and the environment. Furthermore, the relationships with socio-demographic data systems have not been elaborated.

The fact that an established system for environmental accounting is not available at the present time seems to justify that the planned SEEA takes as *starting point* only the well-established system for economic accounting, the SNA. The non-market valuation of the economic use of the natural environment is introduced in addition to the monetary data of the national accounts. The physical data of environment statistics, the natural resource accounts and of the material/energy balances are connected with the respective data in monetary terms in the national accounting system.

Employing the established economic accounting system does not necessarily lead to a dominance of economic aspects. On the contrary, it can reveal possibilities of stressing ecological aspects. Ecological aspects can be introduced in economic thinking and in economic decisions only if ecologists and economists are using the same language. If ecological aspects could be translated into money terms the possibilities of economic decisions taking environmental problems into account would be much greater. The aim of the SEEA should be to establish a suitable data basis for a policy of sustainable development. This development can only be influenced by economic decisions. There is, therefore, higher priority to introduce ecological aspects into the

sphere of economic activities than to monitor only economic impacts on the natural environment without economic valuation.

## 2. SEEA as satellite system to the national accounts

During the last twenty years, proposals have been made to modify the national accounting system with regard to environmental aspects (cf. Baltensperger, 1972; Bartelmus, 1974, 1987 and 1989; de Boo, Bosch, Gorter, Keuning, 1991; Eisner, 1988; Fickl, 1991; Franz, 1988, 1989; Hamer, 1974; Harrison, 1989a, 1989b, 1992; Hueting, 1980; Levin, 1990; Marin, 1978; NNW Measurement Committee, 1973; Nordhaus, Tobin, 1973; OECD, 1971; Olson, 1977; Peskin, 1980, 1989; Richter, 1989; Uno, 1989, 1990; Reich, 1991; Reich, Stahmer, 1983; Thage, 1990, 1991). It has been argued that it is not sufficient to focus the accounting system on market transactions and to describe non-market activities only if they are connected with observable costs (e.g. in the case of government and non-profit institutions' activities). The results of this debate have shown that the majority of experts rejects substantial changes to the traditional national accounts but would prefer to establish a *special system* outside the traditional framework to describe environmental-economic relations (cf. the comprehensive analysis of Chr. Saunders in United Nations, 1977, and Adler, 1982; Carson, 1989; Denison, 1971; Drechsler, 1976; Herfindahl, Kneese, 1973; Stone, 1972; United Nations, 1974, prepared by R. Stone; United Nations, 1979, 1980; United Nations, ECE, 1973).

The *traditional national accounts* seem to be a *sine qua non* for analyzing problems of a *market economy*. There are many applications for which the restriction to market transactions is not a disadvantage but rather an advantage (see Reich, 1989, 1991). Short-term economic policy needs data on labor, commodity and financial markets. National accounting systems are effective because the data fulfill two preconditions: They are suitable and they are observable. The degree of necessary estimations is low because most data of the national accounts can be directly observed from household and enterprise surveys.

The urgent need to describe the interrelationship between the environment and the economy should therefore not invalidate national accounting systems, but should lead to a special data system which, though being separate, should be closely linked to the traditional national accounts. This approach necessitates two systems, the traditional national accounts used as a *core system* and a special data framework which has the character of a *satellite system* (or satellite accounts) (Hamer, 1986; Lemaire, 1987; Reich, Stahmer, et al., 1988; Schäfer, Stahmer, 1990; Teillet, 1988; Vanoli, 1989; Weber, 1983, 1989). The preconditions for the success of such a construction are twofold:

- The concepts of a satellite system should have higher degrees of *freedom* than those of national accounts. They should be chosen in

such a way that they can both give a comprehensive picture of the environmental-economic interrelationship and take into account the ecological point of view. It should also be possible to use valuation methods which might have a weaker data basis than the traditional national accounts. Furthermore, the possibility should be offered to test different methods and to describe different options. The complex problems of the use of the environment for economic activities can not be reduced to one specific approach. The most comprehensive measures of economic-environmental relations represent at the same time concepts which have the weakest data basis. The experimental character of possible environmental accounting systems should, therefore, be stressed. A satellite system should certainly present a consistent framework. But such framework should as far as possible take into account different schools of thinking.

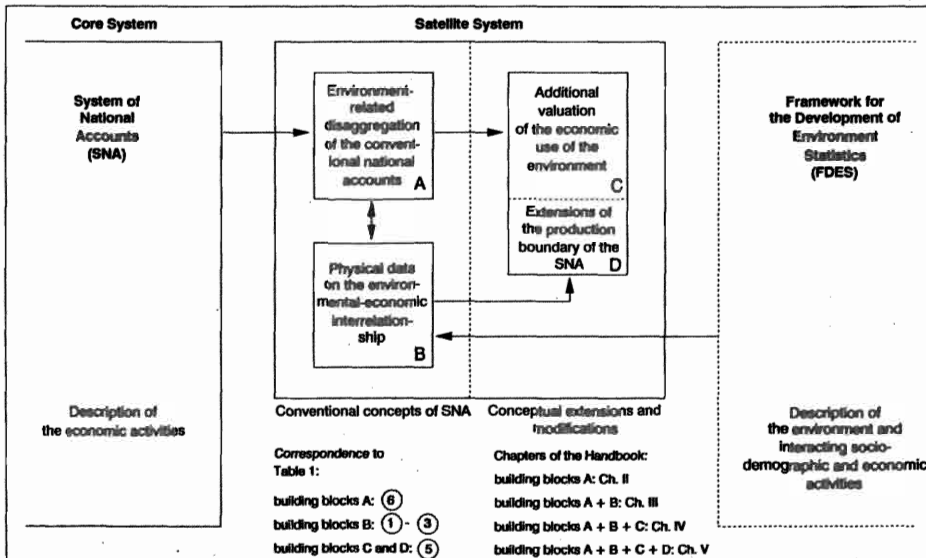
- The aim of the accounting system can not be restricted to describing environmental deterioration caused by economic activities. The system should become a data basis for integrated environmental and economic policies. This aim can only be achieved if both the direct and indirect impacts of the economic use of the environment on economic activities can be analyzed. This implies close connections between the traditional economic accounting system and the new satellite system. The *links* between the two data systems could be used to establish comprehensive economic models which comprise not only economic but also environmental data.

At first sight the two preconditions for developing a suitable concept for the SEEA seem to be mutually exclusive. Close linkages to the national accounts seem to prevent an ecological orientation and an experimental design of the satellite system. It is obvious that this possible conflict can only be solved by developing a system with a high degree of *flexibility* (cf. the considerations of the "Dutch school": Bochove, Tuinen, 1986). The system should comprise modules or building blocks which are linked to the traditional accounting system in differing degrees (see e.g. Friend, 1991). As far as possible, the same concepts should be used for both the core system and the satellite system. In cases where different concepts are required, bridge tables are necessary which explicitly show the conceptual differences and which could be used as links between the new data sets and the traditional national accounts.

The SEEA (see United Nations, 1992b) comprises four types of *building blocks* which follow the concepts of the SNA (see United Nations, 1992a) to a differing extent (see Table 2):

- The first type of building blocks for constructing the SEEA is the production part of the SNA which contains a description of production and consumption activities (supply and disposition tables), and the accounts of non-financial assets (*building blocks A* of the SEEA). The production part of the SNA is sometimes called

Table 2 – SNA Satellite System for Integrated Environmental and Economic Accounting (SEEA)



the *input-output part* because it comprises the data basis for input-output tables with uniform row and column classifications. The input-output framework seems to be the most suitable economic framework for analyzing environmental-economic relations because it could easily be extended by including flows of natural resources from the natural environment as inputs of economic activities and the flows of residuals of production and consumption activities as unwanted outputs delivered back into the natural environment. The starting point for the *natural asset accounts* of the SEEA are the non-financial asset accounts of the SNA which also comprise non-produced natural assets in the revised version (see United Nations, 1992a).

The SEEA contains the above mentioned parts of the SNA partly in an aggregated version, and partly in a more disaggregated form. *Disaggregation* seems to be particularly necessary to identify the environmental protection activities which should prevent an environmental deterioration or should restore an already deteriorated natural environment, and the repercussion (damage) costs (health expenditures, material corrosion costs) caused by a deteriorated environment. In the case of non-financial assets, further disaggregation of stocks and volume changes of natural assets is required.

- A second type of building blocks of the SEEA (in Table 2: *building blocks B*) comprises a description of the interrelationship between the natural environment and the economy in *physical terms*. This part of the SEEA which applies the conceptual considerations and empirical

experiences of natural resource accounting, material/energy balances, and input-output compilation is closely linked to the monetary flows and assets of the SEEA derived from the production part of the SNA. These extensions could be made without modifying the concepts of the SNA.

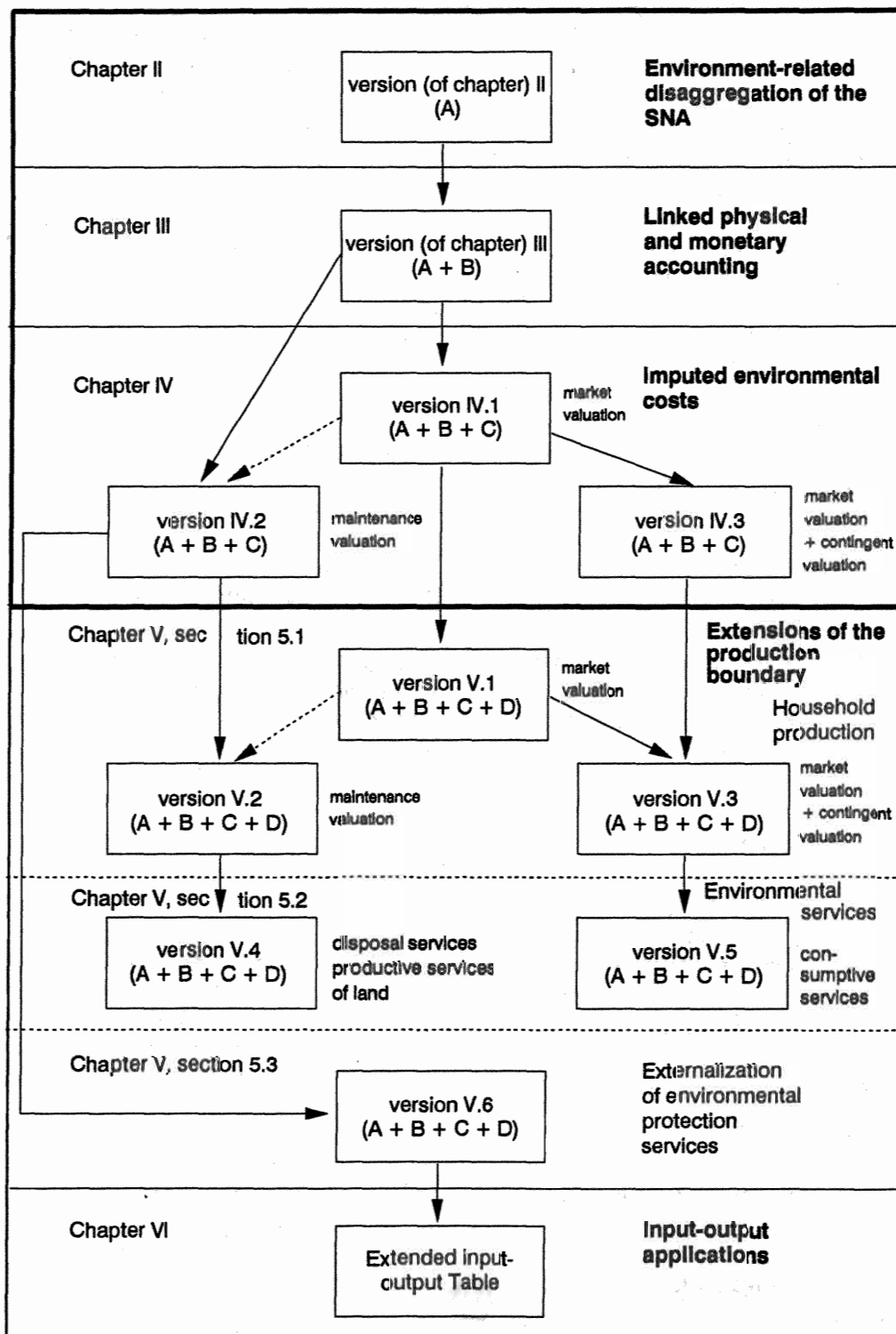
- In a third part of the SEEA (in Table 2: *building blocks C*), different approaches are discussed for estimating the *imputed costs* of the economic use of natural assets. In this context, three different valuation methods are used:
  - market valuation according to the concepts of the non-financial asset accounts in the SNA maintenance valuation which tries to estimate the costs necessary to sustain at least the present quantitative and qualitative level of natural assets
  - contingent valuation which could be applied especially for estimating the value of the "consumptive services" of the natural environment.
- A fourth type of building blocks of the SEEA (in Table 2: *building blocks D*) contains additional information which could be obtained by *extending the production boundary* of the SNA. These extensions have been applied especially in the case of household activities whose detailed analysis is necessary for understanding the impacts of household activities on the natural environment and the welfare aspects of the deteriorated nature. Furthermore, the consequences of treating economic functions of the natural environment as production of "environmental services" are discussed. A third method for extending the production boundary of the SNA refers to treating both internal and external environmental protection activities as production activities.

### 3. Versions of the SEEA in the Handbook

The different types of building blocks of the SEEA are not described as separate entities but as extensions of a *common accounting framework*. Each stage of extension comprises the data of the preceding stages as long as the valuation methods are not mutually exclusive. In Table 3, the dependencies between the different versions of the SEEA are shown. Furthermore, the chapters of the Handbook dealing with the specific SEEA *versions* are also indicated.

- In *Chapter II*, possible environment-related disaggregations of the SNA are described: version (of Chapter) II of the SEEA with *building blocks A*.
- In *Chapter III*, the monetary data of part A of the SEEA are linked with environment-related information in physical terms: version (of Chapter) III of the SEEA with *building blocks A + B*.

Table 3 – Versions of the SEEA in the Handbook



- In *Chapter IV*, imputed environmental costs (part C) are added: versions (of Chapter) IV of the SEEA with *building blocks A + B + C*. Three different types of valuation of imputed costs are discussed: market valuation (version IV.1), maintenance valuation (version IV.2) and contingent valuation in addition to market valuation (version IV.3).
- In *Chapter V*, extensions of the production boundary of the SNA are described in combination with differing types of valuation of imputed environmental costs: versions (of Chapter) V of the SEEA with *building blocks A + B + C + D*. The extended record of household production activities has been applied in combination with the three valuation methods of imputed environmental costs already applied in Chapter IV (versions V.1, V.2 and V.3). Environmental services have been treated as production in the case of disposal services (part of version V.4) which describe the use of the natural environment as sink of economic residuals, in the case of productive services of land, landscape and ecosystems (part of version V.4), and in the case of consumptive services for households (version V.5). Versions V.4 and V.5 take into account not only environmental production but also an extended concept of household production. Thus, they are derived from versions V.1, V.2 and V.3, and represent a further stage of extension. The "externalization" of internal environmental protection activities (version V.6) is described on the basis of the concepts of version IV.2 (maintenance valuation).
- A product-based symmetric input-output table with environment-related extensions is derived from version V.6 and described in *Chapter VI*. This table is used as a conceptual basis for applications of the SEEA in input-output analysis.

#### 4. SEEA Matrix

The SEEA is presented mainly in matrix form comprising a description of both flow and asset accounts. Table 4 shows the *SEEA matrix* in different stages of extension. For facilitating the description, the record of monetary data is described only. Flow data are recorded in rows 2 to 12; the asset accounts, in columns 5 to 7. Flow and asset accounts are linked to each other by the volume changes of assets described in rows 2 to 10 and columns 5 to 7. The classification items refer to the basic row and column classifications used throughout in the Handbook.

The structure of the *columns* has some similarities with the columns in input-output tables. The first three columns comprise different production activities; columns 4 to 8, final uses. Differing from input-output concepts, the record of gross capital formation (columns 5 to 7) has been supplemented by complete asset accounts including stock data.

The structure of the *rows* reflect a combination of items necessary for establishing asset accounts, and items used for recording flows.



Rows 1 and 13 to 15 are relevant only in the context of asset accounts. The structure of rows 2 to 12 is again similar to that of input-output tables. It comprises the use of products and assets, net value added (net domestic product) and gross output.

The SEEA matrix is used for describing all versions of the SEEA in different *stages of extension*. Three stages are differently hatched in Table 4:

- Version II (see Chapter II of the Handbook) of the SEEA refers to data according to the *conventional concepts* of the SNA. These data are further disaggregated to reveal environment-related activities and the monetary flows and stocks connected. In version II, domestic production activities comprise only the production activities of industries (column 1), and, therefore, produced assets contain only assets of these industries (column 5). The use of products is limited to products of industries (row 2); and the use of assets to the use of produced fixed assets of industries (row 3). The asset accounts comprise opening stocks (row 1), net capital formation (rows 2 and 3), other volume changes (row 13), revaluation due to market price changes (row 14) and closing stocks (row 15) which are the column totals of the recorded asset accounts.
- The versions of Chapter IV of the Handbook which describe different approaches of valuing *imputed environmental costs* imply a recording of additional costs associated with different economic activities (production, final consumption, use of produced assets), and with reverse sign, costs associated with volume changes of natural assets deteriorated by economic activities (see row 7). Adjustment items are introduced (rows 9 and 10) which balance imputed environmental costs against the conventional figures of Net Domestic Product (column 1) and the corresponding volume changes of natural assets against other volume changes and the closing stocks of natural assets still valued according to the SNA concepts.
- A third stage of development of the SEEA implies *extensions* of the *production boundary* of the SNA (see Chapter V of the Handbook). The extended concept of household production activities is reflected in the SEEA matrix as an additional record of production activities (see "other household activities" in column 2) and products (row 4). The corresponding extension of the concept of produced assets entails the introduction of asset accounts of consumer durables and the record of corresponding user costs (row 5). If environmental services are treated as production activities, a further extension of the concept of domestic production is necessary (see column 3 and row 6). The conceptual implications of externalizing internal environmental protection services are not explicitly shown in Table 4 for sake of simplicity. If externalized such approach would imply modifications of the concepts of industries.

Hatched elements of the SEEA indicate that they could, at least theoretically, contain figures in monetary terms. In Chapters II, IV and V

Table 4 – SEEA matrix in different stages of extension - monetary data

■ disaggregation of the SNA  
(Chapter II)

⊙ imputed environmental  
costs (Chapter IV)

▨ extensions of the production  
boundary (Chapter V)

Ser. no.		1. Domestic production			2. Final con- sumption	3. Non-financial assets			4. Exports	Σ Total uses
		1.1 Indu- stries	1.2 Other house- hold ac- tivities	1.3 Environ- mental services		3.1 Produced assets		3.2 Non- produced natural assets		
						3.1.1 indu- stries	3.1.2 consumer durables			
1	1. Opening stocks					▨	▨			
2	2.1 Use of products of industries	■	▨			▨	▨	■	■	■
3	3.3.1 Use of produced fixed assets of industries	■	▨			▨	▨	■	■	■
4	2.2 Use of other household outputs		▨				▨			▨
5	3.3.2 Use of consumer durables		▨				▨			▨
6	2.3 Use of environmental services		▨							▨
7	3.1 Use of non-produced natural assets	⊙	▨	▨	⊙	⊙	▨	⊙	⊙	⊙
8	3.2 Economic treatment of residuals	⊙	▨	▨	⊙	⊙	▨	⊙	⊙	⊙
9	4.1 Adjustments due to market valuation	⊙	▨	▨	⊙	⊙	▨	⊙	⊙	⊙
10	4.2.1 Eco margin	⊙	▨	▨	⊙	⊙	▨	⊙	⊙	⊙
11	4.2.2 Net value added/Net Domestic Prod.	■	▨							
12	Σ 5. Gross output	■	▨	▨						
13	6. Other volume changes					▨	▨			
14	7. Revaluations due to market price changes					▨	▨			
15	Σ 8. Closing stocks					▨	▨			

of the Handbook, the different versions of the SEEA matrix are shown in detail.

## 5. Implementation of the SEEA

In order to adapt the SEEA to different environmental and socio-economic conditions in countries, the SEEA has been designed to be as comprehensive, flexible and consistent as possible.

The aim of *comprehensiveness* refers not only to a variety of different patterns of economic development or categories of environmental deterioration, but also to alternative theoretical approaches which can be applied for analysing the economic and environmental situation. Physical accounting is used as well as differing types of monetary valuation.

Comprehensiveness in the SEEA does not imply the use of the whole range of possibilities to describe environmental-economic interrelations. The specific environmental and economic problems of a particular country have to determine the choice of the main fields which should be taken into account. Furthermore, data availability and possibilities of further improvement of the data base restrict the application of SEEA concepts. These constraints necessitate a *flexible* system which should comprise a variety of building blocks which could be used independent of each other (see the proposals of van Bochove, van Tuinen, 1986).

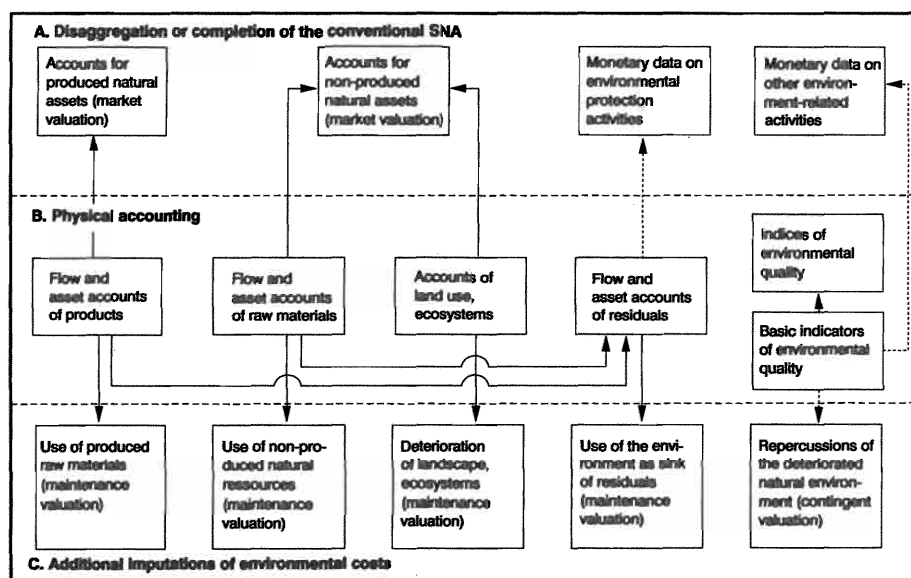
This necessary flexibility of the SEEA should not affect the *consistency* of the system. A consistent data system is guaranteed if the versions of the SEEA remain an extension of the national (economic) accounts and apply the accounting rules of extended accounts. These rules imply for instance that supply and destination of products, natural resources and residuals should be balanced in the flow accounts and that complete asset balances should be established. Therefore, the concept of flexibility permits the selection of high priority flow and asset accounts but should not encourage the development of incomplete accounts.

The implementation of the SEEA should focus on high-priority concerns and related economic activities. Implementation will, however, be limited by *data availability*. Therefore, it seems useful to start with implementing that part of the SEEA which has both high priority and a sufficient data basis. The data basis should be improved in parallel to the implementation of initial building blocks of the SEEA with a view to achieving a more complete version of the SEEA in the future.

In Table 5 an overview is given of possible statistical building blocks of the SEEA. Of course, each building block comprises a variety of specific items that could be compiled separately (e.g. accounts for different types of products, raw materials or residuals).

The building blocks are grouped according to the three mentioned *types of data* in the SEEA:

Table 5 – Building blocks for implementing the SEEA



- (a) Disaggregation or completion of the *conventional SNA* with regard to environmental issues (building blocks A of version II of the SEEA): This part of the SEEA comprises, in particular, building blocks describing the accounts of natural assets (market valuation), and actual (observable) monetary data connected with environment-related defensive activities (e.g. environmental protection activities or defensive activities against the repercussions of a deteriorated natural environment);
- (b) *Physical accounting* (building blocks B of version III of the SEEA): This part of the SEEA comprises accounts for products, raw materials and residuals, as well as land use accounts, environmental quality indicators and other (more aggregated) indices.
- (c) *Imputed environmental costs* with regard to the impacts of economic activities on the natural environment (building blocks C of versions IV of the SEEA): This part of the SEEA comprises estimates of the prevention costs necessary to maintain the qualitative and quantitative level of the natural assets (Bartelmus, Stahmer and van Tongeren, 1991) and the imputed costs of the repercussions of the deteriorated natural environment (using contingent valuation methods, see OECD, 1989; Pearce, Markandya and Barbier, 1989; and Stahmer, 1991).

The arrows in Table 5 show **dependencies** in compiling different building blocks: The empirical implementation of some building blocks require the implementation of other parts of the system. This is especially true of the monetary data (building blocks A and C in Table 5) which – in many cases – can be compiled only on the basis of sufficient

physical data (building blocks B). The compilation dependencies between the different building blocks in monetary terms are not so strong. These data can mainly be compiled independently. Nevertheless, imputed environmental costs (building blocks C) can be usefully analyzed only in comparison with actual (observable) data (building blocks A). These compilation dependencies among the different parts of the SEEA support the view that first priority should be given to physical accounting. Monetary data could then be estimated in a second step. This procedure does not exclude the immediate implementation of monetary building blocks which are more or less independent of physical data. This is especially true of the estimation of expenditures connected with environmental protection activities and the, more controversial, application of contingent valuation.

Flexibility of the SEEA would permit an implementation of the SEEA limited to building blocks A and B (version III of the SEEA). This limitation implies that the concepts of the traditional national accounts would remain completely unchanged because building blocks A and B only record a disaggregation and completion of conventional data or, in the case of physical accounting, additional environment statistics which provide further information without affecting traditional concepts. On the other hand, a limited presentation of details of the environmental-economic interrelationships in physical terms is questionable. If the results of the SEEA are to support an integrated environmental and economic policy, a sort of *weighting procedure* for condensing the details is needed because political decisions are often based on a few highly aggregated figures. The estimation of imputed environmental costs allows such aggregation. Of course, aggregated physical indicators, for instance on changes in quality of specific environmental media, have to supplement this monetary information.

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**III SESSION**  
**BALANCING NATIONAL ACCOUNTS**

Chairman: Ignazio Visco (*Bank of Italy*)



# A "NEGLECTED PROBLEM" REVISITED THE CONSISTENCY, BALANCING AND RELIABILITY OF NATIONAL ACCOUNTS

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## 1. Introduction

The definitive role played by Richard Stone in the development of national accounting is undeniable. Certainly, Stone did not invent the idea of national accounting (as he himself pointed out several times), but he succeeded in turning this notion into an original synthesis of the accounting concept of "double entry", the notion of "general economic circulation" and the system of Keynesian equations. He thus gave national accounting a remarkable orientation, often called "Anglo-Saxon" (as opposed to the so-called "French" [42] and "Soviet" [53], [54] approaches). Let us add that along with his qualities as a theorist, Stone was also a fundamentally pragmatic man of action; I shall try to follow his example here.

However, an aspect of his impressive work has, according to Stone himself, been "neglected" his concern for theoretically justifying the consistency of national accounts, and for balancing national accounts by a method taking into account the precision of the various constitutive aggregates of the accounting tables.

It is a problem with which Richard Stone was concerned throughout his career, and to which he returned many times, in particular in 1941 [56], 1975 [64], 1984 [66], 1987 [69] and 1988 [70]. But one cannot say that he ever really succeeded in being heard by the community of national accountants, except for some rare research workers. The initiative of the organizers of the present conference in devoting a session to the problem of balancing national accounts must therefore be greeted with pleasure.

I propose to present, from a critical point of view, Richard Stone's ideas on the consistency and balancing of national accounts, by emphasizing those elements that bring me closer to or move me further away from Stone's theses. I will finish by discussing the reliability of national accounts, setting forth some of my personal views on the matter,

and, at the end, sketching some possible solutions to these problems, which are obviously very complex.

## 2. The Problem of the Consistency of National Accounts

### 2.1. The concept of articulation in Stone

It is because national accounts are supposed to be consistent that there is a genuine problem involved in balancing them.

For Richard Stone, national accounting is, first of all, a system of balanced accounts. This idea comes across, in particular, in his Nobel Prize reception speech in 1984. Stone cites there the very first works on national accounting, that of William Petty and that of Gregory King, and then says: "*After this brilliant start all thoughts of balanced accounts seem to have evaporated*" [67], p.9; Stone does not pick up the thread of this history again until he comes to the Soviet work carried out in 1924 by the team directed by P.I. Popov, the then President of the Central Statistical Department [52] (Stone does not mention the work completed by Lavoisier in 1791, but that is explained by the old and general misunderstanding which surrounds Lavoisier's well-known essay [24]).

In Stone, the theoretical foundation of overall economic consistency is, undoubtedly, the general economic circulation diagram, in particular, that of Keynes. The guiding idea of this diagram is that of a flow without leakage from one vertex to another; but a second, and hardly obvious, principle is that the total of all flows entering a vertex is equal to the sum of those leaving (one must be careful with analogies from physics). From which comes the concept of *articulation* defined by Stone as follows: "*Dans un système fermé de comptes construits sur cette base, chaque poste correspond exactement à un autre poste du système, puisque chaque transaction implique un paiement et un encaissement. On peut dire qu'un système de ce genre est articulé*" [59] p. 29 (see also [57] p. 29, [47] p. 45, [58] p. 5, etc.). In short, articulation is using the principle of double entry to write down a set of flows characterizing the national economic activity for a given period; it amounts to applying the accounting procedure of double entry to the general economic circulation diagram. In this sense Stone is really the inventor of the *accounting* approach to national accounting. Another way of understanding this is to consider national accounting as a totality of individual accounts (see § 5) [16].

The idea of articulation is present in Stone's writings as early as 1941 [46], although it is barely even assumed in the first British accounts (those of 1941 [76]). It becomes completely explicit in the second series of British accounts (which began in 1947 [77]) and in the first American accounts, which began at the same period [73]. It was set up as a paradigm in 1952 in the United Nations handbooks [49] and in those of the Organization for Economic Cooperation and Development (then the O.E.E.C. or Organisation for European Economic Cooperation) [50],

[61]. The Dutch work (1940/1941) must be mentioned here, as it is also based on articulation [74], [75].

"Articulation" is only the first and principal component of Stone's notion of consistency (corresponding to what the Soviets called "accounting equilibrium" – *sbalansirovanost*). The second component is the Keynesian equations (resembling the Soviets' "aggregates proportionality" – *proporsional'nost*) and is harder to understand. First, let us see the problem.

Once all flows have been recorded, using the double entry rule, in a single account, that of the nation, this global account is articulated and, as a consequence, it is *necessarily* balanced in income and expenditures. Stone divides this overall account into five subaccounts connected by these flows. Obviously the goal of this procedure is to display the Keynesian aggregates and the famous equations linking them (Keynes *dixit*) by adding up the different subaccounts.

But why should each of these five accounts be automatically balanced in income and expenditure? There is a simple reply to this question: Let us balance (in the usual accounting sense of the word) all the subaccounts except for one (say the last one, that of "national investment"), set up the four resulting balances as flows, and post the four corresponding counter-entries to the last subaccount, which will then necessarily be balanced in income and expenditure (the fifth balance is necessarily zero). The five accounts are thus balanced *by construction*: This is doing for the flows, at the level of the account of the nation, what one does for entries in the daybook at the level of the overall account of a company (if you like: a one account ledger) [16], [9].

The idea of accounts balanced by construction constitutes one of the guiding principles of Stone's approach; but it is difficult to maintain that Stone adheres explicitly to the formulation that I just made. The fact is that Stone holds the Keynesian equations as absolute truth (but without actually saying so, as far as I know): an implicit postulate which for him takes the place of a demonstration. But unlike Keynes, Stone is not only a logician of national accounts, but also a statistician [60], and it is at this point that logic deviates from statistical reality. Indeed, Richard Stone is not satisfied with a simple hypothesis of the existence of flows; he intends as well to really observe them, while understanding clearly that: "*L'adoption d'une méthode de comptabilisation implique celle d'une méthode systématique de rassemblement des informations relatives aux totaux généraux du système*" [57] p. 26. However, *the metrological base* (the set of primary sources, the microdata base), adopted by Stone, *is that of individual accounts*; this naturally leads Stone to postulate the existence, at a theoretical level, of a universal accounts system enabling the mailing of a standard questionnaire to each individual economic unit [59] p. 53 (giving rise to problems of aggregation that I will not discuss here [16]).

Consequently, all the flows of Stone's (sub-)accounts (now including the accounting balances) have to correspond precisely to concretely observable accounting microdata. In particular, the "balances" could no longer be regarded as pure accounting balances and must now be

completely and solely justified by individual accounting data; otherwise, they would only be adjustment macrocategories, without a very clear microeconomic significance, and thus nothing more than the results of the usual accounting balancing, which we will later discuss at length.

## 2.2. A critical examination of the Stone theses

A simple solution to all these difficulties would have been to choose a basis for observing the flows where each flow is measured only once; then to adopt a suitable nomenclature for flows, allowing one to build five (or more) sub-accounts, possibly described as Keynesian, then to simply balance them, in the accounting sense of the term. The balances would then have a purely macroeconomic meaning, as a "semantic algebra", disconnected from any microeconomic meaning [23], [22], [17], [14], [11]: in such a way one could build "functional" accounts, for example.

This is not the solution adopted by Stone, nor by many others afterwards, among them the first American national accountants (from this arose the controversy with Simon Kuznets who considered flows as measured only once, as in the old measurements of isolated aggregates, and, consequently, did not see the relevance of presenting traditional national income statistics in an accounting framework, which he considered a bit pedantic in the absence of its adding any new information – in this Kuznets was both right and wrong [41], [39]).

There are two major reasons for these mistaken ideas. The first is a difficulty in distinguishing between "defining" and "observing" something which is only defined. Thus, the S.A.M. (Social Accounting Matrices) formalism indeed gives one the advantage of representing by only one entry what in accounting requires two. But does such a formalism also mean reducing to only one observation something that normally requires two? This is the crux of the ambiguity of matrix representations in national accounting (see [63], [71], [72]).

The second, of particular importance, is ignorance of the phenomena of aggregation. In particular, Stone supposes that the passage from micro- to macrodata is carried out by simple addition, in a harmonious way, with consistency preserved at each step (in connection with which see the difficulties uncovered by Allais, beginning with the first pages of [1]). Indeed, Stone wants disaggregation "... *to show the precise relationship between the individual transactions of the system and the aggregates which are represented and found useful in economic analysis*" [58] p. 5. And he sticks to *the postulate of infinite divisibility* that he states as follows: "*The system itself may be elaborated indefinitely by increasing the number of classes of transactor and transaction until each transactor and transaction has a class for itself*" [58] p. 5 (see also [67] p. 18 and [45] p. 167). In short, in this passage Stone sees a *link* [59] p. 27, where I would see a *break* [23].

And now the question is: is it possible to observe each flow in a one-to-one manner (i.e. without ambiguity) in the books of the individual economic units? My reply to this question is negative. Indeed, an economic operation, in the common sense meaning of the term, is



always a complex chronology of events, which can extend outside the period under consideration and give rise to accounting entries in two or even several individual accounts [9], [7], [6]. These entries can vary in number and in kind for the same type of operation; they are obviously not synchronous, nor are they all relative to the period under consideration. In short, a flow cannot be defined in terms of a standard succession of standard entries. Or, if it is defined in that way, it is only a convention. In other words, if a flow is defined by two standard accounting items, one of which is the source and the other the target, the contents in terms of economic operations of these two items will be qualitatively and quantitatively different. What has just been said for a flow is *a fortiori* true for a *biflow* (or *economic operation* [3] p. 779, or *quadruple entry*), i.e. for the entity formed by the flow and its counterpart [16] p. 176.

Therefore, if a flow is measured twice, at two fixed and predetermined points in the accounting, one generally obtains two different values, each one denoting a different and protean microeconomic reality. For this reason, Stone's system of national accounts is fundamentally not articulated, and his national accounts, have to be adjusted, even in the absence of observational errors, because they are essentially inconsistent. By the way, the same conclusion may be drawn, for the same reasons, about any system of national accounts which aims at combining two radically incompatible approaches, that of flows and that of individual accounts (I have in mind the French system of T.E.E., *Tableau Economique d'Ensemble*, and the accounting system of Keynes' *General Theory*).

In brief, Richard Stone's demonstration of consistency needs to be re-examined and improved at several points.

### 2.3. The problem of the "Economic State of the Nation"

What is the reason for my criticism of Stone's demonstration of the consistency of national accounts, since, obviously, observation errors alone are sufficient to explain the inconsistency of the input data and thus the necessity for adjusting them? The reason is the following: if the accounts were consistent by their very nature, the national economic reality described by them would at all times be characterized by one and only one system of balanced accounts. That is to say, there would exist a single "true" (multidimensional) value  $y$ , both consistent and to some extent "tangible", which one should try to approximate as closely as possible. This notion has given rise to the widespread idea that the consistency of the accounts allows one to improve upon the precision of the observed data, which are always unbalanced; or the equivalent idea, that the more precise the initial data, the more balanced will be the raw national accounts (where "raw" means "unadjusted").

That is exactly what Lavoisier implicitly believed and, in the last analysis, that is exactly what Stone and most national accountants after him implicitly think (I might paraphrase Albert Einstein by saying: "do not listen to what national accountants say, but rather look at what they do").

And they act as if there were only one balanced system of "true" aggregates, aggregates so real that a good national accountant can "feel" them). That is why Stone sees in the problem of the reliability of national accounts only a metrological problem of the traditional kind, and that is also why he is never able to formulate the question of the reliability of national accounts in all its breadth and complexity.

If on the contrary, as I believe to be the case, the accounts are basically unbalanced, there is no longer a unique "true" and consistent multidimensional value, directly and concretely observable and thus "tangible". The nature of the problem of balancing (and reliability) now changes, since one must first settle the question of defining, or of agreeing upon, what is the "true" quantity to be observed. This is a crucial issue for choosing the right method for adjusting the raw data, i.e., the right method for balancing the accounts. It is equally crucial for the reliability of national accounts, since "error" is a discrepancy between the observed value (raw) – or, maybe, the adjusted – and the "true value". This alternative point of view, that the raw accounts are always unbalanced is the one which I adopt here, and consequently my approach will no longer be quite the same as Stone's.

### 3. The Problem of Balancing National Accounts

#### 3.1. Richard Stone's approach

Stone encountered the problem of balancing national accounts starting in 1941. The solution adopted in the Cmd 6261 White Paper is very pragmatic: The accounts are balanced in income and expenditure by introducing an adjustment category, for the year 1938, and, for 1940, by closing the accounts with the item "Internal and External Net Investment". That is as far as the Stone articulation goes. But now the accounts are explicitly unbalanced in the sense of Keynes, since savings are not equal to investment (see [76], p. 13, note 1 of Table C): this is the starting point for what was subsequently called the "Statistical Discrepancy", since  $R - Y = S - I \neq 0$ .

Beginning at the same date, 1941, Stone took up, this time at a theoretical level, the problem of balancing the accounts in detail. Since in his thinking adequately defined and properly measured aggregates without errors would lead to perfectly balanced accounts, the problem of balancing accounts reverted to the traditional statistical problem of estimating a quantity by using an observed value. Thus, Stone naturally resorted to least squares [56], [66], [70]. In such an approach, the more precise the observed aggregates, the closer one comes to the true value, i.e. the closer the system of observed aggregates is to being balanced. In other terms, the adjustment problem is that of making the best possible observation of a quantity, and then giving it a little push by means of least squares. That this is Stone's thinking is confirmed by what he wrote when envisaging the possibility of adjusting aggregates

"by balance" to make them more accurate, provided that "...the measured items are substantially accurate" [70] p. 1, [68] p. 455.

In short, Stone would have liked to proceed as the physicists were already doing at that time, when they improved the consistency of their universal constants using least squares (with, however, at least one difference: the physicists do not consider the relations connecting physical constants as absolutely true, whereas Stone and many national accountants consider the accounting equations to be necessary truth [32]).

It is at this point that Stone's work deserves to be studied and deepened, notably by clarifying the hypotheses he used, in order to find a better justification for the use of least squares in the special domain represented by national accounting. Why here, in particular, should least squares yield a better estimate of aggregates? Is it because of the greater precision of observations (as Stone thinks) or because of the large number  $n$  of these observations? Here the snag is that  $n$ , if it is indeed large (200 to 300, for the usual tables of national accounting), is not the number of independent observations of the same quantity, but a single observation of the  $n$  components of a vector  $x$  representing the national accounting table we are trying to estimate. Stone was aware of this difficulty, and clearly realized that it was not a Bernoulli problem he was dealing with, but that he had to work within a Bayesian context (although he was not on this track at the beginning, as shown by the analogies he made in his 1942 text [56] p. 114) [64] p. 60.

But even now, Stone has scarcely been heard. Surprised by this lack of comprehension, Stone wanted to know why, and he examined the reasons why many people remained opposed to a method of adjustment by least squares [64] p. 59 ff. To tell the truth, his detractors' objections may just be pretexts, concealing (as it is often the case) something more profound, something which it was difficult for Stone to see or, at least, to refute, owing to his theoretical preconceptions, shared until now by the majority of national accountants. My feeling on this question is as follows: it is necessary to distinguish prejudices against an adjustment method which is imposed *a priori*, and prejudices against measuring the reliability of accounts. The two are undoubtedly linked, though in a manner which should be clarified. With regard to reliability, the reservations are numerous [15], in particular in America (see, for example [40]). Let us quote Oskar Morgenstern: "[This reluctance to put error computation into practice] either shows that one is lacking in clear ideas and procedures or does not dare to use them since they would show up the tremendous limitations of the figures which the government itself uses freely in the pursuit of its business " [48] p. 253. It is obvious that one unreliable procedure can bring to mind another. With direct reference to arbitration (the term usually employed in France for balancing the national accounts), the refusal to use a method imposed *a priori* may be a refusal to use uncontrolled variables which do not fall within specified limits – a phenomenon which does not fit into Stone's theoretical framework ("variables outside specified limits" are adjusted aggregates having "inadmissible" values: see § 6.2). Indeed, national

accountants prefer "empirical" balancing methods (sometimes more or less confidential), whether or not assisted by computer, precisely because they avoid creating such variables. This as a matter of principle, because one thinks that an experienced national accountant has a better feeling for the "right" value (consistent, of course) than does any "blind" mechanical method. In France, as already noted, the name "arbitration" is used for such adjustment methods, and that for good reason. A second reason, more sociological, would seem to militate against using a fixed *a priori* balancing method: the adoption of such a method would modify considerably the work of national accountants and, therefore, the way it is organized.

But one might ask: could not an arbitration method be codified until it itself became an *a priori* method? Let us now examine this possibility.

### 3.2. Another approach to the problem of balancing National Accounts

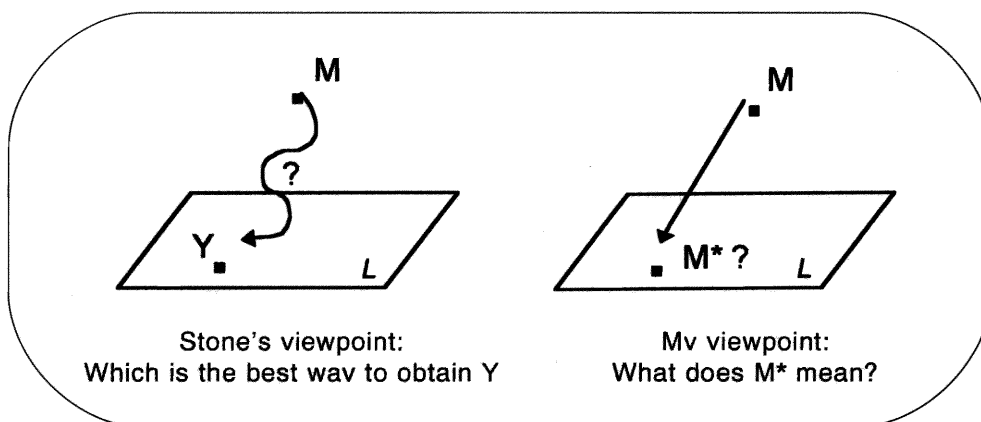
In 1968, as a technical adviser in Cameroon, I had to solve the problem of handing over my office to my successor. More precisely, that involved codifying the empirical balancing method used previously (the "manual" method, as one called it in Africa at that time), so that my successor could employ it again in the same way, thus ensuring the continuity of arbitrations (and that of the series of the accounts itself, since it is by now obvious that the definition of an accounting table remains incomplete until the method used to balance the table is specified).

Using a computer solved the problem of providing for a convenient and faithful transmission of instructions. However, it very quickly turned out to be difficult, if not impossible, to codify the procedure (except by using the method criticized in 4.3). The programming problem thus turned into the search for a simple algorithm; for example, projection using "generalized least squares", as Malinvaud called it [44]. In fact, I was rediscovering the method proposed by Stone in 1941 (though at the time I was unaware of his 1942 text [56]), but as a matter of pure expediency, since all methods seemed to me to be equally valid, in the absence of a "true quantity" to be estimated [5], [8], [12], [4].

Could one and can one justify using the method of least squares without making the hypothesis that the national accounts are unambiguously and exactly balanced? Yes one can, by thinking of it as a problem of description rather than estimation: what is the consistent point  $y$  which best represents the observed point  $x$ ? At a purely geometrical level, the method of least squares answers the question by giving the "nearest" point, in the Euclidian sense of that term. By postulating the vector  $x$  of the  $n$  observed aggregates to be a random vector in  $n$ -dimensional space  $R^n$ , with a mathematical expectation  $y$  situated in the linear manifold  $L$ , defined by the accounting equations, I thought I could provide a better justification for using least squares [12]. This amounted to assuming that the set of all individual entries for the period in question, for the whole country, was structurally random but on average consistent: the accounting state  $y$  representing the national

economy, then became an abstraction, while the tangible, measured reality itself remained inconsistent. This was an assumption I did not make without some hesitation, for I considered this (wrongly) as a lazy way to justify using a traditional formula.

In short, my approach was formally that of Stone, with an incidental use of least squares, but it was fundamentally very different: for me, the state of the economy as represented in the national accounts is either simply a convention or else a very abstract reality governed by a law of probability which only implies consistency on average. My hesitation arose from my reluctance to look at national accounting as nothing more than a simple convention, and an equal reluctance to avoid that by making an *a priori* hypothesis, a convenient hypothesis rather than a working hypothesis, as long as it lacked justification.



#### 4. Which Arbitration Method?

##### 4.1 First a problem formulation

The preceding discussion of the parallel between Richard Stone's approach and mine highlights the complexity of the problem of arbitration, which initially requires a precise formulation rather than a quick solution. Let us examine it in greater detail.

##### 4.1.1. *The national accounts as actually observed are never balanced*

The essential point to note from now on is that once we admit that the national economy is never balanced (in the sense of national accounts), the problem of balancing the national accounts cannot be viewed as searching for a "tangible" equilibrium which really exists, by simply improving the precision of the raw data. In fact, the problem is basically one of representation, and thus requires an adequate formulation (throughout the present section, aggregates will be supposed to be judiciously defined and observed without error).

#### 4.1.2. A basic model of balancing

Let  $x$  (corresponding to the point  $M$ ) designate the vector of  $R^n$  whose  $n$  components  $x_i$  are the  $n$  observed aggregates which constitute the national accounting table we want to balance. The equations which describe the accounting consistency of this table (linear consistency) define in  $R^n$  a linear manifold  $L$  which passes through the origin  $O$  of the space. In general, the observed vector  $x$  will be outside  $L$ ; this is the essential difference between Stone's approach and mine. In short, since one cannot or does not want to do without a consistent overall description of the national economy, the problem of balancing the actual accounts  $x$  is equivalent to choosing in  $L$  the point  $x^*$  (or  $M^*$ ) which "best" represents the point  $x$  (or  $M$ ). The choice of the "best" balancing method is in this case the choice of the "best" correspondence  $F$  which, in  $R^n$ , carries point  $M$  to the point  $M^*$ :

$$x^* = F(x).$$

Let us suppose that the linear manifold  $L$  is defined by  $m$  independent linear equations linking the  $n$  constitutive aggregates of the national accounting table. The equation for  $L$  is then  $AX = \theta$ ,  $A$  being the  $m$  by  $n$  matrix (assumed, for simplicity, to be of maximum rank) of the parameters appearing in the equations linking the aggregates,  $X$  the column vector corresponding to  $x$  and  $\theta$  the column vector of  $m$  zeros. We want  $F$  to be defined everywhere and single valued, so there is one and only one solution  $x^*$  for a given  $x$ . Then in matrix terms the general problem of balancing the accounts is reduced to solving the matrix equation:

$$\{AX = \theta, \quad (1)$$

whose general solution is

$$X^* = (I - A - A)Z(X), \quad (2)$$

where  $A^-$  is any generalized inverse matrix of  $A$ ,  $Z(X)$  an arbitrary vector of  $R^n$  which depends on  $X$ , and  $I$  the unit matrix. Of course,  $X^*$  is the column vector associated with  $x^*$ .

This solution is too general because one usually wants the discrepancy  $E = X - X^*$  to appear explicitly, i.e. to solve the system:

$$\begin{cases} AX^* = \theta \\ X = X^* + E. \end{cases} \quad (3)$$

The general solution of this system is now:

$$X^* = (I - A - A)X + (I - A - A)U(X), \quad (4)$$

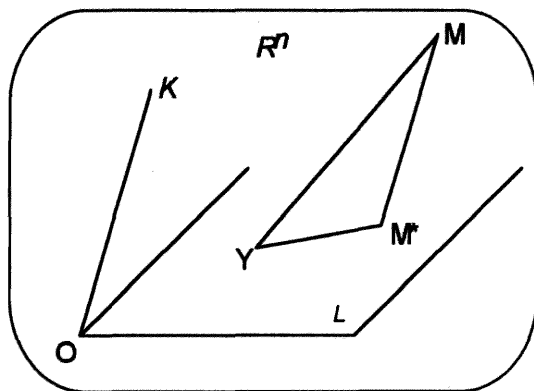
where  $A^-$  is a given (arbitrary) inverse and  $U(X)$  an arbitrary vector function of  $X$ . (one can employ a unique inverse matrix  $A^-$  by using the formula:

$$I - A_0 - A = (I - A - A)(I - A_0 - A),$$

valid for any two inverses  $A^-$  and  $A_0^-$  of  $A$ , since  $A - AA_0^- - A = A - A$ , by definition of generalized inverse).

### 4.1.3. Remark

This outline of the process of balancing makes us prefer to speak of *balancing* rather than *adjusting* the national accounts. Indeed, it implies a global balancing, with the "adjustment" of each individual aggregate a consequence of the overall balancing. If one looks at the process of balancing employed by Stone, one can regard it as a succession of adjustments of the different flows (each time harmonizing one debit with one credit), disregarding Keynesian consistency.



$$L = \text{Ker } A \bar{A} = \text{Ker } A$$

$$= \text{Im } (I - A \bar{A})$$

$$K = \text{Im } A \bar{A} = \text{Ker } (I - A \bar{A})$$

$$R^n = L \oplus K$$

## 4.2. Linear balancing by projection

The general formulas (2) and (4) give mathematical expression to the balancing methods which can be employed; they say nothing about the criteria for choosing one method rather than another. To do this one must specify what one means by the "best representation".

### 4.2.1. Geometrical approach

A first solution, very intuitive, consists in using the point  $M^*$  of  $L$  which is "nearest" to the observed point  $M$  (this is simply the original idea of Gauss on approximations). This purely geometrical intuition leads, within ordinary Euclidean space, to projecting  $M$  orthogonally onto the linear manifold  $L$  using the following formula:

$$X^* = (I - A^0(AA^0)^{-1}A)X. \quad (5)$$

In a Hilbert space, the formula becomes

$$X^* = (I - G^{-1}A^0(AG - 1A^0) - 1A)X, \quad (6)$$

where  $G$  is the positive definite matrix which determines the scalar product (or norm) in the Hilbert space. Here  $A^0$  denotes the transpose of the matrix  $A$ . It is this method I used spontaneously in 1969.

### 4.2.2. Probabilistic approach

Since reality is inconsistent, the need for consistency leads us to model it. Therefore, a possible solution consists in assuming that  $x$  is a random  $n$ -dimensional vector which has a mathematical expectation  $y$

and a covariance matrix  $Q$ , which we take to be positive definite. The problem of choosing a balancing method then becomes that of finding the best estimator  $x^*$  of  $y$ .

In particular, one would like an unbiased estimator:  $E(x^*) = E(x) = y$ ; this in practice excludes formulas of type (4) and leads us to prefer those of the type

$$X^* = (I - A - A)X. \quad (7)$$

These formulas comprise all the linear projections of  $R^n$  onto the linear manifold  $L$ . They also seem to provide an intuitive geometrical solution, but this is only true if the discrepancy between  $M$  and  $M^*$  is minimum in a specific sense. This is the case in Hilbert or other normed spaces, where there exists a single solution  $x_0^*$  minimizing  $\|x - x^*\|$ , and formula (7) establishes a link between such norms and the inverse matrices of  $A$ .

The formula of least squares,

$$X^* = (I - QA^0(AQA^0)^{-1}A)X, \quad (8)$$

where  $Q$  is the covariance matrix of  $x$  and where  $QA^0(AQA^0)^{-1}$  is an inverse in  $A\{1,2,3\}$  (Penrose notations, cf. [34] p. 125-126, according to [51] p. 406-413), responds particularly well to the two previous requirements.

Indeed, from the probabilistic point of view, this is an unbiased estimator of  $y$ , with a minimum dispersion in a certain well defined sense [44] p. 177, within the class of all centered linear estimators of  $y$  having the property that  $AX = \theta$  implies  $X^* = X$  (all the projections of the form (7), as well as  $x$  itself, are estimators of this class).

One notes that from a geometrical point of view,  $\|x - x^*\|$  is a minimum with respect to the norm  $G = Q^{-1}$ . Incidentally, from a probabilistic point of view, the important thing is a minimum dispersion and not a minimum of  $\|x - x^*\|$ ; but here the two coincide. This is the second reason which led me (not spontaneously this time) to use, or rather to justify the use of least squares in 1969. However, saying that the method is the best among many others tells us nothing about its actual precision, unless we declare ourselves resolutely Bayesian and clarify completely the *a priori* law for  $x$  – which is a delicate task.

Note, finally, that the method of least squares establishes a link between the precision of the aggregates and the balancing method – a link which is obviously not essential, and which can even be criticized in certain respects.

There are other projections of type (7) which characterize methods with a rather different motivation from that just described. The two which follow are of particular interest.

#### 4.3. Empirical balancing, or "arbitration"

I explained earlier (cf. § 3.2) how a concern to escape from "manual" balancing methods, which are not easy to transmit (and are thus likely to fluctuate from one year to the next) led me to seek methods which



were simple, explicit, and given *a priori*. That it be given *a priori* serves to guarantee that the method will be objective; that it can be transmitted is a necessary practical condition for this objectivity. In order to be transmitted, the method has to be, at least in part, partial recursive [43]. The uniformity as a function of time of a series of successive balancings is a less evident notion, and the following example will help to clarify it.

For a benchmark year, input data  $Z$  are balanced to obtain balanced accounts  $Z^*$  using a complex empirical method, not very explicit, or at least difficult to make explicit. In order to maintain homogeneity in time, one decides to balance the accounts for the following years according to an "as before" balancing formula which consists in projecting onto  $L$  parallel to the vector  $U = Z - Z^*$  (a simple and radical way to clarify *a posteriori* any empirical method). The formula is particularly simple and attractive:

$$X^* = (I - U(AU)^{-1}A) X, \quad (9)$$

where  $A^- = U(AU)^{-1}$  is an inverse in  $A\{1,2,3\}$ .

It is important to note that this adjustment method is valid if and only if  $L$  is a hyperplane.

The lesson that one can learn from this example (which in the end seems naive) is that the use of an unknown balancing method, however tempting, requires that there always be a solution, i.e., that the function  $F$  be everywhere well defined [30], p. 8-9. In summary, a method can give a solution for one year but not work the following years; as a result, a new *ad hoc* method must be used each year.

Now, the point is that *all empirical methods are like the "as before" method, i.e. they are not defined everywhere*. It is easy to show that: Being given any empirical method, suppose it be defined everywhere; it then should work with any vector  $x$  and, in particular, with  $x$  being observed without error. Since  $x$  nevertheless remained unbalanced, out of specified limits variables should necessarily appear, while all empirical methods intend to suppress such variables; the method, therefore, is not defined everywhere Q.E.D.

This is probably the explanation of the "erratic" character of "empirical" methods, because one can doubt whether a succession of such methods, adjusted each year to fit the circumstances, could ever be synthesized into a single method, given once and for all and capable of being transmitted, which for each year would coincide with the empirical method used that year.

#### 4.4. The Stone-Chaumont balancing method

##### 4.4.1. The principles of the method

The method I called "Stone-Chaumont" [36] is of the same type as the two discussed previously, (the inverse  $A^-$  is still in  $A\{1,2,3\}$ ). Its formula is [31]:

$$X^* = (1/p) (\sum_{k=1}^p (I - P(k) J(AP(k) J)^{-1}A) X, \quad (10)$$

where  $J$  is an  $n \times m$ , matrix consisting of an upper block  $I_{mm}$  and a lower block  $\theta_{n-m}$ , and where  $P(k)$  is a suitable permutation matrix of  $I_{nn}$ .

Stone envisages such a method as simply a palliative for estimating missing data ("missing entries") [70] p. 1: this is often the method used by national accountants. However, it is possible to go much further. Thus, Chaumont takes up the method from a different certain and calculates the  $m$  remaining variables by means of the consistency equations, repeats this process  $p$  times,  $k = 1$  to  $p$ , counting different sets of variables as certain and then calculates the average of the results so obtained. This is nothing but the generalization of adjustment by ordinary accounting balances (mentioned several times by Stone) [30] p.10-11, [36], p. 7 to 11, [31], p. 13 to 15.

#### 4.4.2. The theoretical ambition

Two ideas underlie the method just described, ideas developed by Chaumont in [35] and [36]. From a technical point of view, the first involves applying something which looks like a bootstrap technique to the (single) observation  $x$  (this approach is completely opposite to Stone's Bayesian approach [64] p.60). The Stone-Chaumont method allows  $p$  duplicates of  $x$ , with  $p$  large if  $n$  is large ( $p$  is at the most equal to the number of combinations of  $n$  objects taken  $m$  at a time). The balance point  $x$  is the average of these duplicates of  $x$  ( $k = 1, 2, \dots, p$ ), while the cluster of the  $p$  duplicates  $x^*(k)$  permits a study of the dispersion of the "experimental" distribution thus created.

The second idea is much more general. Earlier I tried to show that the hypothesis that the national accounts aggregates can be perfectly observed (i.e. without error) is incompatible with the hypothesis that they can be perfectly modelled (i.e. they are already consistent at the start). Chaumont chooses to retain the idea of a perfect observation; i.e. he thinks that each economic unit has its own *rational* and *certain* vision of national accounts (that is to say, without error), implying that the postings of all the other agents should conform perfectly to this individual rationality, which is evidently not the case. Thus, Chaumont encounters the problem of aggregating  $p$  individual points of view into a collective one, and he solves it by using an average. This solution is "dictatorial", in the sense of aggregation theory, but is not entirely arbitrary since Chaumont is constantly guided by certain Bernoullian mental reservations.

#### 4.5. By way of conclusion: a problem of aggregation

Thus, the problem of balancing national accounts can receive various formulations: the search for "the nearest" consistent point  $M^*$  (i.e. national accounting is, after all, only a judicious convention) – the search for a unique state of the national economy which really exists (Stone, followed by many national accountants) – the estimation of some hidden characteristic of the behaviour of aggregates – or a straight out

agglomeration of a plurality of individual perceptions to form a rational global view of the national economy (for example, Chaumont).

One can add to these proposals yet another which is more "linguistic", which I formulated in [23]: national accounting as just an organization of social discourse, or at least an aid thereto. The balancing of accounts is then both a semantic break between the micro- and macroeconomic levels and the creation of a new semantics at the latter level. From this point of view, balancing is a means of overcoming an unavoidable paradox. Such a statement could lead to extreme conclusions; it is moderated by adding that a genuine language does not have to be completely disconnected from reality. What is involved is a credible modelling of reality and "true" speech; relative to this last remark, recall that *all statistics are false: they are only more or less precise*. This brings us to that other very neglected problem: the reliability of national accounts.

## 5. National Accounting as an economic model <sup>(1)</sup>

### 5.1. Models already considered

National accounting, considered in terms of the equations which express its consistency, already constitutes a model for the national economy, simple but excellent (it seems that input data, despite their all too numerous imperfections, are globally very close to an accounting equilibrium [5], [8], [12], [28]). If many people are not yet convinced about this point, it is because they wrongly consider national accounting equations to be "identities".

Richard Stone's national accounting is really a model, by means of flows, of the general economic circulation paradigm. It is an attempt to give a microeconomic explanation of the surprising overall consistency of the accounts, regarded as a consistency of the national economy. But this attempt stumbles over the difficulties of aggregation that Stone seems to have deliberately ignored [13], [14], [10].

A related model is that of a national economy thought of as characterized by a set of biflows. One thus tries to reconcile the paradigm of circulation and that of individual accounts. National accounting then becomes "quadruple entry accounting" [16]. Such a model was proposed in one of the International Monetary Fund handbooks thirty years ago, if I remember correctly. This model is particularly well adapted to the analysis of flows and of their counterparts, especially monetary (Irving Fisher's equation of exchange and the velocity of circulation of money) [6], [7], [12].

Modelling the economy solely by giving its consistency and the method of balancing is superficial, as it gives no criterion for defining

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(1) For further details see my "Introduction à la comptabilité nationale: qu'est ce que l'économie nationale?". Editions Ellipses, Paris 1995 (chapter 12).

"true values". However, it has the virtue of recognizing the difficulty of passing from the micro- to the macro-economic level. The probabilistic model of § 4.2.2 does not go any further, except for postulating a fluctuating economic reality which is consistent on average. If this postulate is true, one still needs to explain why it is true.

Finally there is the traditional model of national accounts as the sum of the corresponding individual accounts (one could cite here, among many others, Alfred de Foville [38], Lenin [16] p.165, etc.). It is a model which would like to link national accounting to individual accounts in a concrete way, and validate the "accounting" consistency of national accounting by the double-entry consistency of individual accounts. The Stone-Chaumont method suggests a particularly original model which takes up and extends these traditional ideas (ideas which if not modified remain somewhat naive).

## 5.2. Models based on individual accounts

From now on, the existence of a universal accounting system of accounts (i.e. universal accounting standards for all economic units *and* the national accountants) will be assumed – an assumption also made by Stone. Thus, if all the accounting entries were entered simultaneously by all the economic units in a perfectly symmetrical way, the national accounts obtained by simple addition would then be perfectly in balance in income and in expenditure, by agent (or sector) and also in the sense of articulation defined by Stone. This hypothesis, purely theoretical, is also the basis for national accounting using biflows.

Let us suppose that, at the level of an individual economic unit  $k$  ( $k = 1, 2, N$ ), the actual entries  $k$  makes in its books determine, at once and without ambiguity, all the corresponding (fictitious) entries which the other  $N - 1$  units should have entered in their own books. Then, the summation of these real or fictitious, but always symmetrical, entries will give perfectly balanced national accounts  $x^*(k)$ , each representing the consistent view of the national economy that unit  $k$  maintains for its own sake – its own view of its own immediate economic reality.

It is now a matter of aggregating in a suitable way these  $N$  individual points of view in order to obtain a consistent collective point of view which can be thought of as the national accounts  $x^*$  of the period in question. One knows that there is no universal solution to this problem, unless one makes clear what one understands by "suitable"; a clarification without logical inconsistency is needed, of course, in order to avoid the difficulties encountered by Arrow [33]. This is why we will consider as "suitable", from a statistical point of view, the use of a simple arithmetic mean to carry out the desired aggregation (no Quetelet paradox here).

Let us recapitulate. From this perspective, the reality corresponding to the national economy is the population of all the  $N$  individual points of view  $x^*(k)$ , i.e. a concrete statistical distribution to be described and analyzed by the usual statistical methods. In particular, let us characterize this distribution by a simple arithmetic mean  $x^*$ . It is then natural in these circumstances to consider the average over that

population as the "true value" which the national accountants try to measure:  $y = x^*$ . To be sure, this "true value" is only a convention, but it is a convention which corresponds to a precise experimental reality.

As  $N$  is obviously quite large, we will, if necessary, study the population of viewpoints  $x^*(k)$  by sampling, using a sample of size  $p$  as large as we need in order to calculate the mean  $x^*(p)$ . The next step, of course, will be to study the asymptotic behaviour of the unbiased random variable constituted by the sample mean, as  $p$  gets larger and larger, since we are now in a Bernoullian context.

Thus, the model just described makes it possible to pass from a Bayesian to a Bernoullian context – a difficulty which had me baffled (and apparently to some extent, Richard Stone as well). Of course, the hypothesis that an individual can by his own entries determine those of all other agents cannot be correct. Nevertheless, this idealization helps one better understand the impact and significance of the following model.

### 5.3. The Stone-Chaumont model

The Stone-Chaumont method is an application of the more general model which has just been briefly described. In this method, a national accountant does not, for obvious reasons, start at the level of individual accounts, but instead considers a very aggregated level, for then the "universal accounting plan" exists and is known, as it is constituted by the system of national accounting equations, and thus the calculation of "fictitious entries" becomes possible. The individual points of view are now those of all possible combinations of  $m$  producers of aggregates, taken from a total of  $n$  such producers. As  $m$  is very small in comparison with  $n$  ( $m/n$  is about a tenth for traditional national accounting tables which comprise between 200 and 300 aggregates in general), the number of these points of view is very large. One can therefore apply the procedure of § 5.2, *mutatis mutandis*, in a workable way.

This method is, therefore, very convincing. It is less convincing, however, when placed in an unfortunate combination with the probabilistic model of § 4.2.2(8) [36]. Indeed, one can easily raise a false problem of convergence of one estimator towards another, when one considers simultaneously the Gauss-Markov least squares estimator and the Stone-Chaumont estimator. There is a risk of logical inconsistency if one forgets that the "true" Gauss-Markov value, which is Bayesian, is not necessarily identical to that of the Stone-Chaumont method, the latter being "experimental" and *a posteriori*. However, a dialog between the two methods

might make it possible to develop a procedure to progressively refine the margins of error in the observations of the aggregates. Estimates of many of these margins of error can only be very rough at first (these margins are "subjective", a word which is the spearhead of objections raised against all attempts to estimate errors in national accounting [40]).

In conclusion, note how an attempt to be rational at the macro-economic level seems to lead to a non-existent determinism at the micro-economic level: for a unit by itself does not determine all of the

postings which are actually made in the country during the period under study.

## 6. Introducing the Problem of the Reliability of National Accounts

The problem of balancing national accounts is closely related to that of reliability through the concept of a "true value". Indeed, a definition of this value  $y$  is necessary in order to be able to choose the method of balancing in a rational way. It is also necessary for defining the overall error made in computing the accounts.

### 6.1. Basic indicators of the reliability of national accounts

#### 6.1.1. Definitions

Having defined what one must understand by the "true value"  $y$ , one can consider two kinds of overall error in national accounts: the vector  $x - y$ , or *the overall error before balancing*, and the vector  $y - x^*$ , *the overall error after balancing*. It will be noted that the value of  $y$  is obviously unknown, and that we have:

$$y - x^* = (y - x) + (x - x^*),$$

or, with a given norm,

$$\|y - x^*\| \leq \|y - x\| + \|x - x^*\| \text{ (triangle inequality).}$$

One can then define the following indicators ([25], [29], [27], etc.):

$$\pi_0 = \|y - x\| / \|x\|, \quad \pi_1 = \|x - x^*\| / \|x\|,$$

$$\pi' = \|y - x^*\| / \|x\|,$$

The value of  $y$  being unknown, one can use the triangle inequality to define *the national accounts reliability indicator* by the formula:

$$\pi = \pi_0 + \pi_1.$$

This indicator is a conservative measurement of reliability, since  $\pi \leq \pi'$ , which corresponds to the usual rule of metrological prudence.

One can define another interesting indicator:

$$\|x - x^*\| / \|y - x\| = \pi / \pi_0,$$

This is *the centering indicator*, closely connected with the reliability indicator since:

$$\pi = \pi_0 (1 + \phi).$$

The centering indicator combines the individual accuracy coefficients of adjusted aggregates (see below variables outside specified limits).

Let us come back to the indicator  $\pi_1$ ,  $\pi_1 = \|x - x^*\|/\|x\|$ . If  $x$  were observed without error and if no particular assumption were made about  $y$  (which might even lack a precise meaning), this new indicator would measure the discrepancy between the "real" value  $x$  and  $x^*$  representing the "consistency determined by L and the associated method of balancing". That is to say  $\|x_i - x_i^*\|$  cannot automatically be regarded as a better margin which improves upon the initial raw margin  $dx_i$ , and which therefore might be substituted *a posteriori* for the latter.

### 6.1.2. Metrological convention

To be able to actually calculate all these indicators, we have to specify the norm used to define the "lengths" and, especially, make assumptions about  $CCx - yCC$ . This second point leads us to make the following assumption (or convention) corresponding to a principle of metrological prudence:

$$\|x - y\| = \|dx\|,$$

where  $dx$  is the vector with components  $dx_i$ , all of which are positive. The justification of this rule is as follows: Given a vector  $\delta$  whose components are  $\delta_i$ :  $\delta_i = dx_i \cdot \text{sgn } \varepsilon_i$  with  $\varepsilon_i = x_i - y_i$ , prudence leads one to suppose that  $\varepsilon = \delta$ , whence  $\|\varepsilon\| = \|\delta\|$ . For norms which are invariant under a change of the sign of their arguments, this is equivalent to  $\|\delta\| = \|dx\|$ . From which comes the convention:

$$\|\varepsilon\| = \|dx\|.$$

This rule arises from two ideas, the first is that the true value  $y_i$  is always in the interval  $x_i \pm dx_i$ , and the second that it is always the most unfavourable case which occurs:  $y_i$  is always the maximum possible distance  $dx_i$  from  $x_i$ .

### 6.1.3. Margins and the framing property of national accounts

The margin  $dx_i$  is here viewed as the national accountant's knowledge of the limits outside of which his figures are not allowed to fall [37] p. XII. This is nothing else than the *framing property* which is often attributed to national accounting, and implies knowing the margins of error with a probability equal to one. In other words, a producer of an aggregate is considered capable of giving, and should give, an estimate of the margin, even if this estimate is fragile.

## 6.2. Variables outside specified limits

Thus, let us suppose that every aggregate  $x_i$  is observed with a known margin of error  $dx_i$  (or  $2dx_i$ , to be more precise, but there will be no ambiguity in the present context), and assume that the true value  $y$  always falls inside the corresponding interval  $x_i \pm dx_i$ .

Since the vector  $x$  representing the raw data is generally inconsistent, the  $n$  – dimensional parallelepiped centered on  $x$ , with sides of length  $2 dx_i$ , could have an empty intersection with  $L$  (the extreme case of observations which are too – or much too – precise). Consequently, one might expect some of the adjusted aggregates  $x_i^*$  to fall outside their respective intervals  $x_i \pm dx_i$ ; such aggregates will be called *Outside Specified Limits* (or OSL). For each aggregate  $x_i$  let us consider, in addition to its precision  $p_i = dx_i/|x_i|$ , its centering coefficient  $r_i$  defined by:  $r_i = |x_i - x_i^*|/dx_i$ . Then a variable is "Outside Specified Limits" if and only if  $r_i > 1$ . Of course, the OSL concept depends on the balancing method used. The centering coefficient  $r_i$  might be compared with the coefficient  $C_p$  of process capability analysis, apart from the fact that that in national accounting, measurements are not, in general, repeatable [55], [21] p. 48 ff.

Just as  $\pi_0$  is the aggregation of individual precisions  $p_i$ ,  $\phi$  is the aggregation of the centering coefficients  $r_i$ . These aggregation functions reduce to classical averages for some types of norms [26] p. 239-240, [30] p. 18-19.

The OSL variables which appear at the end of the balancing process become a major element in the critical discussion of the results which will then take place between the national accountant and the producers of the various aggregates  $x_i$ , each aggregate having been given with an estimate of its associated margin  $dx_i$ . The national accountant and the producers constitute what I have elsewhere called a *metrological circle*. Thus an aggregate is OSL, either because the initial rough estimate of its margin was too optimistic (the natural point of view of the national accountant), or because the method of balancing has treated this variable unfairly (the natural point of view of the producer of the aggregate). The participants in the metrological circle will, of course, keep in view during their discussions the values of the various overall indicators, such as  $\pi$  or  $\phi$ .

The interesting point is that the discussion will result in re-examining the estimates of some margins, and "adjusting" them by replacing  $dx_i$  by  $dx_i^*$ . Since an OSL variable may itself be an aggregate already previously adjusted in another metrological circle, the revision of the method for estimating the margin will have successive repercussions, thus generating a process which I have previously called "metrological" [26] p. 246 ff, [29] p. 347 ff. If the initial shake-up comes from the metrological circle made up at the very aggregated level of the T.E.E. (Tableau Economique d'Ensemble), the metrological process will initially be top down.

### 6.3. Metrological process and the metrological institution

I have just suggested that the creation of a "metrological circle" at a point in the "National Statistical Information System" would generate a metrological process throughout the system – in particular, by means of OSL variables (regarding extrapolating the concept of an information system to the national economy, see [18]). The environment for such a



process might be described as a set of metrological circles with a more or less hierarchical organization, each circle corresponding to a particular representation of the national economy, as I termed it in [19], [20]. It is obvious that the creation of a metrological institution is necessary, if one does not want this process to grow chaotically or (more likely) die out quickly.

The role of such an institution would, evidently, be to encourage the formation of metrological circles and to coordinate them, thus keeping up with and channeling the metrological process, and helping it converge. The necessity of creating such an organization raises numerous practical as well as theoretical difficulties.

One of these is to make clear what is meant by *convergence* of the process. This requires a collective vision of the process; to quote, once again, one of Stone's penetrating remarks: "*Working with what we have usually means that our data come from a number of sources intended generally to serve separate purposes and not requiring common definitions, classifications and conventions. So, what we have is likely to be in some measure inaccurate, inconsistent and incomplete*" [65] p. 292. Thus, metrological convergence supposes a common purpose for all the data constituting economic and social information. The overall process has to be incorporated into a consistent whole, and not be just a simple juxtaposition of individual purposes.

Another problem is to define the difference between primary sources (the "microbase") and "aggregated" data. This makes it possible to clarify the concept of *simple aggregation*, by which I mean any function of the microdata. They are data provided by primary sources and conventionally supposed to have been observed, or, rather, *collected without error* (that is, whatever aggregation occurs at this level is done without any attempt at balancing, or requirement for consistency as defined at a higher level.) Margins of error appear when these "simple" aggregates are considered by some metrological circle and subjected to a requirement of consistency previously ignored. These margins of error (which could be termed "primary") will thus be worked out by the appropriate metrological circle and will be refined as the metrological process proceeds. It is these margins which make it possible for the primary sources of data to have an appropriate effect upon the aggregates at the meso- and macro-economic levels, with all the precision which is possible and desirable [19], [20].

The last point I will consider is the difficult problem of defining the overall consistency of all the margins established at the various levels of aggregation in the "national statistical information system". One of the functions of the metrological institution would be to create and manage a data base for these margins. The integrity of this base (in the sense of data processing) is the practical definition of the "consistency of the margins". Establishing this integrity *i.e.* the adoption of adequate checking procedures for data processing, will probably be a question of patience and pragmatism, a pragmatism dear to Richard Stone who adopted as watchword "*solvitur ambulando*" [62] p. 9 (see also p. 84 *in fine*): this is a lesson to ponder and an example to follow.

## Appendix 1

Bulletin de l'A.C.N. N°18, 190/D006.1, November 21, 1991 (excerpt)

### *Tribune libre:*

*Sur le modèle de la comptabilité nationale  
et les propositions d'O. Arkhipoff concernant sa fiabilité (1)*

Jean Chaumont, administrateur de l'I.N.S.E.E.

La comptabilité nationale est un modèle de représentation de l'économie. Ce modèle fait notamment l'hypothèse que l'ensemble de tous les comptes de l'économie obéit globalement aux mêmes équations comptables qu'un seul compte: le nom même de "comptabilité nationale" vient de là.

Comme dans les autres sciences humaines, les hypothèses faites sont très fortes et une saine démarche scientifique est de chercher d'abord à expliciter ces hypothèses pour les éliminer progressivement ensuite lorsqu'elles paraissent trop fortes, au point d'être visiblement falsifiées au sens de Popper.

Ainsi, le problème de la comptabilité nationale semble-t-il être dominé par trois hypothèses de ce genre:

- H1: l'hypothèse d'indépendance entre les  $n$  agrégats observés (les cases du TEE brut, par exemple),
- H2: l'hypothèse de simultanéité de tous les comptes,
- H3: l'hypothèse d'observation parfaite.

La modélisation de la comptabilité nationale, c'est donc la formulation puis l'élimination progressive des hypothèses, en commençant par H1 (celle de l'indépendance), en proposant un système d'équations linéaires ("comptables") que les agrégats doivent vérifier, mais cela implique malheureusement que H2, la deuxième hypothèse, soit vraie: la simultanéité des comptes.

Ensuite, le modèle tente d'éliminer H3 (l'hypothèse de l'observation parfaite) au moyen d'un "arbitrage" qui prétend corriger les erreurs d'observation. Dans sa tentative de modélisation de la praxis des comptes nationaux, O. Arkhipoff s'est longuement attardé sur la question du choix de la méthode d'arbitrage, en envisageant en particulier un modèle probabiliste, en relation avec le problème d'une caractérisation numérique globale de la fiabilité globale des comptes nationaux.

Le noeud de la difficulté est, comme il l'a souligné, la non-répétabilité des mesures qui fait qu'on ne dispose que d'une *seule* observation (à  $n$  dimensions): le vecteur du TEE brut. Il s'agit donc d'un

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(1) Voir la communication d'Arkhipoff au dernier colloque de l'ACN de mars 1991 "Fiabilité des comptes nationaux: un processus convergent?". {This text is given here under the reference [27]}.

problème statistique très particulier, non bernoullien mais bayésien, où la loi des grands nombres n'intervient pas dans l'estimation du TEE équilibré. Si donc le vecteur aléatoire du TEE brut possède bien un ellipsoïde indicateur, celui-ci reste malheureusement inconnu et, semble-t-il, inaccessible à l'observation (j'essaierai plus loin de tourner cette difficulté en proposant un mode d'arbitrage particulier). Pourtant c'est un lieu commun de dire que les  $n$  agrégats du TEE équilibré ont une meilleure précision que ceux du TEE brut, comme si l'ajustement avait les mêmes vertus que l'estimation d'un échantillon aléatoire bernoullien de  $n$  observations parfaites et indépendantes! Mais je suis bien d'accord avec O. Arkhipoff pour dire que l'un des problèmes est de donner un sens précis et vrai à cette affirmation des comptes nationaux (et à d'autres).

Revenons sur les trois hypothèses du problème de la comptabilité nationale: on a vu que l'élimination des hypothèses H1 et H3 a conduit à poser l'hypothèse H2, manifestement fausse pourtant. Il en résulte que l'arbitrage sert, avant toute chose, à compenser les effets de cette hypothèse: ainsi l'écart entre TEE brut et TEE équilibré n'a-t-il pas le sens d'une erreur d'observation mais celui d'un mélange inextricable entre erreur d'observation et erreur du modèle lui-même.

Avancer dans la connaissance de la fiabilité des comptes, cela demande, me semble-t-il, la distinction de ces deux sortes d'erreurs, de significations très différentes, puisque l'une d'entre elles est, en quelque sorte, *voulue* par construction!

Nous pensons y parvenir en inversant la démarche habituelle: Éliminons l'hypothèse de simultanéité (H2) avant de toucher à l'hypothèse d'observation parfaite, que nous maintiendrons dans un premier temps. Alors, l'arbitrage change complètement de nature.

En effet, éliminer l'hypothèse de simultanéité, c'est renoncer à exiger que l'*agrégation* de comptes – *parfaitement observés* – satisfasse au système des équations comptables. Cependant, le système reste satisfait pour chaque compte particulier, ainsi que pour la vue que l'on a des autres comptes de l'économie à partir de ce compte-là. Mais cette vue des autres comptes est différente des comptes réels, parce que les informations que l'on a sur eux ne sont pas instantanées et que la période de référence comptable (un an) est trop courte pour effacer les différences.

A chaque vue, donc à chaque compte, correspond ainsi un TEE partiel, mais équilibré et différent. Il y a ainsi un grand nombre de TEE partiels, *relatifs* à une vue, et équilibrés.

L'hypothèse de simultanéité est ainsi remplacée par une sorte de principe de relativité des TEE, qui n'est pas sans rappeler celui de la physique. Mais comment construire ces TEE relatifs?

A défaut de disposer de chaque compte et de sa vue, nous partons des agrégats du TEE brut pour construire tous les TEE équilibrés qui lui sont associés lorsqu'on considère que les observations sont parfaites. La méthode proposée revient *grosso modo* à considérer que  $n - p$  variables brutes sont connues et à calculer les  $p$  variables restantes par résolution

du système des  $p$  équations comptables ( $n$  étant le nombre total des agrégats). Ce qui donne un nombre de solutions au plus égal au nombre des combinaisons des  $n$  agrégats pris  $p$  à  $p$ .

Ces solutions sont représentées dans l'espace vectoriel  $R^n$  par un nuage de points situé dans la variété linéaire du système des équations comptables.

Ce nuage de points est donc obtenu à partir d'un seul point: celui du TEE brut observé, au moyen d'un procédé qui évoque celui du bootstrap statistique et transforme le problème de Bayes en problème statistique classique. Parmi tous ces points, on distingue alors le centre de gravité du nuage, *le point moyen*. On lui donne, *par définition*, le sens de TEE "absolu", traditionnel.

Cette méthode pourrait donner un sens au terme "précision des comptes", car la variance du nuage de points s'identifie à *l'erreur du modèle* classique de la comptabilité nationale, lorsqu'on conserve l'hypothèse de l'observation parfaite (H3): c'est cela l'intérêt de l'affirmation commune évoquée plus haut.

Il reste maintenant à éliminer cette dernière hypothèse en estimant une erreur d'observation pour chaque agrégat du TEE brut. Cette estimation doit être faite indépendamment pour chaque agrégat avant d'être réintroduite dans le modèle. On pourrait envisager de demander à chaque responsable d'un agrégat d'en faire directement l'estimation grossière, au jugé.

Enfin se pose le problème pratique de la construction du nuage de points: en comptabilité nationale,  $n$  et  $p$  ont des valeurs élevées ( $n = 300$  et  $p = 40$ , par exemple). Aucun ordinateur, si puissant soit-il, ne pourrait faire en un temps raisonnable le calcul de l'ensemble des solutions!

Un calcul approché s'impose donc. Nous proposons d'utiliser pour cela le pseudo-inverse de Moore et Penrose, afin de donner directement une valeur approchée du point moyen sans construire effectivement ce nuage <sup>(2)</sup>. Bien entendu, la justification de cette approximation reste à démontrer. Cela pourrait être vérifié sur un petit nombre choisi arbitrairement. Qu'en pensent nos collègues?

### **Réponse rapide à la "Tribune Libre" de Jean Chaumont: Faut-il céder au charme du bootstrap comptable?**

Il est évident pour tous que la difficulté du problème conjoint de l'arbitrage et de la fiabilité des comptes vient de la non-répétabilité du

(2) Le pseudo-inverse de Moore et Penrose a déjà été utilisé pour la construction du TES, mais dans le contexte totalement différent d'un problème bernoullien dont l'échantillon, le nuage de point, était directement observé et où l'hypothèse de simultanéité des comptes n'intervenait pas. On ne saurait donc comparer cette méthode avec celle du TEE (voir: "Une nouvelle approche dans l'élaboration des tableaux entrées-sorties: la démarche probabiliste" par J. Chaumont in "La comptabilité nationale face au défi international" – E. Archambault et O. Arkhipoff (éditeurs) – Economica 1990). [This text is given here under the reference [35]].

mesurage. Cette difficulté entraîne une formulation ardue de l'énoncé même du problème en termes statistiques rigoureux. Cependant, comme je l'ai déjà dit, cette difficulté ne devrait pas nous arrêter.

La cohérence de la comptabilité nationale permet-elle d'améliorer les statistiques de départ? J'ai répondu par l'affirmative dans le sens suivant: l'arbitrage Gauss-Markov a son ellipsoïde indicateur contenu dans celui du vecteur aléatoire brut: nous ne connaissons pas ces ellipsoïdes (platoniciens dirait probablement Chaumont), mais nous savons déjà quel est celui qui est le "plus petit". Jean Chaumont visiblement cherche à donner un sens moins bayésien à l'affirmation traditionnelle des comptes nationaux évoquée ci-dessus. Ce sens est-il radicalement différent? La "Tribune" de J. Chaumont nous invite tous à examiner la question. Mon opinion, à vérifier, est qu'en pratique il n'y a aucune divergence, les divergences se situant plus au plan philosophique: Bayes contre Bernoulli.

La méthode façon "bootstrap" proposée par Chaumont me paraît être stimulante à plusieurs égards. On pourrait déjà en faire une application au T.E.E. français où partie haute (le T.E.E. *stricto sensu* ancienne manière) et partie basse (le T.O.F.) sont séparés par des postes d'ajustement qui mesurent, sinon la qualité des comptes, du moins un certain degré de désaccord.

Cette méthode me paraît ensuite permettre une première formulation d'un autre principe aussi traditionnel qu'implicite chez les comptes nationaux: un tableau est d'autant mieux assuré qu'on le connaît davantage dans le détail.

Cette méthode bootstrap me paraît encore constituer non seulement une généralisation intéressante de la bonne vieille méthode de la "Statistical Discrepancy" proposé par Richard Stone en 1941, mais encore et surtout apporter une interprétation *économique* de la Statistical Discrepancy tranchant agréablement avec ce qui avait été écrit naguère sur cette méthode (méthode semble-t-il fort déconsidérée aujourd'hui: aucune communication présentée à la Conférence IARIW 1989, à Lahnstein, n'y faisait allusion, dans la session consacrée à la fiabilité des comptes, – une session qui connut un succès aussi grand qu'inattendu. Ce que propose Chaumont, c'est peut-être, et enfin, un statut théorique décent à la méthode de l'erreur résiduelle. Affaire à suivre.

Dernière remarque. Si la conjecture de Chaumont, quant au calcul approché par le pseudo-inverse pouvait être confirmée, la méthode de l'erreur résiduelle de Stone, généralisée et interprétée par Chaumont fournirait une nouvelle interprétation, cette fois-ci économique, de l'arbitrage Gauss-Markov en tant qu'il est aussi bien arbitrage Moore-Penrose. Autre affaire à suivre, en distinguant bien les deux problèmes: calcul approché de la solution Chaumont de l'erreur résiduelle, et coïncidence éventuelle de cette solution avec la solution Gauss-Markov, ou, plus modestement: éventuelle convergence de celle-là vers celle-ci, en un sens qui serait à préciser.

## Appendix 2

**Bulletin de l'A.C.N. N°19, 59/D006.1, February 10, 1992 (excerpt)**

*Tribune libre:*

*Formalisation et analyse de la méthode d'arbitrage Stone-Chaumont*

Oleg Arkhipoff, administrateur de l'I.N.S.E.E.

Dans la Note ACN précédente (n° 18), Jean Chaumont proposait une méthode d'arbitrage intéressante et posait différentes questions à ce sujet. Nous proposons ici une formalisation mathématique de la méthode; après cela, nous ferons quelques premiers commentaires.

### **Préliminaires**

Les comptes nationaux arbitrés peuvent se formaliser en une équation matricielle  $AX^* = \theta$ , représentative de  $m$  équations liant  $n$  agrégats  $x_i^*$  ( $m < n$ ). Chaumont propose d'arbitrer les agrégats bruts  $x_i$  ( $AX \neq \theta$ ), comme suit: on pose que  $n - m$  agrégats bruts (les  $n - m$  derniers par exemple) sont exacts et on calcule les  $m$  autres (les  $m$  premiers, ici) à l'aide des équations comptables. On recommence en choisissant un autre jeu de  $n - m$  agrégats bruts, etc. On fait enfin la moyenne des solutions: cette moyenne  $X^{ch}$  est donc l'arbitrage proposé par Chaumont.

### **Formules générales**

Nous partitionnerons nos matrices comme suit:

$$A = [A_{mm} \mid A_{m \ n - m}],$$

$$X = \begin{bmatrix} X_m \\ X_{n - m} \end{bmatrix}, \quad X = \text{vecteurs des données brutes,}$$

$$X(0) = \begin{bmatrix} X(0)_m \\ x(0)_{n - m} \end{bmatrix}, \quad \text{première solution,}$$

$$J_{nm} = \begin{bmatrix} I_{mm} \\ \theta - m \ m \end{bmatrix}, \quad K_{n \ n - m} = \begin{bmatrix} \theta m \ n - m \\ I_{n - m} \ n - m \end{bmatrix}.$$

La première solution  $X(0)$  consiste à se donner comme exactes les  $n - m$  derniers agrégats et à calculer "par solde" (pour utiliser une

terminologie familière aux comptables nationaux) les  $m$  premiers. Soit donc à résoudre le système d'équations matricielles

$$\begin{cases} AJJ^T X(0) = -AKK^T X \\ X(0) = JJ^T X(0) + KK^T X \end{cases} \quad (1)$$

Quand la solution  $X(0)$  existe et est unique, on obtient, tous calculs faits:

$$X(0) = (Inn - J(AJ)^{-1}A) X \quad (2)$$

Une solution quelconque numéro  $k$ ,  $X(k)$  (quand elle existe et est unique), s'obtient par une permutation convenable à la fois des colonnes de  $A$  et des lignes de  $X$ , faisant intervenir une matrice de permutation convenable  $P(k)$  ( $P(0) = Inn$ ). On trouve alors la formule:

$$X(k) = (Inn - P(k)J(AP(k)J)^{-1}A)X \quad (3)$$

L'arbitrage proposé par Jean Chaumont consiste à prendre la moyenne simple de toutes les solutions  $X(k)$  ainsi obtenues:  $k = 0, 1, 2, \dots, N$ ; soit donc la formule finale:

$$X^{ch} = (X(0) + X(1) + \dots + X(N))/(N + 1) \quad (4)$$

### **Analyse des résultats obtenus**

Un simple T.E.E. compte 30 à 40 équations et environ dix fois plus de variables. Le nombre  $N$  est évidemment colossal et défie les ordinateurs les plus puissants. Jean Chaumont se demande si la solution exacte  $X^{ch}$  n'est pas approchée par la solution  $X^*$  Moore-Penrose, voire si  $X^*$  n'est pas précisément cette solution exacte  $X^{ch}$ .

Sérions les problèmes: le *hic* dans la formule (3) est que  $P(k)$  apparaît deux fois, et surtout, fort malencontreusement, dans la parenthèse intérieure. De plus, dans le cas général, la solution  $X(k)$  peut ne pas exister ou bien être multiple. On peut donc craindre qu'il n'existe aucune formule simple de  $X^{ch}$ , c'est-à-dire pouvant être calculée par ordinateur en un temps raisonnable.

Peut-on trouver une approximation simple  $X^e$  de  $X^{ch}$  (en d'autres termes:  $\|X^{ch} - X^e$  tend vers zéro quand  $N$  croît? Je ne peux malheureusement pas répondre à cette question qui reste ouverte (dans l'état de mes connaissances).

Cette solution approchée pourrait-elle être  $X^*$ ? On peut faire ici une première remarque. La matrice  $A(k) = P(k)J(AP(k)J)^{-1}$  est un inverse normalisé de  $A$ , c'est-à-dire très près d'être le pseudo-inverse  $A^+$  Moore-Penrose de  $A$  ( $A^+ = A^T(AA^T)^{-1}$ ). Mais si l'on fait la moyenne (simple ou pondérée) d'inverses normalisés, la moyenne reste bien un inverse généralisé mais n'est plus nécessairement normalisée (elle continue cependant à vérifier la propriété particulière de l'hermiticité de l'invariant à droite); cependant, il se trouve ici que cet inverse reste normalisé. Quoi

qu'il en soit, l'arbitrage Chaumont est bien du type:  $(I - A^{-1})X$  où  $A^{-1}$  est un inverse généralisé, variable selon la moyenne adoptée (selon N, notamment) et, semble-t-il, difficilement calculable quand N croît.

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# BALANCED NATIONAL ACCOUNTS FOR THE UNITED KINGDOM, 1920-1990

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## 1. Introduction (i)

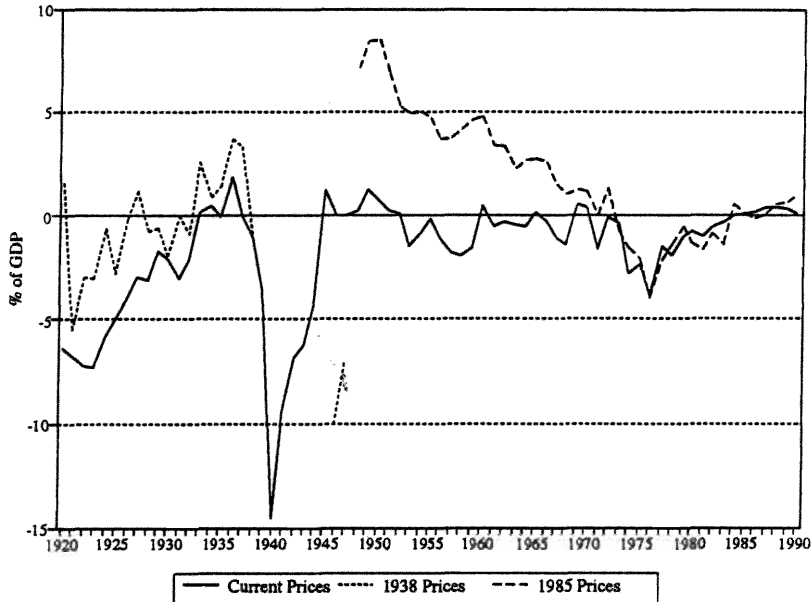
There are, traditionally, three measures of GDP, expenditure, output and income. If these measures are calculated independently, they cannot be expected to be equal, because errors will inevitably creep in at the various stages of their calculation. In practice the measures are reduced to two for each of constant and current prices. Factor incomes are not, at least in the United Kingdom, measured independently of the industries in which those incomes are earned. In constant prices output and expenditure are measured separately, but the income measure is calculated by applying the expenditure deflator to current price income, and it does not offer a third independent measure of output. Thus there are discrepancies between income and expenditure measured at current prices and output and expenditure measured at constant prices. This paper discusses the question how these different measures should be reconciled and applies appropriate methods to the UK national accounts for the period 1920-1990. Estimates of the standard deviations of the measurement errors associated with some of the key variables of interest are also presented.

Figure 1 shows the discrepancies between expenditure and income at current prices and expenditure and output measured at constant prices as a percentage of the expenditure measure (1). There are no output measures of GDP during the period 1939-45 and therefore no constant price residual can be calculated for this interval. It can be seen that the problems are worse with constant than with current price data, but in neither case are they trivial. The constant price estimates are calculated to 1938 prices for the period 1920-47 and to 1985 prices for

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(1) Or, more accurately as  $100 \times \{\log(\text{Income or Output}) - \log(\text{Expenditure})\}$ .



**Figure 1 – The Residual Errors in Constant and Current Price GDP**

the period 1948-90. This means that in 1938 and in 1985 the discrepancies are the same in constant and current prices. In the period after 1948 there is a tendency for the two constant price measures to drift apart which is absent from the current price data, while there seems to be a considerable amount of drift in both residuals before 1938. During the second world war the two current price measures of GDP showed very substantial discrepancies.

## 2. The Average Estimate of GDP

The conventional way of resolving this inconsistency is to take a simple arithmetic average. For many years the mean of income and expenditure was the recommended measure of GDP at current prices, while in constant prices the average of expenditure, income and output was used.

Recently the presentation of official statistics has changed. National Accounts statistical adjustments are introduced into the data reflecting the views of the statisticians as to where the major measurement errors belong. As far as the constant price measures are concerned, the output measure is regarded as defining movements of GDP in the short term, but in measuring output movements between base years (1948, 1958, 1963, 1970, 1975, 1980, and 1985) no weight at all is put on the output index, so that long-term movements in real GDP are estimated from the average of the income and expenditure data deflated by the expenditure GDP deflator. The methods by which the output index is elided into the

deflated income/expenditure index are described by *Economic Trends, No 420*.

The current price data are treated slightly differently. The most recent estimates presented in the 1992 *Blue Book* show no residual at all between 1985 and 1989. This does not mean that the estimates are exact. The notes to the *Blue Book* explain that there were residuals (shown in Figure 1) of between £105mn and £3454mn in this period. They have been allocated equally between income and expenditure, but no clues are given as to the components of income/expenditure to which the adjustments have been made.

Neither of these methods of dealing with the discrepancy is completely satisfactory, for a number of reasons. The method of averaging does not, on its own, allocate the discrepancy across the components of national income. If expenditure is above income, then averaging brings down aggregate expenditure, but it offers no indication as how much of the adjustment should be allocated to consumption, investment, government consumption or trade flows. The current method of adjustment raises the same problem at one remove. The broad aggregates of income and expenditure are adjusted, but the adjustment to consumption is not allocated to the components of consumption. One can reasonably worry about an estimation methodology which leaves statisticians aware that low estimates of consumption were leading to discrepancies, but apparently unaware which components of consumption are understated. Thus the second method does not resolve the issue and, in any case it is not applied to most of the data before 1983.

There is another problem which arises from the interaction of constant and current price data. It may well be the case that, in current prices expenditure is above income, while in constant prices it lies below output and deflated income. If the current price data are adjusted independently of the constant price data there may be large adjustments placed on the implicit deflator. One would be happy with a downward adjustment of current price expenditure if there were a similar adjustment in constant price expenditure. After all, many of the constant price data are simply measured by deflating the current price data. The existing methods of reconciliation do not seem to pay enough attention to the links which ought to exist between the constant and current price data.

### **3. Adjustment on the Basis of Data Reliability**

#### **3.1. Least-squares/ Maximum-likelihood Adjustment**

This paper describes an alternative, and in our view more satisfactory way of reconciling inconsistent data sources. Intuitively it makes a great deal of sense that those adjustments needed to create accounting balance should be borne in some inverse relation to the degree of confidence one has in the data. Any adjustments to expenditure data should be made disproportionately to variables like

stockbuilding because it is known that this cannot be measured satisfactorily. The knowledge that the accounting constraints must be satisfied by the true data provides extra information which can be used to refine the accuracy of the data collected through conventional means.

The solutions to the problem of data inconsistency used hitherto can, of course be seen in this context. If one believed that income and expenditure data were of equal reliability, then the arithmetic average would be the result produced by adjustment with reference to reliability. But the concept of reliability could be applied to the income/expenditure components and used to allocate the corrections across the series which make up the national accounts. This would lead to a fully-consistent set of income/expenditure data.

Such a data set may lead to important changes in the way economic variables are perceived. For example, in the period 1920-24 a negative personal sector savings ratio is shown by Feinstein (1972). But at the same time the expenditure measure of GDP is well above the income measure. Since personal consumption is the largest component of expenditure and wage income the largest type of factor income, any reconciliation will almost certainly reduce consumption and raise personal income, thereby raising the savings ratio. Adjustments of this type, which do no more than reflect accounting identities, cannot be made unless the adjustments are allocated across the income/expenditure components.

The reliability with which changes in data are measured is of course closely related to that with which levels are measured. The link between the two depends on the degree of serial correlation in the measurement error. If there are large measurement errors in the output index, but they are strongly serially correlated, then it makes sense to put more weight on the output index in the short run than in the long run. But to give no weight to it in the long run, as is current practice, implies that it conveys no information at all about long-run movements in output; this seems inherently unlikely.

The idea of adjusting data with reference to reliability has to be given a precise meaning. The most usual way of doing this is to minimise the sums of the squared changes of the data, weighted by the variances of the measurement errors of the data: this gives balanced data which are a linear combination of the initial data. Thus, if the variance of expenditure were half that of income,  $\frac{2}{3}$  of the adjustment would be borne by income and  $\frac{1}{3}$  by expenditure. As is implicit in this example, it is the relative and not the absolute variances of the data which determine the allocation of the adjustments. The least-squares estimates which emerge from this process have the attraction that, if the original measurement errors are normally distributed, they are also maximum-likelihood estimators: this is a general property of least-squares estimators.

It is a property of estimators calculated by minimising the sum of squares that they are the linear estimators with the lowest variance (or best linear unbiased estimators). Not only can one calculate the variances of the reconciled data, but one can also be assured that they



are lower than those which would be delivered by some other linear combination, such as the arithmetic average.

Obviously, this conclusion is dependent on starting with the correct data variances. If incorrect variances are supplied it is no longer possible to be sure that the reliability of the data is enhanced. Where nothing is known about data reliability, it might be sensible to start from an assumption of equal reliability, but it is still necessary to decide whether this means equal proportionate reliability or equal absolute reliability.

The basic result can be derived by the method of least-squares, as originally suggested by Stone, Champernowne and Meade (1942) or as a maximum-likelihood estimate (Weale, 1985). It is assumed that there is a data vector  $\mathbf{x}$ , which should satisfy the accounting constraints,  $\mathbf{Ax} = \mathbf{s}$ . In many cases  $\mathbf{s}$  will be the zero vector. There are, however, some circumstances in which it may be non-zero. Perhaps the best-known of these is the situation in which the components of an input-output table are being adjusted to meet known marginal totals <sup>(2)</sup>. However, because of the problem of measurement errors, in fact  $\mathbf{Ax} = \mathbf{r}$ , where  $\mathbf{r}$  is a vector of accounting residuals. It is also assumed that the observed data are distributed without bias around the true data  $\mathbf{x}^*$  with known variance matrix  $\mathbf{V}$ , so that

$$\mathbf{x} = \mathbf{x}^* + \varepsilon \quad (1)$$

with

$$E(\varepsilon) = 0 \quad (2)$$

and

$$E(\varepsilon\varepsilon') = \mathbf{V} \quad (3)$$

The least-squares problem is then one of finding a vector  $\mathbf{x}^{**}$  which satisfies the accounting constraints,  $\mathbf{Ax}^{**} = \mathbf{s}$ , and is as close as possible to the observed data,  $\mathbf{x}^*$ . Closeness could be defined in a number of ways. Here we choose to define it as the sum of the squared deviation of each element of  $\mathbf{x}^{**}$  from  $\mathbf{x}$ , weighted by the data reliability matrix,  $\mathbf{V}$ .

**Proposition 1** *The least-squares estimator,  $\mathbf{x}^{**}$  of the  $\mathbf{x}$  which satisfies the accounting constraints  $\mathbf{Ax}^{**} = \mathbf{s}$  is*

$$\mathbf{x}^{**} = (\mathbf{I} - \mathbf{VA}'[\mathbf{AVA}']^{-1}\mathbf{A})\mathbf{x} + \mathbf{VA}'(\mathbf{AVA}')^{-1}\mathbf{s} \quad (4)$$

**Proof** The problem is that of minimising

$$(\mathbf{x}^{**} - \mathbf{x})'\mathbf{V}^{-1}(\mathbf{x}^{**} - \mathbf{x}) \quad (5)$$

subject to the constraint

(2) See Bacharach (1971) for an account of the *rAs* method of achieving the same end. The *rAs* method does not, however, meet any clear statistical criteria.

$$\mathbf{Ax}^{**} = \mathbf{s}. \quad (6)$$

This can be set out as a Lagrangian

$$\text{Min } L = (\mathbf{x}^{**} - \mathbf{x})' \mathbf{V}^{-1} (\mathbf{x}^{**} - \mathbf{x}) - \lambda (\mathbf{Ax}^{**} - \mathbf{s}) \quad (7)$$

Differentiating with respect to  $\mathbf{x}^{**}$

$$\frac{dL}{d\mathbf{x}^{**}} = 2\mathbf{V}^{-1} (\mathbf{x}^{**} - \mathbf{x}) - \mathbf{A}'\lambda \quad (8)$$

Setting this equal to zero we find

$$\mathbf{VA}'\lambda = 2(\mathbf{x}^{**} - \mathbf{x}) \quad (9)$$

Premultiplying by  $\mathbf{A}$  and rearranging

$$\lambda = 2(\mathbf{AVA}')^{-1} \mathbf{A}(\mathbf{x}^{**} - \mathbf{x}) \quad (10)$$

and, since  $\mathbf{Ax}^{**} = \mathbf{s}$

$$\lambda = -2(\mathbf{AVA}')^{-1} (\mathbf{Ax} - \mathbf{s}) \quad (11)$$

Substituting this in (9), we find

$$\mathbf{x}^{**} = [\mathbf{I} - \mathbf{VA}'(\mathbf{AVA}')^{-1} \mathbf{A}] \mathbf{x} + \mathbf{VA}'(\mathbf{AVA}')^{-1} \mathbf{s} \quad (12)$$

It can be seen that this estimator is unaffected by premultiplication of the variance matrix  $\mathbf{V}$  by any scalar constant. This implies that the balancing process depends on relative but not on absolute data reliability.

It follows from (12) not very surprisingly, that this estimator is unbiased. Since  $E(\mathbf{x}) = \mathbf{x}^*$  and  $\mathbf{Ax}^* = \mathbf{s}$ ,

$$\begin{aligned} E(\mathbf{x}^{**}) &= [\mathbf{I} - \mathbf{VA}'(\mathbf{AVA}')^{-1} \mathbf{A}] E(\mathbf{x}^*) \\ &\quad + \mathbf{VA}'(\mathbf{AVA}')^{-1} \mathbf{s} = \mathbf{x}^*. \end{aligned} \quad (13)$$

Balancing of the data also leads to a reduction (or at least not an increase) in the data variance. This can be demonstrated by evaluating the variance matrix of the balanced data.

**Proposition** *The variance of  $\mathbf{x}^{**}$ ,  $\mathbf{V}^{**}$  is*

$$\mathbf{V}^{**} = \mathbf{V} - \mathbf{VA}'(\mathbf{AVA}')^{-1} \mathbf{AV}. \quad (14)$$

**Proof**

$$\begin{aligned} \mathbf{V}^{**} &= E(\mathbf{x}^{**} - E[\mathbf{x}^{**}]) (\mathbf{x}^{**} - E[\mathbf{x}^{**}])' \\ &= E\{(\mathbf{I} - \mathbf{VA}'(\mathbf{AVA}')^{-1} \mathbf{A}) (\mathbf{x} - E[\mathbf{x}]) \\ &\quad (\mathbf{x} - E[\mathbf{x}])' (\mathbf{I} - \mathbf{A}'(\mathbf{AVA}')^{-1} \mathbf{AV})\} \\ &= \mathbf{V} - \mathbf{VA}'(\mathbf{AVA}')^{-1} \mathbf{AV} \end{aligned} \quad (15)$$

since  $AE[\mathbf{x}] = \mathbf{s}$  because the expected values of the observed data satisfy the accounting constraints.

Since  $\mathbf{VA}'(\mathbf{AVA}')^{-1}\mathbf{AV}$  is a positive semi-definite matrix, it follows that the process of least-squares balancing has the effect of not making the data less accurate. In normal circumstances, when there are discrepancies in the original data,  $\mathbf{VA}'(\mathbf{AVA}')^{-1}\mathbf{AV}$  will be positive definite, and balancing results in a gain in reliability.

### 3.2. Covariances and Serial Correlation

The discussion so far has assumed that the measurement errors in one variable are independent of those in other variables. This may often be the case but it is not universally so. For example domestic sales of motor cars may be measured with reasonable accuracy, but it may not be so easy to allocate these sales between consumption sales to purchases and investment sales to businesses. In such a case there will be negative covariance between the errors in consumption and those in investment.

At the level at which we have worked, there is not much evidence of problems of this type. However, there are two very important ways in which off-diagonal elements arise. The first is from the presence of constant and current price data. If the constant price data are calculated by deflating current price data, then there will be a covariance between the two equal to the variance of the current price data. The second arises from serial correlation. This means that the error in a variable in one period is correlated with that error in the same variable in a different period. It can be represented by stacking all the observations of all the variables as a single data vector and weighting the adjustments to this data vector by the data covariance matrix, with serial correlation showing up as off-diagonal elements in this matrix.

### 3.3. Validation of the Variance Matrix and the Mahalanobis Criterion

The procedure of section 3.2 is normally carried out using an estimate of the variance matrix provided by those responsible for the data. The researcher is, nevertheless, not left completely at the mercy of those providing the data variance matrix. It is possible to carry out two tests to see whether the variance matrix is consistent with the data. From the variance matrix it is possible to calculate the variance matrix of the accounting residuals. This is simply a linear transformation of the data variance matrix. In the simplest case in which there is only one accounting residual the variance matrix is a scalar equal to the sum of the income and expenditure variances. One can then test the actual variance of the residual against the hypothesized variance calculated from the statisticians' estimates of data reliability. The related hypothesis that the measurement errors are normally distributed implies that the residuals are themselves normally distributed. The observed residuals can be tested to see whether this is true by means of a standard goodness of fit test.

Alternatively one may use the *ex post* variance of the accounting residuals as a means of calibrating the variance matrix after Mahalanobis (1936). Consider a vector  $\mathbf{x}$  which may represent a number of annual observations stacked on on to of another.  $\mathbf{V}$  is the variance-covariance matrix of this vector with its off-diagonal elements representing serial correlation in the measurement errors. Over the whole period there are  $N$  accounting constraints. Since it is known that the  $N$  independent accounting constraints must be true, the  $\chi_N^2$  statistic.

$$Z' = (\mathbf{x} - \mathbf{x}^{**}) \mathbf{V}^{-1} (\mathbf{x} - \mathbf{x}^{**}) \quad (16)$$

which defines the distance of the balanced data from the raw data, can be regarded as a measure of the accuracy with which  $\mathbf{V}$  is measured. In repeated samples we would expect  $Z'$  to be as likely to be on either side of the median value of  $\chi_N^2$  which is just below  $N$  itself and converges to  $N - 0.67$  in large samples. One can therefore (see Stone, 1982, Kenny, 1992) assume that  $Z' \mathbf{V} / \chi_N^2$  (50%) offers <sup>(3)</sup> a better estimate of the variance matrix than does  $\mathbf{V}$  on its own. Scaling of the variance matrix does not, of course, change the balanced data estimates.

If we substitute in (16) using the expression for  $\mathbf{x}^{**}$  given in (12) then the statistic simplifies to

$$Z' = \mathbf{x}' \mathbf{A}' (\mathbf{A} \mathbf{V} \mathbf{A}')^{-1} \mathbf{A} \mathbf{x} \quad (17)$$

so that the Mahalanobis score is simply an assessment of the accounting residuals  $\mathbf{A} \mathbf{x}$  against the variance of the residuals  $\mathbf{A}' \mathbf{V} \mathbf{A}$  implied by the variance matrix  $\mathbf{V}$ .

While this statistic provides only a limited amount of information it does nevertheless allow an assessment of whether the  $\mathbf{V}$  matrix is consistent with the published data, and this provides a very important objective review of the whole exercise.

### 3.4. Unobserved Components

It might be thought that, in the national accounts, there would be accounting constraints applying to the sectoral data as well as to the components of GDP. For example, the current account of the balance of payments, identified as the gap between exports and imports of goods and services (including property income), should be offset exactly by the capital flows identified in the financial accounts. This would suggest that the observed gap between the two is a residual like the residual in the GDP account.

Unfortunately this is not correct. Observed financial transactions do not cover all financial transactions. Large items such as flows of trade credit are not recorded and it is unlikely that it will ever be possible to record them in full sectoral detail (Begg and Weale, 1992). This means that the sectoral financial balances do not offer extra usable constraints. They must be calculated as residual items; the fact that balancing

(3) The median  $\chi^2$  statistic is the maximum-likelihood value.

ensures that the GDP identity is satisfied means, nevertheless, that these residual balances will satisfy the constraint of adding to zero across all the sectors. More generally, one must be careful to distinguish genuine accounting residuals from figures which combine them with unobserved data.

### 3.5. Unknown Variances

The discussion so far has focussed on the need for some estimates of data reliability in order to make the least-squares adjustment possible. Is it possible to make the adjustments without knowledge of data reliability? The answer turns out to be yes (Satchell, Smith and Weale, 1992, Weale, 1992). If the residuals are regressed on the data, then the regression coefficients converge asymptotically to the weights which would be used to allocate the residuals if the true error variance matrix were known. Furthermore, it is possible to carry out statistical tests on the weights to test, for example, the hypothesis that two different observations of the same variable should be given equal weight.

The analysis can be extended to the case where the measurement errors are proportional to some linear combination of the data. One would not expect the absolute standard deviation of money consumption to be the same now as it was in 1948, but it may well be a constant proportion of nominal GDP. Thus problems of heteroscedasticity can be avoided.

Nevertheless, this result is not as helpful as might be thought, at least with the sort of dataset we consider here. It turns out that well over 100 observations are needed before any degree of precision can be given to the estimated weights. In view of this, we considered that it would be better to use published information on data reliability, which we then assessed using the tests of the data variances summarised above.

## 4. The Effect of Autocorrelation on Measurement Error

References to the reliability of both historical and contemporary national accounting aggregates frequently point out that changes from one year to the next are measured more accurately than the data themselves. If the measurement errors were uncorrelated this would not be the case; the variance of the difference between two observations would equal the sum of the variances of the two observations. It is the presence of positive autocorrelation in the measurement errors which confers this property. A simple example can demonstrate this.

Suppose that data are constructed by extrapolating away from a benchmark using an indicator variable, a method of data construction widely applied to both historic and current data. Working with proportionate or percentage errors (which assumes that the errors are in fact multiplicative), then if  $\varepsilon_N$  is the percentage error in the benchmark variable in year  $N$ ,  $v_N$  is the percentage error in the indicator variable in

year  $N$  and  $v_t$  is the percentage error in the indicator variable in year  $t$ , the total error for period  $t$  is

$$u_t = \varepsilon_N + v_t - v_N \quad (18)$$

and its variance is, with  $\varepsilon_N$  independent of  $v_t$  and  $v_N$  given as

$$E(u_t^2) = E(\varepsilon_N^2) + E(v_t^2) + E(v_N^2) - 2E(v_t v_N) \quad (19)$$

It is likely that the measurement error in the indicator variable  $v_t$  follows an autocorrelated process.

A simple example demonstrates this. The number of cars produced may be used as an indicator of the constant price value added in the car industry; in the short term changes in the number of cars produced are likely to be a fairly good indicator of changes in the constant price value added of the car industry. In the longer term quality changes and changes in product mix will almost certainly make the indicator less good.

We may then specify the autocorrelation process followed by  $v_t$  as

$$v_t = \rho v_{t-1} + w_t \quad (20)$$

If  $E(w_t^2) = \sigma_w^2$ , then the variance of  $v_t$ ,  $E(v_t^2)$  is given by the asymptotic formula,  $E(v_t^2) = \sigma_v^2 = \sigma_w^2 / (1 - \rho^2)$ . It then follows that

$$E(u_t^2) = \sigma_{\varepsilon_N}^2 + 2\sigma_v^2(1 - \rho^{t-N}) \quad (21)$$

and it can be seen that, in the simple case in which  $\rho = 0$ , there is a step increase in the variance outside the benchmark observation from  $E(\varepsilon_N^2) = \sigma_{\varepsilon_N}^2$  to  $\sigma_{\varepsilon_N}^2 + 2\sigma_v^2$ . If  $\rho$  is positive, this upper value is approached asymptotically rather than in one step. For given initial and asymptotic variances, the variances in the intervening periods are lower, the higher is the degree of autocorrelation.

The error in the change in the variable from one period to the next is given, in percentage terms as

$$u_t - u_{t-1} = v_t - v_{t-1} = (\rho - 1)v_{t-1} + w_t \quad (22)$$

so that

$$\begin{aligned} E(v_t - v_{t-1})^2 &= (\rho - 1)^2 E(v_{t-1}^2) + \sigma_w^2 \\ &= \sigma_w^2 \left( 1 + \frac{[\rho - 1]^2}{1 - \rho^2} \right) = \frac{2\sigma_w^2}{1 + \rho}. \end{aligned} \quad (23)$$

Depending on the various parameter values, this variance may well be below  $E(u_t^2)$ . In particular, for a given value of  $\sigma_v^2 = \sigma_w^2 / (1 - \rho^2)$  a high value of  $\rho$  will reduce the variance of the growth rate relative to that of the level.

Interpolation creates a different, and much more complicated pattern of autocorrelation. Nevertheless, it remains the case that it results in

measurement errors varying with the time from the benchmark years and in which changes are measured more reliably than levels. The presence of autocorrelation also, of course, means that adjustments to data for one period imply adjustments to data in other periods. This may have an important impact on the results of any data reconciliation exercise.

For these two reasons, the presence of autocorrelation in measurement error cannot be neglected. The subsequent sections describe the way in which the data of our study are constructed and the autocorrelation to which this is likely to give rise.

## 5. Patterns of Autocorrelation and Covariance

### 5.1. Extrapolation

If data are extrapolated from a single reference point, the error structure is simple. The relation between the indicator variable,  $i_t$ , and the true value,  $b_t^*$ , is given as

$$i_t = kb_t^*(1 + v_t) \quad (24)$$

where  $v_t$  is an error term with expected value 0. This corresponds exactly to the assumption made about the error process of the indicator described above. If the reference or benchmark year is  $N$ , then

$$b_t = b_N(i_t/i_N) \quad (25)$$

offers an extrapolated estimate of  $b_t^*$ . If  $b_N = b_N^*(1 + \varepsilon_N)$ , with  $\varepsilon_N$  being a percentage error term in  $b_N$  and  $E(\varepsilon_N) = 0$ , then, to a first-order approximation, as in equation (18)

$$u_t = \frac{b_t}{b_t^*} - 1 = \varepsilon_N + v_t - v_N \quad (26)$$

where  $u_t$  is the proportionate error in  $b_t$ ,  $b_t = b_t^*(1 + u_t)$ . It is now possible to calculate the autocorrelation between  $u_t$  and  $u_{t+k}$ . This is shown in a more general form which allows for the possibility that  $u_t$  is calculated with reference to benchmark  $N_1$  while  $u_{t+k}$  is calculated with reference to benchmark  $N_2$ . Such a situation can arise in some cases, and there is no loss of generality since, with only one benchmark point  $N_1 = N_2$ . The autocorrelation is

$$\begin{aligned} E(u_t u_{t+k}) &= E(\varepsilon_{N_1} \varepsilon_{N_2}) + E(v_t v_{t+k}) - E(v_{t+k} v_{N_1}) \\ &\quad - E(v_t v_{N_2}) + E(v_{N_1} v_{N_2}) \\ &= \sigma_{\varepsilon_{N_1}, \varepsilon_{N_2}}^2 \\ &\quad + \frac{\sigma_w^2}{1 - \rho^2} (\rho^k + \rho^{|N_1 - N_2|} - \rho^{|t+k - N_1|} - \rho^{|t - N_2|}) \quad (27) \end{aligned}$$

on the assumption that the two sources of error  $\varepsilon_N$  and  $v_t$  are independent of each other. This expression should be interpreted bearing in mind that

$E(\varepsilon_{N_1}, \varepsilon_{N_2}) = \sigma_{\varepsilon_{N_1}, \varepsilon_{N_2}}^2 = 0$  unless  $N_1 = N_2$ . Approximating the data by the observations, we may then write

$$\text{Cov}(b_p, b_{t+k}) = b_t b_{t+k} E(u_t u_{t+k}) \quad (28)$$

as the covariance between the measured values of  $b_t$  and  $b_{t+k}$ .

## 5.2. Interpolation between benchmarks

Frequent use is made of interpolation in data construction. This can be described in the following way. A trend line is drawn between the two successive observations of the benchmark variable,  $b_{N_1}$  and  $b_{N_2}$  and the variable used for interpolation  $i_{N_1}$  and  $i_{N_2}$  for observations in benchmark years  $N_1$  and  $N_2$ . The ratio of the indicator variable  $i_t$  to the trend value,  $[(N_2 - t)i_{N_1} + (t - N_1)i_{N_2}]/(N_2 - N_1)$  is then applied to the trend value of the benchmark  $[(N_2 - t)b_{N_1} + (t - N_1)b_{N_2}]/(N_2 - N_1)$  so that the interpolated value is given as

$$b_t = \frac{i_t [(N_2 - t)b_{N_1} + (t - N_1)b_{N_2}]}{[(N_2 - t)i_{N_1} + (t - N_1)i_{N_2}]} \quad (29)$$

The true values of the benchmarked data are denoted by  $b^*$  and it is, once again assumed that

$$b_N = b_N^*(1 + \varepsilon_N) \quad (30)$$

in a benchmark year, while the indicator is related to the true data as

$$i_t = kb_t^*(1 + v_t) \quad (31)$$

We may then write the estimated data as

$$b_t = b_t^*(1 + v_t) \left[ \frac{(N_2 - t)b_{N_1}^*(1 + \varepsilon_{N_1}) + (t - N_1)b_{N_2}^*(1 + \varepsilon_{N_2})}{(N_2 - t)b_{N_1}^*(1 + v_{N_1}) + (t - N_1)b_{N_2}^*(1 + v_{N_2})} \right] \quad (32)$$

To a first-order approximation we may then replace  $b_{N_1}^*$ ,  $b_{N_2}^*$  by  $b_{N_1}$ ,  $b_{N_2}$  and calculate the proportionate error in  $b_t$ ,

$$u_t = \frac{b_t}{b_t^*} - 1 = v_t + r_t \{ [N_2 - t]b_{N_2}\varepsilon_{N_1} + [t - N_1]b_{N_1}\varepsilon_{N_2} - [N_2 - t]b_{N_1}v_{N_1} + [t - N_1]b_{N_2}v_{N_2} \} \quad (33)$$

where

$$r_t = \frac{1}{(N_2 - t)b_{N_1} + (t - N_1)b_{N_2}} \quad (34)$$

From (32) it is a straightforward, if involved, matter to work out the autocorrelation between the measurement errors in estimates of  $b_t$  and  $b_{t+k}$  where  $t$  lies between  $N_1$  and  $N_2$  and  $t+k$  lies between  $N_3$  and  $N_4$ .



$$\begin{aligned}
E(u_t u_{t+k}) &= E(v_t v_{t+k}) - r_t b_{N_1} (N_2 - t) E(v_{N_1} v_{t+k}) \\
&\quad - r_t b_{N_2} (t - N_1) E(v_{N_2} v_{t+k}) - r_{t+k} b_{N_3} (N_4 - t - k) E(v_{N_3} v_t) \\
&\quad - r_{t+k} b_{N_4} (t + k - N_3) E(v_{N_4} v_t) \\
&\quad + r_t r_{t+k} \{ b_{N_1} b_{N_3} [N_2 - t] [N_4 - t - k] E(\varepsilon_{N_1} \varepsilon_{N_3} + v_{N_1} v_{N_3}) \\
&\quad \quad + [t - N_1] [t + k - N_3] b_{N_2} b_{N_4} E(\varepsilon_{N_2} \varepsilon_{N_4} + v_{N_2} v_{N_4}) \\
&\quad \quad + [N_2 - t] [t + k - N_3] b_{N_1} b_{N_4} E(\varepsilon_{N_1} \varepsilon_{N_4} + v_{N_1} v_{N_4}) \\
&\quad \quad + [t - N_1] [N_4 - t - k] b_{N_2} b_{N_3} E(\varepsilon_{N_2} \varepsilon_{N_3} + v_{N_2} v_{N_3}) \} \\
&= \sigma_v^2 \{ \rho^k - r_t b_{N_1} (N_2 - t) \rho^{t+k-N_1} - r_t b_{N_2} (t - N_1) \rho^{t+k-N_2} \\
&\quad - r_{t+k} b_{N_3} (N_4 - t - k) \rho^{N_3-t} - r_{t+k} b_{N_4} (t + k - N_3) \rho^{t-N_4} \} \\
&\quad + r_t r_{t+k} \{ b_{N_1} b_{N_3} [N_2 - t] [N_4 - t - k] \sigma_{\varepsilon_{N_1}, \varepsilon_{N_3}}^2 + \sigma_v^2 \rho^{|N_1 - N_3|} \\
&\quad \quad + [t - N_1] [t + k - N_3] b_{N_2} b_{N_4} [ \sigma_{\varepsilon_{N_2}, \varepsilon_{N_4}}^2 + \sigma_v^2 \rho^{|N_2 - N_4|} ] \\
&\quad \quad + [N_2 - t] [t + k - N_3] b_{N_1} b_{N_4} [ \sigma_{\varepsilon_{N_1}, \varepsilon_{N_4}}^2 + \sigma_v^2 \rho^{|N_1 - N_4|} ] \\
&\quad \quad + [t - N_1] [N_4 - t - k] b_{N_2} b_{N_3} [ \sigma_{\varepsilon_{N_2}, \varepsilon_{N_3}}^2 + \sigma_v^2 \rho^{|N_2 - N_3|} ] \} \quad (35)
\end{aligned}$$

on the reasonable assumption that the  $v_N$  and  $\varepsilon_N$  are uncorrelated. It may also be assumed that the measurement errors in the benchmarks,  $\varepsilon_{N_1}$ ,  $\varepsilon_{N_2}$ ,  $\varepsilon_{N_3}$  and  $\varepsilon_{N_4}$  are uncorrelated, so that the terms in the final two lines of expression (35) only make a contribution to the covariance when the interpolation periods coincide or are consecutive.

### 5.3. Autocorrelation between Interpolated and Extrapolated Data

The benchmarks used in interpolation will not, typically fall at the ends of the data period and it may in practice be the case that benchmarking does not extend to a time outside the data period of interest. For example, if the interwar data are considered in isolation (and this makes a great deal of sense because the wars led to major data discontinuities), then the earliest benchmark year is 1924, with the latest 1935 for many data series of interest. The implication is that the estimates before 1924 or after 1935 are constructed by extrapolation rather than interpolation. The autocorrelation in the measurement error between an interpolated estimate and an extrapolated estimate is given by a combination of the error specifications of the previous two sections. Suppose that  $t$  lies between  $N_1$  and  $N_2$  so that  $b_t$  is calculated by interpolation, while  $t+k$  lies above  $N_3$  so that  $b_{t+k}$  is constructed by extrapolation. The autocorrelation of the measurement errors is then given as

$$\begin{aligned}
E(u_t u_{t+k}) &= E(v_t v_{t+k}) - E(v_{N_3} v_t) \\
&\quad + E(\varepsilon_{N_3} \varepsilon_{N_1}) r_t b_{N_1} (N_2 - t) + E(\varepsilon_{N_3} \varepsilon_{N_2}) r_t b_{N_2} (t - N_1)
\end{aligned}$$

$$\begin{aligned}
& + E(v_{N_3}v_{N_1})r_t b_{N_1}(N_2 - t) + E(v_{N_3}v_{N_2})r_t b_{N_2}(t - N_1) \\
& - E(v_{N_1}v_{t+k})r_t b_{N_1}(N_2 - t) + E(v_{N_2}v_{t+k})r_t b_{N_2}(t - N_1) \\
& = \frac{\sigma_w^2}{1 - \rho^2} \{ \rho^k - \rho^{|t - N_3|} \\
& + \rho^{|N_3 - N_1|} r_t b_{N_1}(N_2 - t) + \rho^{|N_3 - N_2|} r_t b_{N_2}(t - N_1) \\
& + \rho^{|t+k - N_1|} r_t b_{N_1}(N_2 - t) + \rho^{|t+k - N_2|} r_t b_{N_2}(t - N_1) \} \\
& + \sigma_{\varepsilon_{N_3}, \varepsilon_{N_1}}^2 r_t b_{N_1}(N_2 - t) + \sigma_{\varepsilon_{N_3}, \varepsilon_{N_2}}^2 r_t b_{N_2}(t - N_1) \quad (36)
\end{aligned}$$

Once again the autocorrelation between different benchmark errors ( $\varepsilon_N$ ) may be assumed to be zero, so that the terms in  $\varepsilon_{N_1}, \varepsilon_{N_3}$  and  $\varepsilon_{N_2}, \varepsilon_{N_3}$  only enter if  $N_3$  coincides with either  $N_1$  or  $N_2$ . It cannot coincide with both.

Finally, if the series is extrapolated at both ends, then the autocorrelation will be given by equation (27), where  $N_1$  and  $N_2$  represent the observations from which extrapolation takes place.

Whichever method is appropriate for calculating the covariance between  $u_t$  and  $u_{t+k}$ , the final covariance between the observed data is given by equation (28).

#### 5.4. Covariance between Constant and Current Price Data

There is inevitably covariance between constant and current price data. If the constant price data are constructed by deflating the current price data by means of a deflator which is calculated separately, then the covariance between the two will equal the variance of the current price data, and the covariance between the constant price data in one period and the current price data in another period will equal the covariance between the current price data in the two different periods. If only a fraction of the volume data,  $x\%$  are constructed in this way, then the relevant variances and covariances will be multiplied by  $x^2$  (where  $x$  is assumed constant although, over long periods it will not be). This was described by Weale (1988) and Solomou and Weale (1991) but, at the disaggregate level considered here, is not a problem. The constant price series can be regarded as constructed either by means of deflation or by using volume indicators with reference to the year of the price base.

However wage estimates are calculated from estimates of employment and wage rates, while output estimates are calculated from estimates of employment. To the extent that the estimates of employment are incorrect, there will be covariance between the output and wage data, while to the extent that they are poor indicators of the change in output, there will not be. Since only relatively small parts of the series which we consider are measured from employment indicators, and since a large part of the output error will arise from the inappropriate use of the employment data, we have neglected this.

There are, however, two aspects of the current price/ constant price relationship which were not dealt with by Weale (1988) and Solomou and Weale (1991). The first arises when the constant price data are

constructed by means of volume indices applied to current price base year data. This means that any error in the base year data is reflected in all the constant price data; it generates a specific pattern of autocorrelation.

The variance of output in the constant price series for the base year must be the same as the variance shown in the current price series, because the numbers are exactly the same. The covariance between the two must equal the variance. If  $\sigma_{val,base}^2$  is the variance for the base year (taken from information on current price data), while  $\sigma_{vol,t}^2$  is the variance implied from the information about output reliabilities provided by statisticians and interpreted by means of the formulae of sections 5.2 and 5.3, then the variances actually used must be adjusted to

$$\sigma_{vol,t}^2 + \sigma_{val,base}^2 - \sigma_{vol,base}^2 \quad (37)$$

Since the base year adjustment is present in all the variances, the same adjustment,  $\sigma_{val,base}^2 - \sigma_{vol,base}^2$ , must be made to the estimates of the covariance between the errors in output in years  $t$  and  $t+k$ . The implication of this, of course, is that any percentage change made to the base year current price data is reflected in all the constant price data for the same variable, because constant price data are always measured relative to base.

The position is complicated further by the presence of autocorrelation in the current price series. Observation  $t+k$  in the current price series will have a covariance with the base year observation in the current price series. Because the base-year error in the current price series is added in to all the constant price data, the covariance between the constant price estimate for  $t$  and the current price estimate for  $t+k$  will equal the covariance between the current price estimate for  $t+k$  and that for the base year.

To the extent that constant price series are constructed by deflation, the issue is more straightforward but still requires some account to be taken of autocorrelation. The variances in the constant price series are the sum of those in the current price data and those in the price series. The covariance between the constant and current price data is then the same as that in the current price data, and the covariance in the constant price data is the sum of that arising from the current price series and from the construction of the price index itself. It is easier to measure short-term price changes than long-term price changes. The errors in the price series are likely to be strongly autocorrelated. By definition the measurement error in prices is zero in the base year. Strong autocorrelation implies that this source of error rises gradually to its asymptotic maximum, while weak covariance would imply a more sudden rise to the asymptotic maximum (4).

(4) The example considered by Weale (1988) assumed that there was no covariance at all, and that therefore prices relative to base were measured with the same reliability for years distant from the base as for those years close to the base.

There is, however, one important difference between the sort of autocorrelation which arises here and that which was found in section 4. There the variance of  $v_t$ , the autocorrelated component of the measurement error was constant,  $\frac{\sigma_w^2}{1 - \rho^2}$ . In the case of deflators, the errors in the price index in the base year are zero. They cumulate as one moves away from the base. If the proportionate error in the price series,  $\pi_t$ , follows the following pattern

$$\begin{aligned}\pi_t &= \rho_p \pi_{t-1} + w_t & t > \text{base year} \\ \pi_t &= \rho_p \pi_{t+1} + w_t & t < \text{base year} \\ \pi_{\text{base}} &= 0\end{aligned}$$

then the covariance between errors in a deflated constant price series is given as

$$\begin{aligned}E(u_t \mu_{t+k}) &= \text{covariance from current price data} \\ &+ \rho^k \frac{\sigma_w^2}{1 - \rho^2} (1 - \rho^{2|t+1 - \text{base}|})\end{aligned}\quad (38)$$

if  $t + k > t > \text{base}$  or  $t < t + k < \text{base}$  and

$$E(u_t \mu_{t+k}) = \text{covariance from current price data} \quad (39)$$

if  $t < \text{base} < t + k$ .

### 5.5. Unit Roots in Constant price Data

In the current price data the assumption that the error processes are stationary makes reasonable sense. Otherwise the errors would be expected to worsen with the passage of time, and there is no obvious reason why that should be the case.

With constant price data the situation is rather different. If the error pattern were stationary, it would follow that the annual average growth rate could be computed with a measurement error which would fall asymptotically to zero. This seems inherently unlikely. While it is reasonable to expect that the standard error with which the annual average growth rate is measured declines over time it should not be expected to decline to zero. The problems of measuring volume and price changes over long periods are well-known and are referred to briefly in section 4. We have thought it sensible to include a small unit root error process in the error attributed to each volume series and implicit price deflator.

### 5.6. Covariance Across Accounts

One important point which must be taken into consideration in the calculation of balanced estimates of national income is the question of covariance between entries on the two sides of the accounts. This can be expected to arise if the same data are used to make both entries. For example, the imputed income from ownership of dwellings is a

component of housing costs and is entered there both as income and as expenditure. Consumers' expenditure on domestic service is measured by employment and wage rates in domestic service and is a component of the output of the 'miscellaneous services' industry. Expenditure on public administration and defence is a part of public consumption. Measurement errors in any of these expenditure side variables will lead to equal errors in the output/income side variables, and any adjustment to one must lead to an equal adjustment in the relevant component on the other. Such an adjustment therefore cannot be used to allocate any residual error.

In this situation it is typically not the case that exactly the same number appears on both sides of the account. Rather an element on one side of the account includes as a component the entry in the other side of the account. The covariance between the two entries is then equal to the variance in the component. For example the covariance between spending on domestic service and output of miscellaneous services is equal to the variance of spending on domestic service. Since these data appear in both constant and current prices, the correction must be made to both. A covariance also exists between the constant price figure for spending on domestic service and the current price wage component of the value added of the miscellaneous services industry. This is equal to the covariance between the constant and current price estimates of domestic services expenditure. A similar covariance exists between current price domestic service consumption and constant price miscellaneous service output.

The examples above present situations in which the covariance between the two sides of the account is equal to the variance of the component concerned. A similar, but more complicated situation arises when consumption data are calculated from the supply side. The variance of the consumption data is equal to the sums of the variances of the output, import and export data plus an allowance for errors in distributive margins, and the covariance between consumption and supply is equal to the variance of the supply estimate concerned.

Nevertheless, the fact is that, even if common sources are used, the consumption and output data tend to be constructed by different people making different assumptions. In view of this we have neglected cross-account variance from this source.

## **6. Sectoral Accounts**

The balancing exercise which we have described has been carried out with income classified by industry. But an important part of national income accounting is the estimation of the incomes accruing to the institutional sectors of the economy, the personal sector, companies, public corporations, the government sectors and the rest of the world. One may also wish to identify current and capital expenditure by the institutional sector making it.

In principle this could have been done simply by extending our classification. Operating surplus could have been split into five categories as accruing to persons, companies, public corporations and central and local government <sup>(5)</sup>. Capital formation could have been decomposed in the same way.

But there were two practical reasons for doing this. First of all the introduction of the extra categories would have made the variance matrix unreasonably large. We already have to deal with over 500,000 non-zero elements. Secondly, it would have been necessary to take account of the covariances between the various sectoral estimates since they are clearly not independent of the industrial estimates <sup>(6)</sup>. These covariances could have been calculated but since all the calculations were performed with a sparse  $V$  matrix, they would, once again have led to a substantial increase in the effective size of the matrix. Accordingly, some way has to be found of allocating the adjustments to income and expenditure to the respective sectors if consistent sectoral accounts are to be produced.

With the major components of income and expenditure there is no problem. All the changes to employment income affect the incomes of the personal sector, and all the changes to personal consumption affect its expenditure. Public consumption accrues to one or other of the government sectors. But adjustments to operating surplus and capital formation must be classified by institutional sector in order to produce sector accounts. In fact this is quite consistent with the general approach we have adopted because there are no independent estimates of sectoral financial saving. Adjustment of factor incomes leads to offsetting changes in the estimates of financial saving and, when the GDP account is balanced, it is also true that financial saving will add to zero across the sectors (including the rest of the world). This means that we can apply a step-by-step approach to data reconciliation. Having calculated the adjustments by industry, these adjustments can then be allocated across the sectors on the basis of estimates of the reliability of sectoral components.

## 7. The Data and their Reliabilities

We have used two main sources of data for this study. Feinstein (1972) provides a series for 1920 (excluding the Irish Free State) to 1967, but we have relied on the more recent official sources for the period 1948-1990, taking our data from the tape consistent with the 1991

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(5) With none accruing to the rest of the world by convention. Income on direct investment is treated as being paid by the company sector and not as a direct payment of factor income.

(6) If they were, operating surplus calculated by industry would not equal the same aggregate calculated by institutional sector.

*Blue Book*. For 1946 and 1947 we have relied on Feinstein (1972) and Lomax (1959) to give information on data series not present on the CSO tape. It should be noted that there are no output estimates of GDP available during the second world war <sup>(7)</sup>.

The data tape does not provide all the necessary information back to 1948. In particular the estimates of value added at current prices by industry are not available before 1971. We have therefore taken the series published in the contemporary *Blue Books*, reclassified them, as far as possible to the 1980 Standard Industrial Classification and scaled them to be consistent with the aggregates which are available on the CSO tape.

Output indices are available back to 1948, but the component series are not consistent with the aggregates before 1983, when the data were first directly estimated in 1985 prices. This inconsistency is a consequence of rebasing. Nevertheless, output at constant prices should equal expenditure at constant prices and output by component should add to aggregate output. We have therefore scaled the components to be consistent with the CSO's aggregate output index. Similar problems arise with the components of consumption and capital formation; these have been dealt with in the same way.

Feinstein (1972) provides estimates of the reliability of his data. We have also referred to the sources on which Feinstein draws- Stone and Rowe (1966) for consumption, Chapman (1953) for wages and salaries and Feinstein (1965) for capital formation - to provide more detailed information. Postwar, data reliability is described by CSO (1985) with Kenny (1988) giving more specific results for the mid-1980s. We have assumed that standard errors lie at the mid-points of the quoted ranges. Where benchmarking is involved, they are assumed to rise asymptotically from the bottom to the top of the relevant range. The rate at which the error rises is determined by the serial correlation parameter.

There then arises the question of calibrating the variance matrices. The information available on data reliability says nothing about serial correlation in the measurement errors, and one might therefore consider using the serial correlation parameter as a means of adjusting the Mahalanobis statistic so that it equals the median  $\chi^2$  value. Obviously there is no guarantee that a satisfactory value of the Mahalanobis statistic will be delivered by a value of the parameter between 0 and 1.

We have in fact carried out the balancing exercise in three stages, reflecting the availability of the data. The periods 1920-38 and 1948-90 were treated quite distinctly. Balanced estimates were produced, with the volume data being calculated to 1938 prices in the first case and to 1985 prices in the second. The data for 1938-48 were estimated, at 1938

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(7) Austin Robinson has suggested that output data were not so readily available then because of potential use to the enemy.

and and current prices, by applying the two relevant accounting constraints and the additional restrictions that the current price data for 1938 and 1948 should be compatible with the results of the two earlier exercises and, additionally, that the 1938 constant price data should be equal to the results from the interwar balancing exercise.

Figures 2 and 3 show the values of the score criteria plotted against the serial correlation parameters for the inter-war and post-war periods.  $\rho_1$  is the parameter associated with the components of output and  $\rho_2$  is

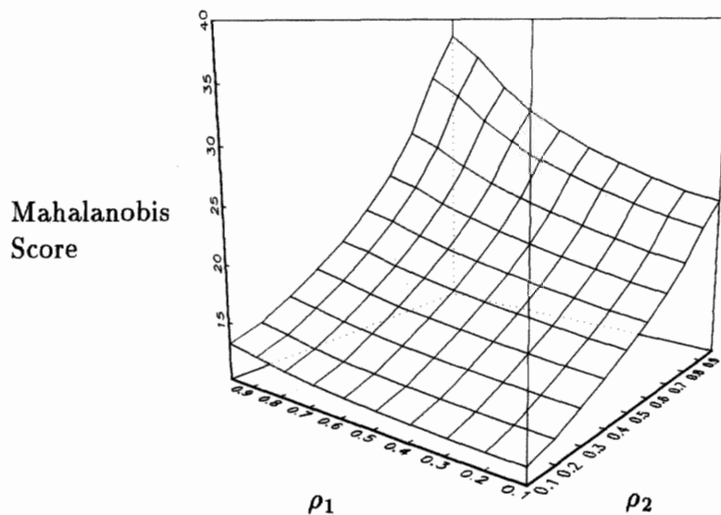


Figure 2 – The Mahalanobis Score, 1920-1938

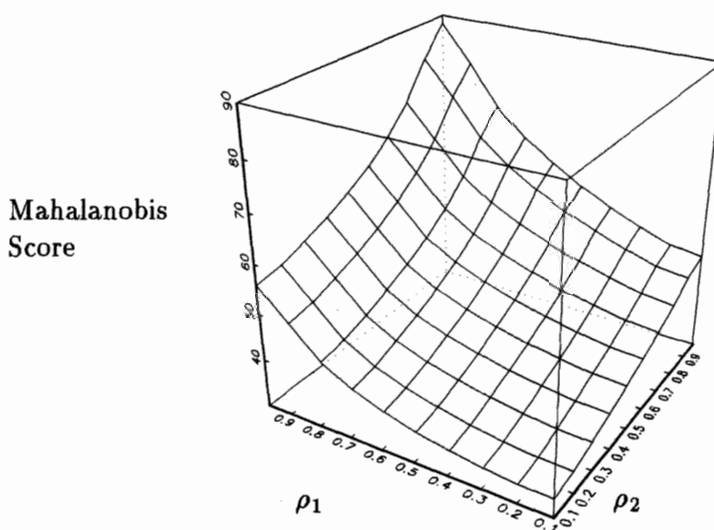


Figure 3 – The Mahalanobis Score, 1948-1990



applied to all the other items in which serial correlation is believed to be present. The period 1938-48 cannot be assessed helpfully in this way because of the extra restrictions needed to ensure that it fulfils its linking role. For the first period, low values of the serial correlation parameter deliver score criteria within the 95% confidence limits <sup>(8)</sup>, while in the second period,  $\rho$  has to take a high value for the score to fall within the 95% confidence limit.

While the standard errors of the year-one year changes in data are sensitive to the serial correlation coefficients, this is much less true once the variance matrix is scaled to yield the Mahalanobis score given by the median  $\chi^2$  statistic. Figures 4 and 5 show the *ex post* estimates of the average standard error of year-on-year growth in the balanced estimate of real GDP which emerges from the calculations after the variance matrix associated with each serial correlation coefficient has been scaled so that the Mahalanobis score takes its median value and compare these with the standard errors of the unbalanced estimates of the changes in GDP(E). There appears to be a certain robustness to the outcome provided the variance matrix is scaled.

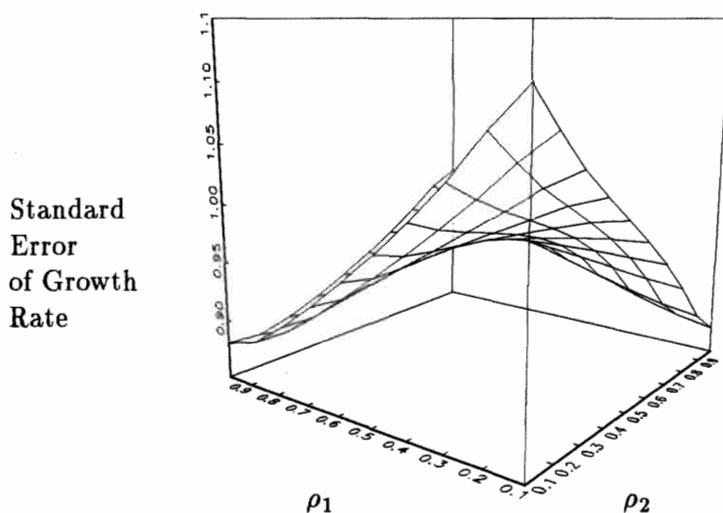


Figure 4 – The Standard Error of Growth in Balanced GDP, 1920-1938

We have settled for a value of 0.8 for the output serial correlation parameter and 0.6 for the expenditure parameter, relying on scaling to achieve the median value of the Mahalanobis criterion. This choice reflects the fact that very high values for the serial correlation coefficients leads to reliability increasing rather than decreasing between benchmark years.

(8)  $\chi^2_{37}(0.025) = 22.1$ ,  $\chi^2_{37}(0.975) = 55.7$ ,  $\chi^2_{85}(0.025) = 61.4$ ,  $\chi^2_{85}(0.975) = 112.4$ .

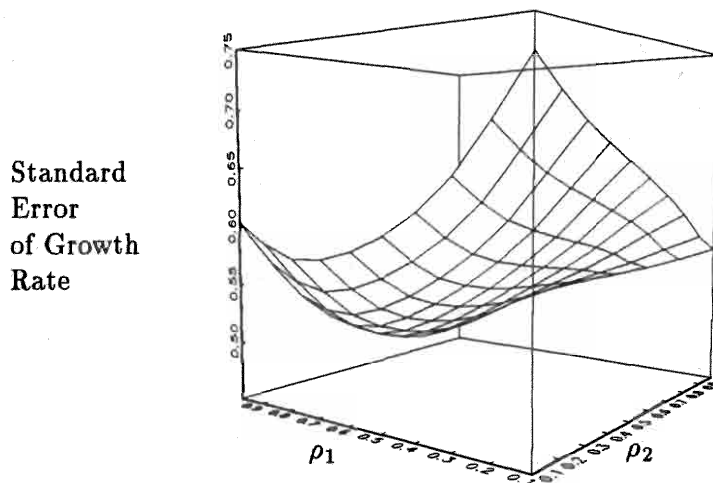


Figure 5 – The Standard Error of Growth in Balanced GDP, 1948-1990

## 8. Results

The output of the exercise is a new set of national accounts for the United Kingdom, 1920-1990 (Sefton and Weale, forthcoming). The dataset comprises full accounts for income and expenditure by sector and estimates of output classified by industry. We have attempted to reclassify the post-war data to the 1980 SIC; this means that, even without the balancing, our dataset offers a longer run of disaggregate post-war data at current prices than is available from existing sources.

Here we do not attempt to present the data tables, but limit ourselves to a graphical display of some of the points which emerge from the exercise. Figure 6 shows the difference between the growth rate as measured by the average measure of GDP and that calculated from our balanced estimate. The largest differences are found in the war-time periods. The reason for this is that, between 1939 and 1945 GDP(A) is calculated simply by deflating the average of income and expenditure by the expenditure deflator, while GDP(B) gives less weight to the income measure. Components of current price expenditure are adjusted to satisfy the accounting identity in current prices and the corresponding constant price components are adjusted through the effects of the covariances in the measurement error matrices. There is no constant price measure of output with which to reconcile the series. Before and after the war the changes are smaller and the average growth rates over long periods are scarcely changed.

One of the virtues of this method of dealing with the discrepancy is that adjustments are made to the components as well as to the aggregates. Figure 7 shows the changes to estimated growth in manufacturing with a gap between 1938 and 1946 when there are no output data available. The adjustments appear to be larger post-war, which might be rather surprising since a gain in accuracy would be

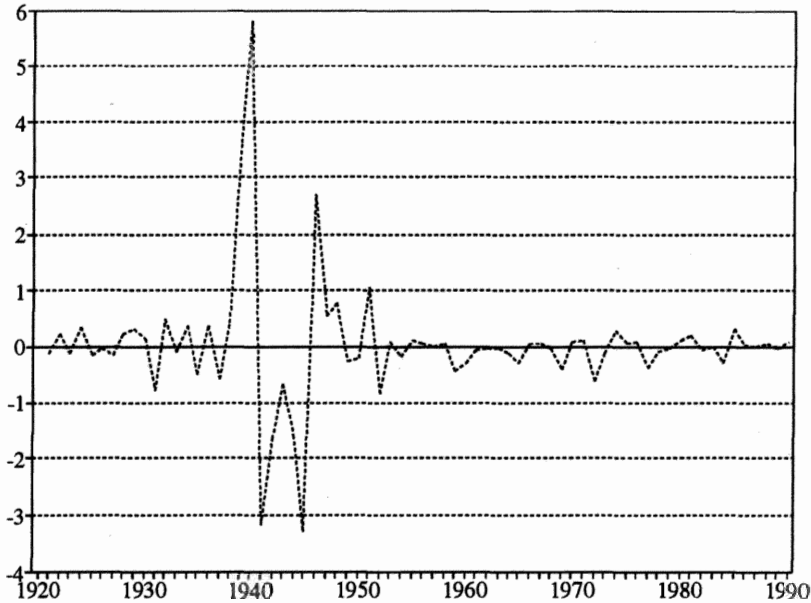


Figure 6 – Change in GDP(B) - Change in GDP(A) (%)

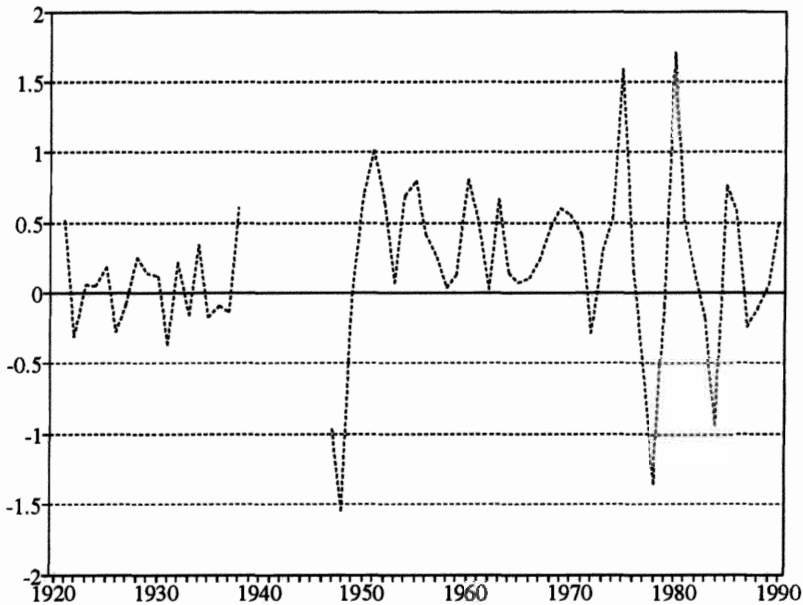


Figure 7 – Balanced - Actual Change in Manufacturing Output (%)

expected. But in fact there is a greater gain in accuracy by the measures of non-manufacturing output after 1948, and this explains why the adjustments to manufacturing become relatively larger. Whereas the balanced estimates of output growth were below the conventional estimate, the balanced estimate of manufacturing output growth is higher

than the official estimate. The explanation of this is quite straightforward. The output indices show lower growth than the expenditure indices; balancing reconciles them. In figure 7 the balanced estimate is compared with what the CSO describe as an average but is in fact probably close to an expenditure estimate (section 2). On the other hand the manufacturing component is compared against the output estimate. This draws attention to the hazards of comparing manufacturing growth with aggregate (GDP(A)) growth using the official data; the relative performance of manufacturing would be better than the official data suggest.

An analysis of the personal savings ratio in figure 8 shows more striking changes to the original data. In particular, during the period 1920-1938 the savings ratio appears to be considerably more stable than Feinstein's (1972) estimates imply. This observation is encouraging,

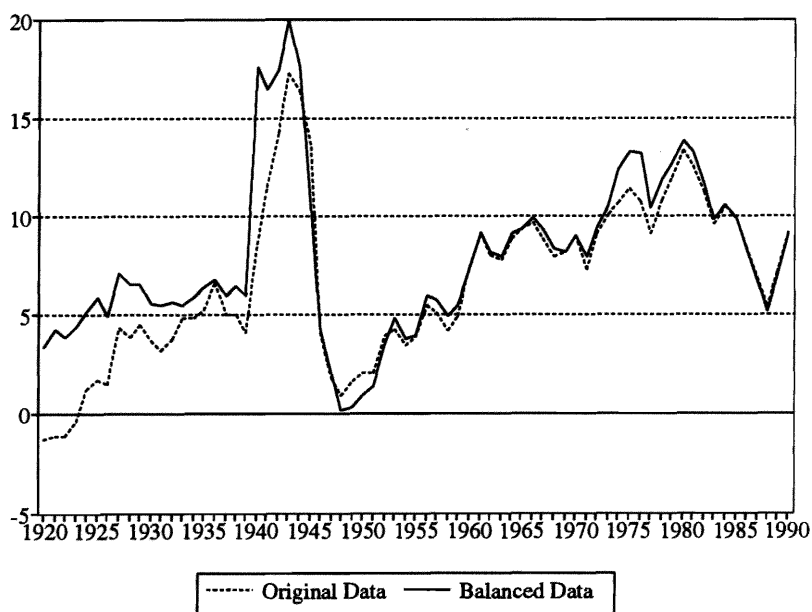
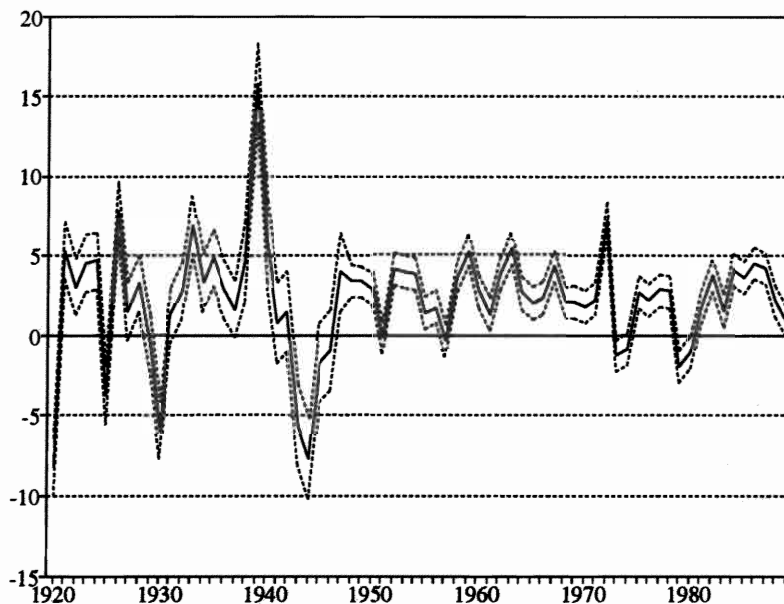


Figure 8 – Original and Balanced Estimates of the Savings Ratio (%)

because there is nothing in the inputs to the reconciliation process which would predispose such an outcome. After the war, the changes to the savings ratio are smaller, because the discrepancies in the current price data are smaller. However, during the mid-1970s there are marked changes to the savings ratio which might be important to anyone trying to fit a consumption function to the post-war data.

Finally, we noted that one aim of this work was to quantify the measurement errors which are likely to exist in the main macroeconomic aggregates and in their rates of change. In Figure 9 we plot the balanced estimate of growth which emerges, together with upper and lower confidence limits represented by twice the *ex post* standard



**Figure 9 – Confidence Intervals for Growth In Real GDP (% p.a.)**

deviations <sup>(9)</sup>. This does not imply that the errors in growth rates are serially independent. They are in fact negatively correlated, so that a value near the bottom of the range in one year will be associated with values near the top of the range in earlier and later years. Nevertheless, the graph provides a useful illustration of data uncertainty.

## 9. Conclusions

We have applied the method of least squares to allocate the discrepancies between the different measures of UK GDP. The results suggest that growth in manufacturing output may have been markedly understated in the period 1948-1970 and that the savings ratio was probably also understated in the 1920s and in the mid-1970s.

Our method allows an assessment of official statements on data reliability and allows us to calculate standard deviations for both levels and growth rates of data. The results suggest that the standard deviation with which growth in real GDP is measured is likely to be almost 0.5% of the level of GDP. Such a standard deviation is by no means trivial and should be borne in mind in assessment and use of the data.

(9) The standard deviations during the period 1939-1947 are calculated by scaling Feinstein's estimates in the same way as was done for the period 1920-1938.

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# ON BALANCING NATIONAL ACCOUNTS

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## 1. Introduction

In an article published in 1942, Stone, Champernowne and Meade (SCM) presented a methodology aimed at reconciling a given set of direct observations of economic items, most probably affected by errors of measurement, with the corresponding set of accounting identities, or, equivalently, with the corresponding set of integrated economic accounts. This method had been elaborated by the authors mainly for the very practical, immediate purpose of providing the first integrated set of national accounts for the British economy. These accounts, drawn up by the authors at the War Cabinet, have been published as an annex to the White Paper, written by John Maynard Keynes, to be circulated with the Budget for 1941 <sup>(1)</sup>.

In the following years, further analyses and applications by Stone, Byron, Weale, and others <sup>(2)</sup> have demonstrated both the theoretical soundness of this method and the extensiveness of its potential applications. The purpose of this paper is to provide a further contribution to the analysis of the theoretical bases of SCM methodology. Moreover, it aims to point out the implications of the SCM approach: National Accounts should be considered as sets of linearly dependent random variables, which serve the purpose of providing a data base for applied economic and econometric analysis. This is hardly possible if data are affected by measurement errors, as is almost always the case.

Therefore, paragraph 2 of this paper is devoted to a short summary of the problems which arise when the explanatory variables of an econometric model are affected by errors of measurement. In paragraph 3, I examine whether, under the assumption of independently, identically

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(1) See Pesaran, 1991.

(2) See, for example, Byron (1975), Weale (1992).

distributed measurement errors, a balanced set of economic time series provides a satisfactory set of instrumental variables for econometric modeling. In paragraph 4, the more complex hypothesis that measurement errors are generated by a stationary VAR(1) stochastic process is taken into consideration. It is proved that Stone's method can be viewed as an application of a time invariant Kalman filter. Maximum likelihood estimators of the state space equations parameters are then derived in the last paragraph of the paper.

## 2. Measurement errors and instrumental variables

Assume that the data base used for econometric modeling is a sequence of  $(l + 1)$  – dimensional random vectors  $\{V_t\}$ , such that

$$V_t \subset X_t \quad 2.1$$

$$X_t \subset R^q \quad 2.2$$

$l < q$ ,  $t = 1 \dots T$ . Assume also that each vector  $V_t$  can be partitioned as:

$$V_t = \begin{bmatrix} y_t \\ Z_t \end{bmatrix} \quad 2.3$$

where  $y_t$  has dimension 1 and  $Z_t$  has dimension  $l$ . Suppose further that from economic theory we know that it is possible to model the process which generates  $y_t$  conditionally to the values of the remaining random variables  $Z_t$ , more precisely as the linear model:

$$y' = Z'\beta + \varepsilon \quad 2.4$$

where  $y'$  is the  $T \times 1$  vector of the observed dependent variable,  $Z'$  is a  $T \times l$  matrix of observed explanatory variables,  $\beta$  is a  $l \times 1$  vector of unknown parameters and  $\varepsilon$  is a  $T \times 1$  vector of regression errors. The assumptions underlying the classical linear regression approach are well known:

- (1)  $[y_t, Z_t]$ ,  $t = 1 \dots T$ , are IID;
- (2)  $Z$  is exogenous, i.e. if  $f(y_t, Z_t; \theta)$  is the joint distribution of  $V_t$ , and  $f(y|Z; \theta)$ ,  $f(Z; \theta)$  are the conditional distribution of  $y$  and the marginal distribution of  $Z$  respectively, we must have:

$$f(Z; \theta) = f(Z; \theta_2)$$

$$f(y|Z; \theta) = f(y|Z; \theta_1) = \prod_1^T f(y_t|Z_t; \theta_1) \quad 2.5$$

$$\text{where } \theta_1 \in \Theta_1, \theta_2 \in \Theta_2, \Theta = \Theta_1 \times \Theta_2$$

As a consequence, the regression residuals  $\varepsilon_t$  are IID and:

$$E(\varepsilon_t|Z_t; \theta_1) = E(\varepsilon_t) = 0 \quad 2.6$$

since  $E(y_t|Z_t; \theta_1) = Z_t' \beta$ ;

$$\begin{aligned} \text{var}(\varepsilon_t|Z_t)(\theta_1) &= \text{var}(y_t|Z_t)(\theta_1) = \sigma^2 \\ E(\varepsilon_t Z_t|Z_t) &= 0 \\ t &= 1 \dots T \end{aligned} \quad 2.7$$

Equations 2.6 and 2.7, in particular the orthogonality condition, together with the condition that  $(ZZ')$  has full rank, ensure that the OLS estimator  $\hat{\beta}$  exists and is unique, that  $E(\hat{\beta}) = \beta$ , and that  $\text{var}(\hat{\beta}) = \sigma^2(ZZ')^{-1}$ . In other words,  $\hat{\beta}$  is Minimum Mean Square Linear Estimator (MMSLE). If, moreover,

$$\varepsilon_t \sim NID(0, \sigma^2) \quad t = 1 \dots T \quad 2.8$$

$\hat{\beta}$  is a Maximum Likelihood estimator, hence it is MMSE. Its asymptotic variance-covariance matrix is equal to the inverse of the asymptotic information matrix, i.e. to the Rao-Cramer minimum

$$\frac{1}{\sigma^2} W_z = \frac{1}{T\sigma^2} \text{plim}(ZZ') \quad 2.9$$

and

$$\sqrt{T}(\hat{\beta} - \beta) \rightarrow_D N(0, \sigma^2 W_z^{-1}) \quad 2.10$$

In this paper it will be assumed that the set  $V$  of direct economic observations is affected by measurement errors, i.e. that

$$\begin{aligned} V_t^* &= V_t - N_{vt} = \begin{bmatrix} y_t \\ Z_t \end{bmatrix} - \begin{bmatrix} v_{yt} \\ N_{zt} \end{bmatrix} \\ \begin{bmatrix} v_{yt} \\ N_{zt} \end{bmatrix} &\sim IID(0, \Sigma_N) \end{aligned} \quad 2.11$$

$t = 1 \dots T$ , and that, instead of the "true" relation

$$y^{*'} = Z^{*'}\beta + \varepsilon \quad 2.12$$

the "wrong" relation

$$\begin{aligned} y^{*'} + v_y' &= (Z^* + N_z)'\beta + \varepsilon = Z^{*'}\beta + \mu \\ \mu &= \varepsilon + N_z'\beta \end{aligned} \quad 2.13$$

is estimated. Assuming that

$$\begin{aligned} E(y_t^* v_{yt}) &= 0 \\ E(Z_t^* N_{zt}) &= 0 \\ \text{cov}(v_{yt}, N_{zt}) &= 0 \end{aligned} \quad 2.14$$

$t = 1 \dots T$ , we have

$$\begin{aligned} E[(y^{*'} + v_y')|(Z^* + N_z)] &= E[y^{*'}|(Z^* + N_z)] + E[v_y'|(Z^* + N_z)] = \\ &= Z^{*'}\beta + E[(\varepsilon + N_z'\beta)|(Z^* + N_z)] \neq Z^{*'}\beta \end{aligned} \quad 2.15$$

Since

$$\begin{aligned} E(Z\mu) &= E((Z^* + N_z)(\varepsilon + N_z'\beta)) = \\ &= E\{(Z^* + N_z)E[(\varepsilon + N_z'\beta) | (Z^* + N_z)]\} = \\ &= \beta' E(N_z N_z') = \beta' \Sigma_{N_z} \neq 0 \end{aligned} \quad 2.16$$

the OLS estimator

$$\hat{\beta} = \beta + (ZZ')^{-1}Z(\varepsilon + N_z'\beta) \quad 2.17$$

is biased and, since

$$\text{plim} \frac{1}{T} (ZN_z') \neq 0 \quad 2.18$$

the bias does not decrease if the number of observations increases over time <sup>(3)</sup>.

The alternative to least squares estimation exploits the possibility of finding a set of instruments  $\widehat{Z}$ , i.e. a set of variables closely enough related to  $Z$ , but linearly independent of their measurement errors  $N_z$ , at least asymptotically. In other words, the matrix  $\widehat{Z}Z'$  must have full column rank, but, at the same time, either  $E(\widehat{Z}\mu) = 0$ , or  $\text{plim} (1/T) (\widehat{Z}\mu) = 0$ . When both the expectation of  $(\widehat{Z}\mu)$  and its probability limit are unknown, they are usually replaced by sample averages, i.e. by

$$\frac{1}{T} \sum_1^T \widehat{Z}_t \mu_t = 0 \quad 2.19$$

If  $\widehat{Z}$  is of order  $(m \times T)$  this is a set of  $m$  equations in  $l$  unknowns. In applied economic analysis, usually  $m \geq l$ . If  $m = l$ , 2.19 has unique solution. The solution to the case where  $m > l$  was first suggested by Sargan (1958). It consists in computing a Generalized Instrumental Variables Estimator of  $\beta$ , that is a value of  $\beta$  which minimizes a normed Euclidean distance from 0 of expression 2.19 <sup>(4)</sup>.

In this paper I will examine the case in which the existence of measurement errors can be detected from the fact that the entire set of available direct observations at time  $t$ ,  $X_t$ , fails to satisfy a given set of accounting constraints  $GX_t = 0$ ,  $t = 1 \dots T$ , where  $G$  is a  $(k \times q)$  matrix of constants, the elements of which are either 1 or 0. It will be proved that, in this case, the balanced set of economic time series obtained by the SCM method provides a set of instruments, that is, a set of time series closely related to the "true" series and linearly independent of the measurement errors affecting direct observations.

(3) See White (1984), pp. 1-12. Notice that the bias of  $\hat{\beta}$  does not depend on  $v_y$ . The same holds for its variance and for  $\hat{\sigma}^2 = (1/(T-1)) \hat{\mu}' \hat{\mu}$ .

(4) See also White (1984), ch. 1; Cappuccio and Orsi (1991), pp. 264-266.

### 3. Balanced Time Series as Instrumental Variables

#### 3.a Definitions and assumptions

As in par. 2, define  $\{X_t\}$  as the sequence of  $(q \times 1)$  random vectors of available direct observations,  $t = 1 \dots T$ . Let

$x$  = vector ( $X$ ) be the column vectorisation of  $X$ ;  
 $G$  be the constant  $(k \times q)$  matrix of accounting constraints;  
 $U$  be the  $(k \times T)$  matrix of observed accounting discrepancies;  
 $U_t$  be the  $(k \times 1)$  vector of the observed discrepancies at  $t$ ;  
 $u$  = vector ( $U$ ) be the column vectorization of  $U$ , and set

$$\Gamma = I_T \otimes G \quad 3.1$$

Hence

$$\begin{aligned} GX &= U \\ \Gamma X &= u \end{aligned} \quad 3.2$$

At least define

$X^*$  as the  $(q \times T)$  matrix of the unknown "true" values of economic items, consistent with the set of accounting identities  $G$ ,  
 $X_t^*$  as the corresponding  $(q \times 1)$  vector at time  $t$ ,  
 $x^*$  = vector ( $X^*$ ), the column vectorization of  $X^*$ ,

$$\begin{aligned} N &= [v_{it}] = [x_{it} - x_{it}^*] \\ i &= 1 \dots q \\ t &= 1 \dots T \end{aligned} \quad 3.3$$

the  $(q \times T)$  matrix of measurement errors,  
 $N_t$  as the corresponding  $(q \times 1)$  vector at time  $t$  and  
 $v$  = vector ( $N$ ), the column vectorization of  $N$ .

I assume that

$$N_t \sim IID(0, \Sigma_N) \quad 3.4$$

and therefore

$$GN_t = U_t \sim IID(0, G\Sigma_N G') \quad 3.5$$

where  $\Sigma_U = G\Sigma_N G'$  is a non singular matrix of order  $k$ . Further I assume that

$$E(v_{it} | x_{it}^*) = 0 \quad 3.6$$

hence

$$\begin{aligned} E(v_{it} | x_{it}^*) &= E[(x_{it} - x_{it}^*) | x_{it}^*] = E(v_{it}) = 0 \\ E(x_{it} | x_{it}^*) &= x_{it}^* \end{aligned} \quad 3.7$$

and

$$\begin{aligned} GE(X_t|X_t^*) &= E(GX_t|X_t^*) = \\ E(U_t|X_t^*) &= E(U_t) = 0 \end{aligned} \quad 3.8$$

$i = 1 \dots q, t = 1 \dots T.$

### 3.b The balancing algorithm

Let

$$B \subset R^{q \times T} \quad 3.9$$

be the separable and measurable sample space of  $x$  and

$$A \subset B \quad 3.10$$

be the subset of points  $x$  which satisfy the set of linear constraints

$$\Gamma x = 0 \quad 3.11$$

hence dimension  $(A) = [k \times T]$  and  $x^* \in A$ . Given the set of direct observations  $x$ , suppose that we want to determine continuous linear functions  $\psi : R^{q \times T} \rightarrow R^{q \times T}$  and  $\phi : R^{q \times T} \rightarrow R^{k \times T}$ , such that

$$\begin{aligned} \psi(x) &= \hat{x} \subset B \\ \phi(\hat{x})x' \Gamma' &= 0 \end{aligned} \quad 3.12$$

Since  $GX_t = U_t \neq 0, t = 1 \dots T$ , 3.12 is verified only if  $\phi(\psi(x)) = 0$ . If we are only interested in the linear continuous transformations of  $x$  which satisfy the set of linear accounting constraints  $g_j \hat{X}_t = 0, j = 1 \dots k, t = 1 \dots T$ , we can express  $\phi$  as the set of all linear continuous functions which satisfy

$$\begin{aligned} \phi_j &= g_j \circ i : R^q \rightarrow R \\ j &= 1 \dots k \end{aligned} \quad 3.13$$

where  $i : A \rightarrow B$  is an inclusion mapping, defined by the correspondence  $i(x) = x$  if  $x \in A$ . Hence  $\phi$  is the restriction of  $\Gamma$  to  $A$ . Given  $x$  there are as many restrictions of  $\Gamma$  as there are points in  $A$ . It is therefore possible to determine  $\psi$  as the solution of any exogenously given optimization problem.

It might be convenient to restrict our attention to the points of  $A$  which satisfy the following constrained maximization problem:

$$\max_x E(\hat{X}_t X_t^* | X_t^*) = E[\hat{X}_t (X_t - N_t)' | X_t^*] \quad 3.14$$

sub

$$E(\hat{X}_t N_t' | X_t^*) = 0 \quad 3.15$$

$t = 1 \dots T$ . Substituting 3.15 into 3.14 we have

$$E(\hat{X}_t X_t^* | X_t^*) = E(\hat{X}_t X_t^* | X_t^*) = X_t^* X_t^* \quad 3.16$$

if

$$E(\widehat{X}_t | X_t^*) = X_t^* \quad 3.17$$

$t = 1 \dots T$ . In other words, the correlation between  $\widehat{X}_t$  and  $X_t^*$  is maximized, if, conditionally to  $X_t^*$ ,  $\widehat{X}_t$  is orthogonal to  $N_t$  <sup>(5)</sup> and the expectation of  $\widehat{X}_t$  is equal to  $X_t^*$ .

The SCM method determines  $\widehat{x}$  as that point in the subset A of B which minimizes the normed Euclidean distance of the observed point  $x$  from the unknown point  $x^*$ , i.e. as the solution to the following optimization problem:

$$\min_{x-x^*} (x - x^*)' (I \otimes \Omega)^{-1} (x - x^*) \quad 3.18$$

sub

$$\Gamma(x - x^*) = 0 \quad 3.19$$

where  $\Omega$  is an appropriately chosen symmetric p.d. matrix of constants. Since  $(x - x^*) = v$ , problem (3.18)-(3.19) can also be formulated as

$$\min_v v' (I \otimes \Omega)^{-1} v \quad 3.20$$

sub

$$\Gamma v = 0 \quad 3.21$$

This is formally equivalent to estimating the expectation of  $x$  conditional to  $x^*$ . Now,  $x^*$  is an unknown vector in this problem; on the other hand we have

$$E(x | x^*) - E(x^* | x^*) = E(x | x^*) - x^* = E(v | x^*) = 0 \quad 3.22$$

and

$$\Gamma E(x | x^*) - \Gamma x^* = \Gamma E(v | x^*) = 0 \quad 3.23$$

Hence an *unbiased* estimate of the expectation of  $x$  conditional to  $x^*$  can be regarded as an estimate of  $x^*$  itself. As is well known, <sup>(6)</sup> the solution to 3.18, 3.19 is

$$\begin{aligned} \widehat{X}_t &= X_t - \Omega G' (G \Omega G')^{-1} G X_t = \\ &= X_t - \Omega G' (G \Omega G')^{-1} G N_t \end{aligned} \quad 3.24$$

$t = 1 \dots T$ . Postmultiplying by  $X_t^{*'}$  and taking conditional expectations we have

$$\begin{aligned} E(\widehat{X}_t X_t^{*'} | X_t^*) &= E[\widehat{X}_t (X_t - N_t)' | X_t^*] = \\ &= E(X_t^* X_t^{*'}) + E(N_t) X_t^{*'} - \Omega G' (G \Omega G')^{-1} E(G N_t) X_t^{*'} = \\ &= X_t^* X_t^{*'} \end{aligned} \quad 3.25$$

(5) Hence  $\text{cov}(\widehat{X}_t, X_t | X_t^*) = \text{cov}(\widehat{X}_t, X_t^* | X_t^*)$ .

(6) See Byron (1978).

From 3.16 and 3.17 we know that if, conditionally to  $X^*$ ,  $\widehat{X}_t$  is unbiased, 3.25 implies that  $E(\widehat{X}_t N_t' | X_t^*) = 0$ . Under the assumptions of paragraph 3.a,  $\widehat{X}_t$  is unbiased, since  $E(N_t' | X_t^*) = 0$ . Hence, under the assumptions of paragraph 3.a, balanced items are unbiased estimators of the expectation of  $X_t$  conditional to  $X_t^*$ , closely related to the "true" economic time series, and orthogonal to their measurement errors. Moreover, conditional orthogonality implies, in this case, marginal orthogonality, since

$$\begin{aligned} E(\widehat{X}_t N_t') &= E[E(\widehat{X}_t N_t' | \widehat{X}_t)] = E[\widehat{X}_t E(N_t' | \widehat{X}_t)] = \\ &= E\{E[\widehat{X}_t E(N_t' | \widehat{X}_t) | X_t^*]\} = E[E(\widehat{X}_t N_t' | X_t^*)] = 0 \end{aligned} \quad 3.26$$

The variance-covariance matrix of  $\widehat{X}_t$  is

$$\begin{aligned} \Sigma_{\widehat{x}} &= E[(\widehat{X}_t - X_t^*)(\widehat{X}_t - X_t^*)'] = \\ &= \Sigma_N - \Omega G' (G \Omega G')^{-1} G \Sigma_N - \Sigma_N G' (G \Omega G')^{-1} G \Omega + \\ &\quad + \Omega G' (G \Omega G')^{-1} G \Sigma_N G' (G \Omega G')^{-1} G \Omega \end{aligned} \quad 3.27$$

independent of  $t$ . Setting  $\Omega = \Sigma_N$ , we have

$$\begin{aligned} \widehat{X}_t &= X_t - \Sigma_N G' (G \Sigma_N G')^{-1} G X_t \\ \Sigma_{\widehat{x}} &= \Sigma_N - \Sigma_N G' (G \Sigma_N G')^{-1} G \Sigma_N \leq \Sigma_N \end{aligned}$$

Pre and postmultiplying both sides of either 3.27 or 3.28 by  $G$  and  $G'$  respectively we obtain

$$G \Sigma_{\widehat{x}} G' = 0 \quad 3.29$$

Hence, in both cases, the entire mass of the distribution of accounting residuals  $U_t$  is concentrated on the single point 0, but only 3.28 is MMSLE. Provided that  $\Omega$  is a matrix of constants, two relevant small sample properties of  $\widehat{X}_t$  – unbiasedness and orthogonality – are unaffected by the choice of  $\Omega$ . However, this choice affects the variance of  $\widehat{X}_t$ ; it is therefore relevant for the eventual applied econometric analysis based on  $\widehat{X}_t$ .

### 3.c The choice of $\Omega$

In general, the "true" errors covariance matrix  $\Sigma_N$  is unknown. Weale (1992) suggests that the sample moment matrix of the data

$$W_T = \frac{1}{T} \sum_1^T \left( X_t - \frac{1}{T} \sum_1^T X_t \right) \left( X_t - \frac{1}{T} \sum_1^T X_t \right)' \quad 3.30$$

be used in place of  $\Omega$  in 3.18. He proves that

$$E(W_T G') = \Sigma_N G' + \frac{1}{T} \Sigma_N G' \quad 3.31$$

and therefore

$$plim(W_T G') = \Sigma_N G' \quad 3.32$$



Calling  $\tilde{X}_t(T)$  the estimator obtained by substituting  $W_T$  for  $\Omega$  in 3.18 and  $\tilde{\Sigma}_{\tilde{X}T}$  the estimator of its variance-covariance matrix, for any given  $t$  we have

$$\begin{aligned} plim [\tilde{X}_t(T)] &= plim [I - W_T G' (G W_T G')^{-1} G] X_t \\ &= [I - \Sigma_N G' (G \Sigma_N G')^{-1} G] X_t = \hat{X}_t \\ plim \tilde{\Sigma}_{\tilde{X}T} &= \Sigma_N - \Sigma_N G' (G \Sigma_N G')^{-1} G \Sigma_N \end{aligned} \quad 3.33$$

Since  $plim (W_T G')$  is a matrix of constants, equation 3.32 implies also

$$G[plim(W_T G')] = plim \frac{1}{T} \sum_{t=1}^T [G(X_t - \bar{X}_T)(X_t - \bar{X}_T)' G'] = G \Sigma_N G' \quad 3.34$$

where  $\bar{X}_T$  is the sample mean of  $X_t$ . We also have

$$\begin{aligned} plim \frac{1}{T} \sum_{t=1}^T (G X_t - G \bar{X}_T)(X_t' G' - \bar{X}_T' G') &= \\ = plim \frac{1}{T} \sum_{t=1}^T (U_t - \bar{U}_T)(U_t - \bar{U}_T)' &= \\ = plim \frac{1}{T} \sum_{t=1}^T U_t U_t' = \Sigma_U \end{aligned} \quad 3.35$$

since

$$plim \frac{1}{T} \sum_{t=1}^T (-\bar{U}_T U_t' - U_t \bar{U}_T' + \bar{U}_T \bar{U}_T') = 0 \quad 3.36$$

Then  $G W_T G'$  and

$$\hat{\Sigma}_U = \frac{1}{T} \sum_{t=1}^T U_t U_t' \quad 3.37$$

have the same probability limit. Equation 3.37 can also be written as

$$\hat{\Sigma}_U = G \left( \frac{1}{T} \sum_{t=1}^T N_t N_t' \right) G' = G \hat{\Sigma}_N G' \quad 3.38$$

or, equivalently

$$\hat{\sigma}_U = \text{vector } \hat{\Sigma}_U = (G \otimes G) \text{vector } \hat{\Sigma}_N = (G \otimes G) \hat{\sigma}_N \quad 3.39$$

Since both  $\hat{\Sigma}_U$  and  $\hat{\Sigma}_N$  are symmetric p.d. matrices a consistent estimate  $\hat{\Sigma}_N$  of  $\Sigma_N$  can be calculated as follows: define  $D$  a  $[k^2 \times \frac{1}{2}k(k+1)]$  matrix, the elements of which are either zero or one, such that

$$\hat{\sigma}_U = D \hat{\sigma}_{U^*} \quad 3.40$$

where  $\hat{\sigma}_{U^*}$  is the  $[\frac{1}{2}k(k+1) \times 1]$  vector of the unconstrained elements of  $\hat{\Sigma}_U$ . Call  $\Delta$  a  $[q \times \frac{1}{2}q(q^2 \times 1)]$  matrix such that

$$\hat{\sigma}_N = \Delta \hat{\sigma}_{N^*} \quad 3.41$$

where  $\hat{\sigma}_{N^*}$  is the  $[\frac{1}{2}q(q+1) \times 1]$  vector of the upper-triangular elements of  $\hat{\Sigma}_N$ . Equation 3.39 then becomes:

$$\hat{\sigma}_{U^*} = D'(G \otimes G) \Delta \hat{\sigma}_{N^*} \quad 3.42$$

where  $D'$  is the Moore-Penrose inverse of  $D$ . Hence the the upper-triangular elements of  $\hat{\Sigma}_N$  are

$$\hat{\sigma}_{N^*} = \Delta' [G'(GG')^{-1} \otimes G'(GG')^{-1}] D \hat{\sigma}_{U^*} \quad 3.43$$

where  $\Delta'$  is the Moore-Penrose inverse of  $\Delta$  and  $G'(GG')^{-1}$  is the Moore-Penrose inverse of  $G$ . Since the probability limit of  $\hat{\sigma}_{U^*}$  is  $\sigma_{U^*}$ ,  $\hat{\Sigma}_N$  is a consistent estimate of  $\Sigma_N$ . Unfortunately, it has an important drawback: it is a singular matrix. Equation 3.43 shows that  $k^2$  elements of  $\hat{\Sigma}_U$  are determined once  $\hat{\sigma}_{U^*}$  is known, and that the remaining ones are determined either by the structure of accounting residuals, or by symmetry constraints, therefore its rank is  $k$ .

$\{N_t\}$  is, by assumption, a stationary and ergodic process, so equation 3.43 must hold for the probability limit of  $\hat{\Sigma}_N$  and of  $\hat{\Sigma}_U$  as well; hence the rank of  $\Sigma_N$  cannot be greater than  $k$  (7). Now, we have seen that  $GW_T G'$  and  $G \hat{\Sigma}_N G'$  have the same probability limit; we may then conclude that  $W_T$  may be a full rank matrix because it is biased. Anyway, it is consistent and, moreover, equations 3.33 imply that, for any given  $t$

$$\tilde{X}_t(T) \rightarrow_D \hat{X}_t \quad 3.44$$

In practice, if  $W_T$  is non-singular, it might be preferable to use  $W_T$  rather than  $\hat{\Sigma}_N$  as a weighting matrix in 3.18 (8), although in either case one obtains estimates  $\tilde{X}_t(T)$  which are distributed asymptotically as  $\hat{X}_t$ .

In general, if the weighting matrix in 3.18 is random, orthogonality of the resulting estimates to measurement errors cannot be proved for small samples. However, if  $\hat{X}_t$  is IID, that is, if  $X^*$  is IID, then

$$\begin{aligned} \text{plim} \left( \frac{1}{T} \sum_{t=1}^T \tilde{X}_t(T) \otimes N_t \right) &= \text{plim} \left( \sum_{t=1}^T \tilde{X}_t(T) \otimes \frac{N_t}{T} \right) = \\ &= \text{plim} \left( \tilde{X}_1(T) \otimes \frac{N_1}{T} \right) + \left( \tilde{X}_2(T) \otimes \frac{N_2}{T} + \dots + \tilde{X}_T(T) \otimes \frac{N_T}{T} \right) = 0 \end{aligned} \quad 3.45$$

This can be proved as follows: from equations 3.33 and 3.44 we know that for any given  $t$  each element of  $\tilde{X}_t(T)$  converges in distribution

(7) This means that the distribution of the  $q$ -vector  $N_t$  is singular. Its probability mass is concentrated on the  $k$ -dimensional linear subspace of accounting constraints. Since a  $k$ -vector of accounting residuals  $U_t$  is associated with each realization of the random  $q$ -vector  $N_t$  and only  $U_t$  is observable,  $N_t$  is a linear transformation of  $U_t$ , say  $G'(GG')^{-1} U_t$ , is a point in a  $k$ -dimensional linear subspace and its distribution is non-singular. Hence the distribution of  $N_t$  is singular and the rank of its covariance matrix cannot be greater than  $k$ .

(8) See also Hansen (1982).

to the corresponding, *given* value of  $\widehat{X}_t$ ; hence, for any given  $t$ , each element of  $\widehat{X}_t(T)$  is at most of order 1 in probability (i.e.)  $o_p(1)$ . Since, for any given  $t$

$$plim \frac{N_t}{T} = 0 \quad 3.46$$

each element of  $(N_t)(T)$  is of order smaller than 1 in probability (i.e. is  $o_p(1)$ ). Then 3.45 is the sum of  $T$   $q^2$ -elements random vectors and each element of each vector is  $o_p(1)$ . Since their sum is also  $o_p(1)$  <sup>(9)</sup>, the orthogonality conditions are satisfied asymptotically.

### 3.d *Balanced time series in econometric applications*

Finally, let us examine the properties of the OLS estimators of an econometric model, the data base of which is a subset  $[\widehat{y}\widehat{Z}]'$  of  $\widehat{X}$ . To simplify the argument, I will assume that  $\widehat{y}_t$  is the first element of  $\widehat{X}_t$  that vector  $\widehat{Z}_t$  corresponds to the elements from 2 to  $(l+1)$ , and that  $(\widehat{Z}\widehat{Z})'$  has full rank. Instead of the true model

$$y^{*'} = Z^{*'}\beta + \varepsilon \quad 3.47$$

where  $y^*$ ,  $Z^*$  and  $\varepsilon$  satisfy the assumptions of the classical linear model, the following model

$$\widehat{y}' = \widehat{Z}'\beta + \delta \quad 3.48$$

is estimated. The OLS estimator is

$$\widehat{\beta} = \beta + (\widehat{Z}\widehat{Z})^{-1}\widehat{Z}\delta \quad 3.49$$

while from 3.47 and 3.48 we have

$$\delta = (\widehat{y}' - y^{*'}) - (\widehat{Z}' - Z^{*}')\beta + \varepsilon \quad 3.50$$

The  $i$ -th element of  $\widehat{X}_t$  can be written as

$$\widehat{X}_{it} = x_{it}^* + v_{it} + \sum_1^q \omega_{ij} v_{jt} \quad 3.51$$

where  $\omega_{ij}$  is the  $ij$ -th element of  $\Sigma_N G' (G \Sigma_N G')^{-1} G'$ , independent of  $t$ . Substituting in 3.50, we have

$$\begin{aligned} \delta_t = & v_{1t} + \sum_1^q \omega_{1j} v_{jt} - \sum_1^l \beta_l v_{(l+1),t} - \\ & - \sum_1^l \beta_l \sum_1^q \omega_{(l+1),j} v_{jt} + \varepsilon_t \end{aligned} \quad 3.52$$

hence, the  $h$ -th element of  $\widehat{Z}\delta$  is

(9) See White (1984), corollary 2.36, p. 26, and lemma 4.6, p. 63.

$$\sum_1^T \widehat{Z}_t (v_{1t} + \sum_1^q \omega_{1j} v_{jt} - \sum_1^l \beta_i v_{(i+1),t} - \sum_1^l \beta_i \sum_1^q \omega_{(i+1),j} v_{jt} + \varepsilon_t)$$
3.53

Orthogonality of  $\widehat{X}_t$  to all measurement errors at  $t$  and the usual assumptions about the distribution of regression errors  $\varepsilon_t$  imply that the expectation of 3.49 conditional to  $\widehat{Z}$  is  $\beta$ , since

$$E[(\widehat{Z}\widehat{Z}')^{-1}\widehat{Z}\delta | \widehat{Z}] = 0$$
3.54

If  $W_T$  is used in place of  $\Sigma_N$ ,  $\widehat{\beta}$  is biased in small samples. To satisfy the assumptions of the classical linear model,  $Z_t^*$  must be IID. From the conclusions of the preceding paragraph we know that in this case

$$plim \frac{1}{T} \widehat{Z}\delta = 0$$
3.55

We also know that  $Z_t^*$  is a subset of  $X_t^*$  such that its covariance matrix has full rank. Since

$$\begin{aligned} plim \frac{1}{T} \widehat{X}\widehat{X}' &= plim \frac{1}{T} \sum_{t=1}^T X_t^* X_t^{*'} + plim \frac{1}{T} \sum_{t=1}^T N_t N_t' - \\ &- plim W_T G' (G W_T G')^{-1} G \frac{1}{T} \sum_{t=1}^T N_t N_t' G' (G W_T G')^{-1} G W_T \quad 3.56 \\ &= \Sigma_N - \Sigma_N G' (G \Sigma_N G')^{-1} \Sigma_N G' + plim \frac{1}{T} \sum_{t=1}^T X_t^* X_t^{*'} \end{aligned}$$

we may conclude that if  $\{X_t^*\}$  is strictly stationary and, therefore, the limit of 3.56 is finite,  $\widehat{\beta}$  is consistent. If  $\{X_t^*\}$  is not strictly stationary, the highly simplifying assumption that  $\{N_t\}$  is not serially correlated can hardly be justified.

#### 4. VAR(1) Measurement Errors

##### 4.a SCM Method

In this paragraph the assumption that measurement errors are serially uncorrelated will be abandoned. Instead, it will be assumed that  $\{N_t\}$  is generated by a VAR(1) zero-mean stationary and ergodic stochastic process. Define  $X_{-1}$  and  $X_{-1}^*$  as the matrices obtained by deleting the  $T$ -th column of  $X$  and  $X^*$  respectively,  $x_{-1}$  and  $x_{-1}^*$  as their column vectorizations. Similarly, define  $U_{-1}$  as the matrix of accounting discrepancies after deleting its  $T$ -th column and  $u_{-1}$  its column vectorization. Assuming that

$$x_{it} = x_{it}^* + v_{it} \quad 4.1$$

$$v_{it} = \rho_{i1}v_{1,(t-1)} + \rho_{i2}v_{2,(t-1)} + \dots + \rho_{iq}v_{q,(t-1)} + \varepsilon_{it}$$

the  $q$  equations for the  $(T-1)$  time intervals may be written compactly as

$$x - Rx_{-1} = x^* - Rx_{-1}^* + \varepsilon \quad 4.2$$

where  $\varepsilon_t$  is assumed to be NID  $(0, \Sigma_\varepsilon)$ , orthogonal to  $v_{-1}$ ,

$$R = I_{T-1} \otimes P \quad 4.3$$

and  $P$  is a  $(q \times q)$  matrix of autocorrelation coefficients. Hence

$$\Gamma x - \Gamma R x_{-1} = \Gamma x^* - \Gamma R x_{-1}^* + e \quad 4.4$$

where  $\Gamma$  is now of order  $[k(T-1) \times q(T-1)]$ , and

$$e = \Gamma \varepsilon \quad 4.5$$

Now,

$$\Gamma x^* = 0 \quad 4.6$$

is true by definition. From the analysis carried on in section 3.c we know that this implies:  $\text{rank}(\Sigma_\varepsilon) = \text{rank}(\Sigma_e) = k$ . It also implies the condition

$$\Gamma R x_{-1}^* = 0 \quad 4.7$$

which <sup>(10)</sup> is verified if the elements of the  $i$ -th row of  $GP$  are

$$\begin{aligned} \sum_1^q g_{ij} \rho_{jl} &= r_i && \text{if } g_{il} \neq 0 \\ \sum_1^q g_{ij} \rho_{jl} &= 0 && \text{if } g_{il} = 0 \end{aligned} \quad 4.8$$

or, equivalently,

$$GP = rG \quad 4.9$$

where

$$r = \text{Diag}(r_1, r_2, \dots, r_k) \quad 4.10$$

This means that the diagonal elements of  $r$  are  $k$  characteristic roots of  $P$  and the rows of  $G$  are the characteristic vectors belonging to them. Therefore, equation 4.4 simplifies to:

$$u - I_{T-1} \otimes ru_{-1} = e \quad 4.11$$

(10) Equation 4.7 is a set of  $k$  restrictions on the column sums of  $P$ , analogous to those imposed in vector autoregressive simultaneous demand systems (see, among others, E.R. Berndt and E.N. Savin (1975)). These restrictions are often ignored in the literature on the construction and balancing of National Accounts, although it can be proved that they necessarily ensue from the assumption that errors are generated by a vector autoregressive process and from equation 4.6. A short proof was given in Antonello (1990). A more detailed proof is given in the appendix of this paper.

Restrictions 4.7 on the column sums of  $P$  might lead one to think that the stationarity of the errors generating process does not depend on the absolute values of  $r_i$ ,  $i = 1 \dots k$ , unless  $r_i$  is a multiple root of  $P$  <sup>(11)</sup>. In general this conclusion is wrong. In the present context, we know from section 3.c that  $N_t$  has a degenerate distribution and that its probability mass is concentrated in a  $k$ -dimensional linear subspace of  $R^q$ . Thus the rank of  $P$  cannot be greater than  $k$ . This means that the random  $q^2$ -vector of the components of  $P$  can be represented as a linear transformation of a random  $k^2$ -vector. Hence the opposite conclusion must hold:  $\{N_t\}$  is stationary if and only if  $\{U_t\}$  is stationary. In order to rule out the trivial case where all components of  $\{N_t\}$  are identically distributed and to make sure that the vector autoregressive error generating process is stationary, the following assumptions are needed:

*Assumption 1:*

$$r_i \neq r_j, \text{ some } i, i \neq j$$

*Assumption 2:*

$$0 \leq |r_i| < 1 \quad \forall i.$$

From the arguments developed in previous paragraphs, we know that only unbalanced items and accounting residuals can be observed. The vector-autoregressive error model can therefore be formulated in a state space form as follows <sup>(12)</sup>:

$$\begin{aligned} U_t &= rGN_{t-1} + e_t = \\ &= rU_{t-1} + e_t \end{aligned} \quad 4.12$$

$$N_t = PN_{t-1} + \varepsilon_t \quad 4.13$$

4.12 and 4.13 are the measurement and the transition equations respectively, and  $r$ ,  $P$  are time invariant coefficients matrices. The disturbance terms of the two sets of equations are, of course, not independent, since

$$e_t \sim NID(0, G\Sigma_e G') \quad 4.14$$

Notice, further, that 4.9 can be written as

$$GPG'(GG')^{-1} = r \quad 4.15$$

where  $G'(GG')^{-1}$  is the Moore-Penrose inverse of  $G$ , and that 4.15 is satisfied by setting

$$P = \Omega G'(G\Omega G')^{-1} rG \quad 4.16$$

(11) This statement can be found in Berndt and Savin (1975). In Antonello (1990) I erroneously accepted it.

(12) See Harvey (1989), ch. 3.

where  $\Omega$  is an appropriately chosen matrix of weights. Therefore, given consistent and asymptotically efficient estimates of  $r$ ,  $P$  can also be estimated.

The contemporaneous variance-covariance matrix of the observations  $U_t$  and the state variables  $N_t$  is

$$\text{var} \begin{bmatrix} N_t \\ U_t \end{bmatrix} = \begin{bmatrix} \Sigma_N & \Sigma_N G' \\ G \Sigma_N & G \Sigma_N G' \end{bmatrix} \quad 4.17$$

where

$$\text{vector}(\Sigma_N) = (I_{q^2} - P \otimes P')^+ \text{vector}(\Sigma_\varepsilon) \quad 4.18$$

and  $(I_{q^2} - P \otimes P')^+$  is the generalized inverse of  $(I_{q^2} - P \otimes P')$ . Then, the updating equation of state variables becomes

$$N_t = P N_{t-1} + \Sigma_N G' (G \Sigma_N G')^{-1} (U_t - G P N_{t-1}) \quad 4.19$$

or, equivalently

$$X_t^* - X_t = P(X_{t-1}^* - X_{t-1}) - \Sigma_N G' (G \Sigma_N G')^{-1} (U_t - r U_{t-1}) \quad 4.20$$

and, taking constraints 4.7 into account:

$$X_t^* - P X_{t-1}^* = (X_t - P X_{t-1}) - \Sigma_N G' (G \Sigma_N G')^{-1} G (X_t - P X_{t-1}) \quad 4.21$$

Now <sup>(13)</sup>,

$$P X_{t-1}^* = \Omega G' (G \Omega G')^{-1} r G X_{t-1}^* = 0 \quad 4.22$$

hence, balanced estimates  $\hat{X}$  of  $X^*$  can be obtained from

$$X_t^* = (X_t - P X_{t-1}^*) - \Sigma_N G' (G \Sigma_N G')^{-1} G (X_t - P X_{t-1}) \quad 4.23$$

if suitable estimates of system parameters  $r$ ,  $\Omega$  and  $\Sigma_\varepsilon$  are available. We may then conclude that:

– Also in the case where measurement errors are partly generated by a stationary vector autoregressive process, balanced estimates can be obtained by the usual methodology, applied in two stages. First, estimates of the vector autoregressive error component are deducted from the direct observations set; second, balanced estimates are obtained by subtracting estimates of the residual, normally distributed error component from the data computed at the first stage.

– Under the assumption of time-invariance of  $P$ , estimates at time  $t$ ,  $t = 2 \dots T$ , do not depend on the initial values of balanced items. Therefore, in practice, the balancing of accounts for the first time interval can be neglected.

(13) Equation 4.22 rules out the possibility that the components of  $X_t^*$  are generated by the same autoregressive process which generates measurement errors. It does not rule out the possibility that the components of  $X_t^*$  be serially correlated.

#### 4.b Estimation of system parameters

$\Sigma_\epsilon$  is square  $q$ -matrix. Let us call  $A$  the diagonal matrix of its non-zero eigenvalues and let  $C$  be the matrix of the right eigenvectors belonging to them. The generalized inverse of  $\Sigma_\epsilon$  is

$$\Sigma_\epsilon^+ = CAC' \quad 4.24$$

and

$$\Sigma_\epsilon^+ \Sigma_\epsilon = I_q \quad 4.25$$

Then, the log-likelihood function of 4.13 is

$$K - \frac{T-1}{2} \ln |\Sigma_\epsilon| - \frac{1}{2} \sum_{t=2}^{T-1} (N_t - PN_{t-1})' \Sigma_\epsilon^+ (N_t - PN_{t-1}) \quad 4.26$$

the last term of 4.26 can be written as

$$-\frac{1}{2} \text{tr} [\Sigma_\epsilon^+ (N - PN_{-1})(N - PN_{-1})'] \quad 4.27$$

and, after substituting from equation 4.16

$$-\frac{1}{2} \text{tr} \{ \Sigma_\epsilon^+ [N - \Omega G'(G\Omega G')^{-1} rGN_{-1}] [N' - N'_{-1} G'r(G\Omega G')^{-1} G\Omega] \} \quad 4.28$$

The ML-estimates of  $r$  and  $\Omega$  are the values which maximize

$$\begin{aligned} & \text{tr} \{ \Sigma_\epsilon^+ \Omega [NN'_{-1} G'r(G\Omega G')^{-1} G] \} - \\ & - \frac{1}{2} \text{tr} [\Sigma_\epsilon^+ \Omega G'(G\Omega G')^{-1} rGN_{-1} N'_{-1} G'r(G\Omega G')^{-1} G\Omega] \end{aligned} \quad 4.29$$

If we set  $\Omega = \Sigma_\epsilon$  equation 4.29 simplifies to

$$\begin{aligned} & \text{tr} [(G\Sigma_\epsilon G')^{-1} (GNN'_{-1} G'r - \frac{1}{2} rGN_{-1} N'_{-1} G'r)] = \\ & = \text{tr} [\Sigma_\epsilon^{-1} (UU'_{-1} r - \frac{1}{2} rU_{-1} U'_{-1} r)] \end{aligned} \quad 4.30$$

and the ML-estimates of  $r$  are:

$$\hat{r} = (U_{-1} U'_{-1})^{-1} U_{-1} U' \quad 4.31$$

Hence, in order to obtain ML-estimates of  $r$ , the contemporaneous covariance matrix of the Gaussian component of measurement errors must be used as a weighting matrix in 4.16.

The ML-estimator of  $\Sigma_\epsilon$  is

$$\hat{\Sigma}_\epsilon = \sum_{t=2}^T \frac{1}{T-1} \hat{\epsilon}_t \hat{\epsilon}_t' \quad 4.32$$



where

$$\hat{e}_t = U_t - \hat{r}U_{t-1} \quad 4.33$$

Consistent estimates of  $\Sigma_e$  can be obtained as suggested in section 3.c:

$$\hat{\sigma}_{e^*} = \Delta' [G'(GG')^{-1} \otimes G'(GG')^{-1}] D\sigma_{e^*} \quad 4.34$$

where  $\hat{\sigma}_{e^*}$  is the  $\frac{1}{2}q(q+1)$  – vector of the upper-triangular elements of  $\hat{\Sigma}_e$  and  $\hat{\sigma}_{e^*}$  is the  $\frac{1}{2}k(k+1)$  – vector of unconstrained elements of  $\hat{\Sigma}_e$ . Then, consistent estimates of the remaining system parameters  $P$  and  $\Sigma_N$  can be calculated simply by substituting  $\hat{r}$  and  $\hat{\Sigma}_e$  into equation 4.16 to obtain  $\hat{P}$  and by substituting  $\hat{P}$  in equation 4.18 to obtain  $\hat{\Sigma}_N$ . These parameters can then be used to obtain  $\tilde{X}_t$ .

Since  $\hat{r}$  is consistent

$$plim(\hat{r}) = r \quad 4.35$$

and

$$\sqrt{T-1} \text{ vector}(\hat{r} - r) \rightarrow_D N(0, \Sigma_e \otimes \Sigma_U^{-1}) \quad 4.36$$

where  $\Sigma_U$  is obtained from

$$\text{vector}(\Sigma_U) = (I_{k^2} - \hat{r} \otimes \hat{r})^{-1} \text{vector}(\Sigma_e) \quad 4.37$$

From paragraph 3.c we know that

$$plim(\hat{\Sigma}_e) = \Sigma_e \quad 4.38$$

$$plim(\hat{\Sigma}_e) = \Sigma_e \quad 4.39$$

then, from equations 4.35 and 4.39 we have

$$plim(\hat{P}) = P \quad 4.40$$

and from equations 4.23, 4.38, 4.39, 4.40 we obtain

$$plim(\tilde{X}_t) = X_t^* \quad 4.41$$

and

$$\tilde{X}_t \rightarrow_D X_t^* \quad 4.42$$

hence

$$plim \frac{1}{T-1} \sum_{t=2}^T (\tilde{X}_t \otimes \varepsilon_t) = 0 \quad 4.43$$

Further, from equations 4.2 and 4.7 we know that

$$X - \hat{P}X_{-1} = X^* - PX_{-1}^* + (P - \hat{P})X_{-1} + \varepsilon = X^* + \varepsilon + (P - \hat{P})\varepsilon_{-1} \quad 4.44$$

therefore

$$\begin{aligned}
 \text{plim } \frac{1}{T-1} (\tilde{X}\tilde{X}') &= \Sigma_\varepsilon - \Sigma_\varepsilon G' (G\Sigma_N G')^{-1} G\Sigma_N - \Sigma_N G' (G\Sigma_N G')^{-1} G\Sigma_\varepsilon + \\
 &+ \Sigma_N G' (G\Sigma_N G')^{-1} G\Sigma_\varepsilon G' (G\Sigma_N G')^{-1} G\Sigma_N + \text{plim } \frac{1}{T-1} X_i^* X_i^{*'}
 \end{aligned}
 \tag{4.45}$$

The assumption that  $X_i^*$  has finite contemporaneous covariances is sufficient to ensure that the probability limit of 4.45 is finite. If this condition is verified, any subset  $\tilde{Z}_i$  of  $\tilde{X}_i$  such that its contemporaneous covariance matrix is non singular can provide a suitable set of instruments for applied econometric analysis.

### Appendix

Assume that:

$$U_t = GN_t = GPN_{t-1} + G\varepsilon_t = GPN_{t-1} + e_t \quad \text{A.1}$$

and

$$U_{t-1} = GN_{t-1} \quad \text{A.2}$$

where  $U_t$ ,  $e_t$ ,  $N_t$ ,  $\varepsilon_t$ ,  $G$  and  $P$  are the vectors and matrices defined in sections 3 and 4 and

$$GP \neq 0 \quad \text{A.3}$$

We shall prove the following proposition:

A matrix  $Q \neq 0$  exists, such that

$$U_t = QU_{t-1} + e_t \quad \text{A.4}$$

and therefore

$$GP = QG \quad \text{A.5}$$

*Proof:*

The autocovariances of  $U_t$  and  $U_{t-1}$  are

$$E(U_t U_{t-1}') = E(GPN_{t-1} N_{t-1}' G' + e_t N_{t-1}' G') = GP \Sigma_N G' \neq 0 \quad \text{A.6}$$

Hence  $U_t$  and  $U_{t-1}$  are linearly dependent. Suppose that  $e_t = 0$ ,  $t = 2 \dots T$ . Then a linear continuous operator  $T$  on  $R^k$  exists, such that  $U_t$  is the image of  $U_{t-1}$  under  $T$ . Since both maps  $GP$  and  $G$  are onto and dimension  $(U_t) = k$ ,  $T$  is one-one and onto. Let us call  $Q$  the matrix of  $T$  in standard coordinates. Then  $Q$  has full rank. We then have

$$U_t = QU_{t-1} \quad \text{A.7}$$

and substituting from equations A.1 and A.2 we have

$$GPN_{t-1} = QGN_{t-1} \quad \text{A.8}$$

$t = 2 \dots T$ . Therefore

$$GP = QG \quad \text{A.9}$$

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**IV SESSION**  
**SOCIAL ACCOUNTING AND STATISTICS**

Chairman: Ugo Trivellato (*University of Padova – Italy*)



# BALANCING HIERARCHICAL REGIONAL ACCOUNTING MATRICES

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## 1. Background

### 1.1. Queensland and the input-output problem

Queensland is the third largest State of the Australian Federation in terms of population and economic activity. The population of the State at December 1992 was 3,068,100 persons, or 17.5 per cent of the national population. The States of Australia have fiscal powers and responsibilities protected by the Constitution, and have been increasingly concerned with questions of economic development and growth for both their States as a whole and the regions within the States.

This has resulted in a substantial demand for official estimates of economic structure and performance. In large part, this has been met by the national statistical agency, the Australian Bureau of Statistics (ABS), which, following the integration of the State Government statistical agencies (originally formed in colonial times), has serviced the statistical needs of both the State and Commonwealth (federal) Governments. While from most points of view the range and quality of statistical collections and services have been adequately and appropriately maintained by the national agency, increasing demands for State specific statistical expertise in the 1970s and 1980s led to the re-establishment in 1984 of the Queensland Government's own statistical agency, the Government Statistician's Office (GSO), which is now part of the Queensland Treasury (1).

Part of this renewal of interest in State statistical needs and priorities has been reflected in a program in Treasury and the GSO to develop economic accounts for the State which extend those provided by the ABS. This program includes the *Queensland State Accounts* (a quarterly

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(1) The ABS is also effectively part of the Federal Treasury Department.

set of State income, expenditure and product accounts) and the *Queensland Input-Output Tables*.

The State input-output tables have three main uses:

- a description of the structure of the State economy;
- an input database for the Queensland Treasury computable general equilibrium model of the State economy (Queensland Treasury (1993a)); and,
- an input database for the calculation of impact multipliers for the measurement of the effects on the State economy and Budget of special events and other projects.

While the main focus of interest for the State Government and Treasury is the structure and performance of the aggregate State economy, there has nevertheless been a considerable focus of interest in the sub-State regions of Queensland, given their diversity in economic structure and performance. This has been reflected in the private sector construction of many individual sub-State regional input-output tables of varying quality. Since these tables have been estimated largely on the basis of individual regions, there is little congruence among them, if any, and consequently there is no guarantee that they will consolidate to agree with the more "accurate" representation of the aggregate State economy.

To ensure that proponents of regional development can access tables which can reasonably be thought to relate accurately to the State economy, both in residual regional parts and in whole, the GSO has undertaken a program to estimate a set of inter-regional input-output tables for the regions of the State. These tables use as much primary data as possible, conform to accepted standards of input-output table construction, and consolidate to, and are therefore congruent with, the aggregate State table.

While many other sets of inter-regional input-output tables have been constructed, not just in Australia, but world-wide, there does not appear to have been much attention directed to the question of congruence in regional hierarchies. Furthermore, the issue of balancing the row and column constraints of individual regional input-output tables appears to have been traditionally handled by either forming residual balancing columns or by RASing the tables.

It is the contention of this paper that these questions are better handled in a Stone-type SAM balancing framework, and that advances in computational software and hardware have made this a feasible, indeed quick and easy, procedure.

This paper reports on the adjustment of a large accounting matrix using Stone's method and Byron's (1978) conjugate gradient algorithm. This now contradicts van der Ploeg's (1984, p. 17) view, that the claim that large systems could be adjusted by these methods was *somewhat over optimistic given current computer technology* and that *the adjustment of large accounting matrices has not been reported in the literature*.



## 1.2. The Stone solution

Ensuring that accounting constraints are satisfied in input-output matrices, and more generally, social accounting matrices (SAMs), has always been recognized as an important, indeed necessary, part of their construction process. The methods of ensuring this compliance, however, are varied. The two main or traditional approaches in regional input-output studies are:

- the RAS technique (Bacharach (1970)) and its variants, including Theil's (1965) information entropy method; and,
- residual sinks or residuals, where a particular row or column bears the entire brunt of the discrepancies in the system of accounts (Jensen and West (1986), West *et al* (1989) and Bayne and West (1989)).

Neither of these solutions appeals as a satisfactory method of constructing or adjusting large systems of inter-regional input-output tables. The disadvantages of RAS have been listed by van der Ploeg (1982, p. 170) as including: *that it is not straightforward to include more general linear restrictions, to allow for different degrees of uncertainty in the initial estimates and restraints, and more seriously the economic interpretation of pro rata adjustments to the accounts is dubious.*

The nature of the second traditional method is even more dubious. The method consists, after generating as many adjusted cells as possible from initial pro-rating or survey procedures, of assigning all of the remaining discrepancies in the accounts to, say, a column of "other final demand" or some such balancing item. Jensen and West (1986, pp. 96-7) broadly describe their procedure as follows,

*Manual balancing of the table usually continues until the relative error (difference between sector gross inputs and outputs) is reduced to an acceptable level, e.g. to about 5 per cent or less. If the analyst has ensured to the best of his ability that the significant cells are accurate, then any remaining errors should not have a significant effect on the results of the impact study. This final error is usually removed by mechanical means.*

*Mechanical balancing of an input-output table can be done in a number of ways. Some tables contain a "residual" sector, such as Gross Operating Surplus or Change in Stocks, which is used to absorb any minor adjustments which inevitably take place over time. A more complex procedure is the RAS, a biproportional method, in which the base matrix is adjusted to fit a set of new row and column totals (Gross Outputs and Inputs) via an iterative procedure involving alternate proportional row adjustments and column adjustments. The RAS method is popular because of its mathematical simplicity, and its theoretical properties are well known. (These are basically: (1) preserve the sign of the coefficient, (2) satisfies minimum distance criteria, (3) cannot make a zero cell non-zero, (4) cannot be used with negative entries.) A variation is the so-called "modified" RAS procedure, whereby cells which have been deemed to be accurately preestimated are locked-in prior to the application of the RAS procedure. In this way, only coefficients of which the analyst is*

*uncertain are allowed to adjust in final balancing. Other mechanical adjustment procedures are available, such as the Friedlander method, but these appear to be rarely used by applied regional analysts...*

Bayne and West (1989, p. 19) are more specific, stating that, in the so-called GRIT procedure, *The final transactions table will contain only three final demand columns: Household Consumption, Exports and Other Final Demand. The other Final Demand column is calculated as a residual, i.e. the difference between total inputs and total outputs.*

The method used in this paper was first outlined by Stone *et al* (1942) and was again suggested in Stone (1975). However, as Stone (1988, p. 23) himself recounts, it was not until Byron's contribution in 1978 that the method was first effectively used, probably due to the difficulty in implementing the method in practice, because of the degree of computational burden involved <sup>(2)</sup>.

As improved computing capability has become available, and using the conjugate gradient algorithms suggested by Byron, the method has increasingly begun to be used in practice, mainly on national income and expenditure accounts, (examples include Stone (1981), van der Ploeg (1982), Weale (1985), Crossman (1988), Solomou and Weale (1991), Crossman *et al* (1992) and Byron (1993)). There has also been progress in the balancing of SAMs using Stone's method, with the balancing of the large scale UK SAM by the Cambridge Growth Project (Barker *et al* (1984) and van der Ploeg (1984)), and other SAMs (eg, Pyatt and Round (1985b)).

In summary, Stone's method uses a mathematical technique to balance the system of accounts (or a SAM) after assigning reliability weights to each item in the system. The mathematical technique involves the minimisation of the sum of squares of the adjustments between original items and balanced items in the system, weighted by the reliabilities, or the reciprocal of the variances of the items, subject to accounting constraints (see Byron (1978) and Stone (1981)). This, as Harrigan (1990) has shown, does improve the information content of the final system of accounts.

## **2. The Queensland Regional Accounting Matrix**

### **2.1. Queensland State table**

In July 1990, the Queensland Government Statistician's Office (GSO) published *Experimental Input-Output Tables, Queensland*,

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(2) Stone (1988, p. 23) also suggested that the idea may not have caught on earlier because of *doubts about subjective estimates of reliability*. However, given the surge of balancing projects which has occurred since Byron's 1978 paper, which made the computational side feasible, it seems that the statistical courage which characterised the 1930s (Patinkin (1976, p. 113-4)) is still present, and has not been the reason for holding back the use of the balancing method.

1985-86. This was a 27 industry table, with six primary inputs and seven final demand categories. The classification of primary inputs in the Queensland input-output table follows the ABS Australian table (ABS(1990)), except that imports are not split into competing and complementary imports. Similarly, indirect taxes were not split between commodity taxes and other indirect taxes.

Details of the primary input categories are set out below:

- P1 Wages, salaries and supplements;
- P2 Gross operating surplus;
- P3 + P4 Indirect taxes (net);
- P5 Sales by final buyers;
- P6 Foreign and interstate imports of goods and services;
- P7 Duty on foreign imports.

The final demand categories follow the Australian table and are set out below:

- Q1 Final consumption expenditure – private;
- Q2 Final consumption expenditure – government;
- Q3 Gross fixed capital expenditure – private;
- Q4 Gross fixed capital expenditure – public enterprises;
- Q5 Gross fixed capital expenditure – general government;
- Q6 Increase in stocks;
- Q7 Foreign and interstate exports of goods and services.

The table records transactions at basic values with direct allocation of imports. That is, imports are recorded in the table as imports to the sector which uses them.

The first stage of compilation of this input-output table involves the estimation of output for the 27 industries on the same basis as the Australian input-output table.

As any input-output table is a detailed dissection by industry of the domestic production account, the next stage involved the estimation of the domestic production account for Queensland. In principle, the total for the primary input rows equates to "gross state product at market prices plus imports"; and the total of the final demand columns equates to "expenditure on gross state product less imports".

Data published by the ABS in *Australian National Accounts, State Accounts* (ABS (1992)), supplemented by data from *Queensland State Accounts* (Queensland Treasury (1993b) <sup>(3)</sup>), form the foundation of the State domestic production account on which the Queensland input-output table is based. Estimates of the value of foreign and interstate exports and imports of goods were obtained from the ABS publication *Interstate and Foreign Trade, Queensland* (ABS(1987)).

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(3) *Queensland State Accounts* is expected to be published by Treasury in late 1993.

The experimental input-output table for Queensland was produced using the modified RAS method <sup>(4)</sup> applied to the 1983-84 Australian table as the base table. The initial table produced by the simple RAS method was modified with a substantial amount of Queensland data. Modifications incorporated into the table include:

- Wages, salaries and supplements for the 27 industries;
- Gross operating surplus for the 27 industries;
- Exports by industry;
- Other significant cells where data was available from ABS and other sources.

The 1985-86 Queensland input-output table has been subsequently further modified in line with the 1986-87 ABS Australian tables and updated to be consistent with revised estimates for *Queensland State Accounts*.

## 2.2. Regional tables

The next stage of the input-output work undertaken by the GSO was to produce regional tables for each of Queensland's ten Statistical Divisions or regions <sup>(5)</sup>.

While it is routinely possible to produce the ten single regional input-output tables using cross industry location quotients based on the updated 1985-86 Queensland table and then the modified RAS method, a procedure was required to ensure the individual regional tables consolidate to the Queensland table.

The approach taken was to formulate the ten regional tables and the State table into a regional social accounting matrix (RAM). As the individual regional tables were initially compiled independently, they of course did not necessarily consolidate to the Queensland table and, therefore the initial RAM did not balance. A balancing method was therefore required.

The method adopted was that of Stone *et al* (1942). The advantage of using the Stone approach is that it allows for the assessment and incorporation of the individual reliability of individual cells, or individual blocks of cells, of the ten single regional tables and the individual inter-regional trade flows, as well as the State table, as these reliabilities are determined or judged by the analyst or statistician.

The procedure adopted for the initial construction of the table was as follows:

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(4) It is expected that subsequent official State tables will use the Stone method, rather than the modified RAS method.

(5) A map showing the ten regions is provided in Appendix A, as well as the domestic production accounts, both un-balanced and balanced, of the ten regions.

- The State table is thought to contain "superior" <sup>(6)</sup> data, with higher degrees of reliability than the regional tables. This arises from the fact that there is considerably more data of better quality for the State as a whole than for the regions of the State. This occurs because of a number of factors, including the design of statistical collections (eg, the use of sample surveys which are designed to produce accurate estimates at a State level, but are subject to higher standard errors and confidentiality restrictions at the regional level) and an increasing movement to produce "regional" data only at the State level.
- This means that hierarchical balancing, or ensuring that consolidation restrictions apply, is a necessary part of the construction of the State regional tables, as well as being important as a precondition for meaningful analysis for policy purposes.
- National accounts estimates of the domestic production accounts of the individual regions were constructed by the GSO. These were balanced or consolidated to the *Queensland State Accounts* total using Stone's method.
- Gross outputs were determined for each of the regions for the twenty-seven industries using economic census (for Manufacturing, for Electricity, Gas and Water, for Agriculture and for Mining), labour force surveys, the population census and other statistical collections (including tourism surveys).
- Location quotient techniques were then used to construct the disaggregation of the four input-output quadrants.
- Inter-regional trade flows were then allocated by industry to regions, and adjusted by the GSO statisticians.
- The statisticians then examined the tables manually, and edit checked large cells, known cells and rest of the world trade flows. The tables were then at a stage where they could be RASed, if thought necessary. Since Stone's method can cope with balancing the individual regional matrices, as well as the consolidation problem, this was unnecessary.
- The balancing programme could then be run. Results were then edit checked by the statisticians. The technique immediately indicated its worth as a powerful aid to the compilers, by identifying areas of concern. In some cases these concerns were negative numbers, in other cases there were relatively major adjustments. The programme quickly identified these areas, enabling the statisticians to concentrate their attention on the critical areas.

The programme could then be successively re-run, until the statisticians were satisfied that no identifiable or serious problems remained. This iterative process, made possible by the method and the solution algorithms and their implementation, have made Stone's vision,

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(6) The term "superior" data was coined by the authors of the GRIT procedure (see Jenson and West (1986)) to describe data sourced from primary data collections (eg surveys and censuses, annual reports etc), compared with data derived from allocation methods using other collections (eg national input-output tables).

of the balancing procedure becoming an important stage in the measurement process, into a reality.

### 2.3. A three region, two industry schema

In order to outline the structure of the RAM, a hypothetical aggregate region,  $Q$ , will be defined to comprise three regions,  $A, B$  and  $C$ , and two industries,  $(\alpha, \beta)$ . Figure 1 below illustrates the schematic input-output table for the aggregate region.

This table is a stylized version of the input-output table produced by the GSO. The major differences are the amalgamation of some of the primary inputs (P3-P5 and P6-P7) and the specification of only two industries instead of twenty-seven.

The construction of the sub-regions commences with a core input-output table for each region similar in structure to Figure 1. However, in addition to imports and exports from interstate and overseas, the schema needs to incorporate imports and exports between regions for usage by industries and categories of final demand. These

**Figure 1 – Input-Output Table for Aggregate Region Q, with Two Industries**

	$\alpha$	$\beta$	Q1	Q2	Q3	Q4	Q5	Q6	Q7
$\alpha$	$Q_{\alpha}^{\alpha}$	$Q_{\beta}^{\alpha}$	$C_{\alpha}^{QPR}$	$C_{\alpha}^{QGG}$	$I_{\alpha}^{QPR}$	$I_{\alpha}^{QPU}$	$I_{\alpha}^{QGG}$	$IS_{\alpha}^Q$	$X_{IF}^{Q\alpha}$
$\beta$	$Q_{\alpha}^{\beta}$	$Q_{\beta}^{\beta}$	$C_{\beta}^{QPR}$	$C_{\beta}^{QGG}$	$I_{\beta}^{QPR}$	$I_{\beta}^{QPU}$	$I_{\beta}^{QGG}$	$IS_{\beta}^Q$	$X_{IF}^{Q\beta}$
P1	$W_{\alpha}^Q$	$W_{\beta}^Q$							
P2	$G_{\alpha}^Q$	$G_{\beta}^Q$							
P3-P5	$T_{\alpha}^Q$	$T_{\beta}^Q$	$T^{QCPR}$	$T^{QCG}$	$T^{QPR}$	$T^{QPU}$	$T^{QGG}$	$T^{QS}$	$T^{QX}$
P6-P7	$M_{IF}^{Q\alpha}$	$M_{IF}^{Q\beta}$	$M_{IF}^{QCPR}$	$M_{IF}^{QCG}$	$M_{IF}^{QPR}$	$M_{IF}^{QPU}$	$M_{IF}^{QGG}$	$M_{IF}^{QS}$	$M_{IF}^{QX}$

In the above table the meanings of the rows and columns are:

$\alpha, \beta$	Industries.
Q1	Final consumption expenditure – private.
Q2	Final consumption expenditure – government.
Q3	Gross fixed capital expenditure – private.
Q4	Gross fixed capital expenditure – public enterprise.
Q5	Gross fixed capital expenditure – government.
Q6	Increase in stocks.
Q7	Exports to interstate and foreign.
P1	Wages, salaries and supplements.
P2	Gross operating surplus.
P3-P5	Indirect taxes (net) and sales to final buyers.
P6-P7	Imports from interstate and foreign and duty on imports.

additional inter-regional trade flows are combined with the basic input-output table as illustrated for region A in Figure 2. The basic input-output table is defined as cells (1,1) to (6,9).

The imports to intermediate usage are entered in cells (row 7, column 1) to (12,2). This block can further be divided into imports from a particular region or stacks. Stack 1 comprises cells (7,1) to (8,2) and would represent all imports to intermediate usage from region A (for the illustrated region A this is obviously null). Stack 2, comprising cells (9,1) to (10,2) represents imports to intermediate usage from region B. The notation for cell (9,1) can be translated as imports to region A industry from region B industry. Stack 3, comprising cells (11,1) to (12,2) represents imports to intermediate usage from region C.

Region A exports for intermediate usage by regions B and C are represented by cells (1,10) to (2,15). Again using the stack concept, cells (1,10) to (2,11) are exports to region A (again null), cells (1,12) to (2,13) are exports to region B and cells (1,14) to (2,15) are exports to region C.

Imports to final demand from other regions occupy cells (7,3) to (12,8) and again employ the same stack concept outlined above. Likewise the exports to final demand of other regions reside in cells (1,22) to (2,33).

In practice columns 10 onwards can be dropped from the Figure 2 representation, if indeed only single estimates of the inter-regional trade flows are available, since they are duplicated in the system in the inter-regional trade rows.

Once the three regional input-output tables and the aggregate input-output table have been organised in this fashion, they can be re-defined as a three dimensional RAM. This is illustrated in Figure 3 (inter-regional exports have been removed to facilitate the illustration). Quite simply the process involves placing or stacking the regions behind each other with the aggregate table at the back.

The benefits of this schema are two-fold. Firstly, it makes conceptualising the RAM simpler, since the representation remains that of the standard input-output model. Secondly, the schema is more easily programmed by utilising the four dimensional matrix outlined below in Table 1.

**Table 1 – Dimensions of Data Structures**

Label	Regions	Rows	Columns	Stacks
<i>XIO</i>	11	31	34	ma
<i>XT_IT</i>	10	27	27	10
<i>XT_FD</i>	10	27	67	10

## 2.4. The ten region, twenty-seven industries schema

### 2.4.1. Equations and restrictions

The data structures outlined in the schema above have been specified as follows:





Figure 3 – Arrangement of the Four Input-Output Tables

			$Q_{\alpha}^{\alpha}$	$Q_{\beta}^{\alpha}$	$C_{\alpha}^{QPR}$	$C_{\alpha}^{QCG}$	$I_{\alpha}^{QPR}$	$I_{\alpha}^{QPU}$	$I_{\alpha}^{QCG}$	$IS_{\alpha}^Q$	$X_{IF}^{Q\alpha}$
			$C_{\alpha}^{\alpha}$	$C_{\beta}^{\alpha}$	$C_{\alpha}^{CPR}$	$C_{\alpha}^{CG}$	$I_{\alpha}^{CPR}$	$I_{\alpha}^{CPU}$	$I_{\alpha}^{CG}$	$IS_{\alpha}^C$	$X_{IF}^{C\alpha}$
	$B_{\alpha}^{\alpha}$	$B_{\beta}^{\alpha}$	$C_{\alpha}^{BPR}$	$C_{\alpha}^{BG}$	$I_{\alpha}^{BPR}$	$I_{\alpha}^{BPU}$	$I_{\alpha}^{BG}$	$IS_{\alpha}^B$	$X_{IF}^{B\alpha}$	$X_{IF}^{C\beta}$	
$A_{\alpha}^{\alpha}$	$A_{\beta}^{\alpha}$	$C_{\alpha}^{APR}$	$C_{\alpha}^{AG}$	$I_{\alpha}^{APR}$	$I_{\alpha}^{APU}$	$I_{\alpha}^{AG}$	$IS_{\alpha}^A$	$X_{IF}^{A\alpha}$	$X_{IF}^{B\beta}$		
$A_{\alpha}^{\beta}$	$A_{\beta}^{\beta}$	$C_{\beta}^{APR}$	$C_{\beta}^{AG}$	$I_{\beta}^{APR}$	$I_{\beta}^{APU}$	$I_{\beta}^{AG}$	$IS_{\beta}^A$	$X_{IF}^{A\beta}$			$T^{Q\alpha}$
$W_{\alpha}^A$	$W_{\beta}^A$										$T^{C\alpha}$
$G_{\alpha}^A$	$G_{\beta}^A$									$T^{B\alpha}$	$M^{Q\alpha}$
$T_{\alpha}^A$	$T_{\beta}^A$	$T_{\alpha}^{APR}$	$T_{\alpha}^{AG}$	$T_{\alpha}^{APR}$	$T_{\alpha}^{APU}$	$T_{\alpha}^{AG}$	$T_{\alpha}^{AS}$	$T_{\alpha}^{AX}$	$M_{IF}^{B\alpha}$	$M_{IF}^{C\alpha}$	
$M_{IF}^{A\alpha}$	$M_{IF}^{A\beta}$	$M_{IF}^{ACPR}$	$M_{IF}^{ACG}$	$M_{IF}^{APR}$	$M_{IF}^{APU}$	$M_{IF}^{AG}$	$M_{IF}^{AS}$	$M_{IF}^{AX}$	$M_{A\beta}^{C\alpha}$	$M_{B\alpha}^{C\alpha}$	
									$M_{A\beta}^{B\alpha}$	$M_{B\alpha}^{C\alpha}$	
									$M_{A\beta}^{C\alpha}$	$M_{B\beta}^{C\alpha}$	
$M_{B\alpha}^{A\alpha}$	$M_{B\alpha}^{A\beta}$	$M_{B\alpha}^{ACPR}$	$M_{B\alpha}^{ACG}$	$M_{B\alpha}^{APR}$	$M_{B\alpha}^{APU}$	$M_{B\alpha}^{AG}$	$M_{B\alpha}^{AS}$				
$M_{B\beta}^{A\alpha}$	$M_{B\beta}^{A\beta}$	$M_{B\beta}^{ACPR}$	$M_{B\beta}^{ACG}$	$M_{B\beta}^{APR}$	$M_{B\beta}^{APU}$	$M_{B\beta}^{AG}$	$M_{B\beta}^{AS}$	$M_{C\alpha}^{B\alpha}$			
$M_{C\alpha}^{A\alpha}$	$M_{C\alpha}^{A\beta}$	$M_{C\alpha}^{ACPR}$	$M_{C\alpha}^{ACG}$	$M_{C\alpha}^{APR}$	$M_{C\alpha}^{APU}$	$M_{C\alpha}^{AG}$	$M_{C\alpha}^{AS}$	$M_{C\beta}^{B\alpha}$			
$M_{C\beta}^{A\alpha}$	$M_{C\beta}^{A\beta}$	$M_{C\beta}^{ACPR}$	$M_{C\beta}^{ACG}$	$M_{C\beta}^{APR}$	$M_{C\beta}^{APU}$	$M_{C\beta}^{AG}$	$M_{C\beta}^{AS}$				

XIO consists of the ten regional input-output tables (1 to 10). The eleventh region (matrix 11) is the official aggregate Queensland table. Each table consists of 27 by 27 industry intermediate transactions, 4 primary inputs (P1, P2, P3-P5 and P6-P7) and 7 final demand categories (Q1 to Q7).

XT\_IT contains the inter-regional intermediate trade flows. It consists of the 10 regions, 27 by 27 industry transactions and 10 stacks of inter-regional information.

$XT\_FD$  contains the final demand trade flows for each regional input-output table. There are 6 of these, Q1 to Q6. Note, however, Q2 and Q6 currently have no information. Further, even though there is information for only 6 industries in Q3, Q4 and Q5, having the full 27 industrial detail present makes the practical application of the restrictions simpler.

The restrictions imposed in the Queensland regional accounting matrix are as follows:

### 1. Input-output table restrictions

#### Queensland

$$\sum_{j=1}^{34} XIO_{r,k,j} = \sum_{i=1}^{31} XIO_{r,i,k} \quad r = 11, \quad k = 1,27 \quad (1)$$

#### Other Regions

$$\begin{aligned} \sum_{j=1}^{34} XIO_{r,k,j} + \sum_{s=1}^{10} \sum_{j=1}^{27} XT\_IT_{s,k,j,r} + \sum_{s=1}^{10} \sum_{q=1}^6 XT\_FD_{s,k-27,q,r} = \\ = \sum_{i=1}^{31} XIO_{r,i,k} + \sum_{s=1}^{10} \sum_{j=1}^{27} XT\_IT_{r,i,k,s} \quad r = 1,10, \quad k = 1,27 \end{aligned} \quad (2)$$

### 2. Consolidation restrictions

#### a) Primary inputs:

$$XIO_{11,i,j} = \sum_{r=1}^{10} XIO_{r,i,j} \quad i = 28,29, \quad j = 1,27 \quad (3)$$

$$XIO_{11,i,j} = \sum_{r=1}^{10} XIO_{r,i,j} \quad i = 30,31, \quad j = 1,34 \quad (4)$$

#### b) Final demands: Q1 to Q6

$$XIO_{11,i,j} = \sum_{r=1}^{10} XIO_{r,i,j} + \sum_{r=1}^{10} \sum_{s=1}^{10} XT\_FD_{r,i,j-27,s} \quad i = 1,27, \quad j = 28,33 \quad (5)$$

#### Q7

$$XIO_{11,i,34} = \sum_{r=1}^{10} XIO_{r,i,34} \quad i = 1,27 \quad (6)$$

#### c) Intermediate transactions:

$$XIO_{11,i,j} = \sum_{r=1}^{10} XIO_{r,i,j} + \sum_{r=1}^{10} \sum_{s=1}^{10} XT\_IT_{r,i,j,s} \quad i = 1,27, \quad j = 1,27 \quad (7)$$

### 3. Dual estimate trade flow restrictions

If initial (unbalanced) information supplied different values of the same inter-regional trade flow separately or independently as an import and as an export, it would be necessary to impose the following restrictions:

$$XT\_IT_{r,i,j,s}^* = XT\_IT_{r,i,j,s} \quad r,s = 1,10, \quad i,j = 1,27 \quad (8)$$

However, in this exercise, there was only one initial estimate of each trade flow, and so the set of restrictions was unnecessary and could not be imposed. In the case of national or provincial accounts, however, consolidating to the respective world or national total, this set of restrictions will be important, and indeed may be central to the purpose of the exercise.

#### Total restrictions

The theoretical total set of restrictions in this exercise (in reality some restrictions may total zero on both sides and therefore are removed) is presented in Table 2.

**Table 2 – Theoretical Number of Restrictions**

Restriction Set	Number of Restrictions	Total
1.	$27 + 10 \times 27$	297
2.a	$2 \times 27 + 3 \times 34$	122
2.b	$27 \times 6 + 27$	189
2.c	$27^2$	729
<b>Totale</b>		<b>1,337</b>

#### 2.4.2. Reliability weights

The reliability weights are formulated in a second regional accounting matrix which has the same dimension and structure as the estimates matrix. Following Stone (1981), the weights take the form of a set of broad reliability indicators from A to E. The method uses these weights to determine by how much an estimate may be changed. A is taken as very reliable and thus the estimate should not change by much if at all, whilst E is extremely unreliable and may vary considerably. The weights are assigned to the estimates within the RAM based on the statistician's assessment of the reliability of an individual cell or range of cells.

Again, following Stone (1981)), the relative coefficients of variation for the weights have been specified as follows:

In the 10 region and 27 sector schema, the assignment of reliabilities was generally organised in ranges of cells, with the exception of some individual cells where the estimate was felt to be quite reliable. In these

**Table 3 – Coefficients of Variation for the Reliability Weights**

Letter	Value	Reliability
A	0.001	Excellent
B	3.000	Good
C	13.000	Average
D	30.000	Poor
E	50.000	Very Poor

cases, the reliability was upgraded only by one level (ie C to B). Since an important objective of the exercise was for the ten regions to consolidate to the Queensland input-output table, the whole of this table was assigned reliability "A". The components of the regional input-output tables were broadly assigned reliabilities as summarized in Table 4.

**Table 4 – Assignment of Reliabilities to Regional Account Matrix**

Range of cells	Reliability
<b>Outgoings</b>	
Wages, salaries and supplements (P1)	B
Gross operating surplus (P2)	B
Indirect taxes less subsidies (P3-P5)	D
Imports from interstate and foreign (P6-P7)	D
<b>Incomings</b>	
Private consumption (Q1)	D
Government consumption (Q2)	D
Private investment (Q3)	C
Public investment (Q4)	C
Government investment (Q5)	C
Increase in stocks (Q6)	D
Exports to interstate and foreign (Q7)	D
Inter-regional trade	E
Intermediate transactions	D

### 3. Balancing the Regional Accounting Matrix

#### 3.1. SAM balancing techniques

Stone's solution to the problem of balancing a SAM was to set up the constrained least squares solution as

$$\hat{x} = \hat{x} - VG'(GVG')^{-1}(G\hat{x} - h) \quad (9)$$

where  $\hat{x}$  are the initial estimate of the vectorised (and compressed) matrix,  $V$  is the corresponding vector (or matrix) of reliabilities,  $G$  defines the restrictions and  $h$  is a vector of constants.

This result emerges from constrained quadratic loss minimisation

$$(x - \bar{x})' V^{-1} (x - \bar{x}) + \lambda' (Gx - h) \quad (10)$$

with respect to  $x$ . The constrained solution is

$$\hat{\lambda} = (GVG')^{-1} (G\bar{x} - h) \quad (11)$$

$$x = \bar{x} - VG'\hat{\lambda} \quad (12)$$

Despite the likely size in a large SAM, this lends itself to solution by the conjugate gradient algorithm, in a method due to Hestenes and Stieffel (1952), since  $(GVG')$  is symmetric positive definite.

$$(GVG')\hat{\lambda} = (G\bar{x} - h) \quad (13)$$

$G$  will be sparse and  $V$  is typically diagonal, so conjugate gradients provides a convenient approach to the problem with convergence guaranteed in a maximum of  $n$  iterations, where  $n$  is the number of Lagrangians (constraints).

If the equations are written

$$r = q - A\hat{\lambda} \quad (14)$$

where  $r$  are the residuals,  $q = (G\bar{x} - h)$  and  $A = (GVG')$ , the conjugate gradient algorithm can be expressed as the following iterative sequence

$$\begin{aligned} p_0 &= r_0 = q - (GVG')\lambda \\ a_i &= \frac{r_i' r_i}{p_i' (GVG') p_i} \\ \lambda_{i+1} &= \lambda_i + a_i p_i \\ r_{i+1} &= r_i - a_i (GVG') p_i \\ \beta_i &= \frac{r_{i+1}' r_{i+1}}{r_i' r_i} \\ p_{i+1} &= r_{i+1} + \beta_i p_i \end{aligned} \quad (15)$$

where  $r$  refer to residuals,  $p$  to projections and  $a$  and  $\beta$  are scalars to ensure conjugacy. The sparsity and composition of  $GVG'$  enable very efficient code to be written.

The rate of convergence of the algorithm may be improved by scaling ("preconditioning"). Several theorems in the conjugate gradient literature point to the same result, the rate of convergence depends on the number of distinct latent roots on  $A$ . As conjugate gradients can be solved by using successive eigenvectors as directions (see Goldfeld and Quandt (1972)), this is not all that surprising. If the problem can be scaled so the norm of the matrix is approximately one, convergence can be accelerated even further.

For equations of the form

$$A\hat{\lambda} = q \quad (16)$$

the idea would be to scale such that the roots were clustered around as few values as possible.

$$[CAC]C^{-1}\hat{\lambda} = Cq \quad (17)$$

$$[CAC]\hat{\mu} = Cq \quad (18)$$

In practice, selecting to ensure the product matrix ( $CAC$ ) has terms which are approximately 1 on the diagonal ensures rapid convergence. This was the approach suggested by Byron (1978) and in light of more recent findings it appears to be justified. The problem which has troubled some authors and on which we will comment later, is the possibility of negative solutions; the prevention of which leads us into the domain of quadratic programming or, following the suggestion of Harrigan (1990), into information theory. In terms of the results observed in this paper, the experience contradicts the comments of van der Ploeg (1984, p. 17); the scaled conjugate gradient programme applied to inter-regional SAMs with 1130 restrictions converged in 61 iterations and 119 seconds on a 50mhz 486 machine. This time may be decomposed into 23.78 seconds read, 75.85 computation and 19.55 write. Part of the trick to achieve speed was to use sparse matrix programming, as mentioned in Byron's original article. The problems referred to in van der Ploeg (1984), on storing a  $GVG'$  matrix where  $G$  is  $301 \times 3012$  matrix, are grossly exaggerated as the adding up conditions described in the article are simple row and column sums with 301 restrictions and 11 per cent of 301 columns or 3300 terms; which as they typically have the value +1 or -1 can be stored in integer format. The complete initial and final estimates can be stored as one (overwritten) matrix which, in van der Ploeg's case, will be of the order of 3300 elements. The upshot is that the alleged storage problem is a Furphy (?) if handled correctly. The issue relating to the storage of  $GVG'$  can be ignored as the full matrix is never stored.  $V$  is diagonal and  $GV^{1/2} = G^*$  may be created with the same structure and sparsity as  $G$ . Any products of the form  $G^*G^*x$  may be formed sequentially by multiplying  $G^*x = y$  and then  $G^*y = z$ . There is a little extra computation in each cycle, but it more than compensates for the storage savings using  $GVG'$ . Conjugate gradients, properly implemented, remain an attractive suggestion.

However, there are several alternative solutions to the quadratic minimisation problem which were not explored by Byron (1978) and may be equally appealing. The Lagrangian equation

$$A\hat{\lambda} = GVG'\hat{\lambda} = q \quad (19)$$

is sparse and would also lend itself to solution by either the Jacobi or Gauss-Seidel methods. Decomposing  $A$  into lower, diagonal and upper

(7) Furphy – a rumour, a false story. [From John Furphy, manufacturer in Victoria of water and sanitation carts, which during World War I were centres of gossip.] *The Australian Macquarie Dictionary*.

triangular components and adding the subscript to denote the iteration count,  $A = (L + D + U)$  and  $(L + D + U)\hat{\lambda} = q$ . Hence

$$D_{n+1}\hat{\lambda} = - (L + U)_n q \quad (20)$$

$$\hat{\lambda} = - D_{n+1}^{-1} (L + U)_n q \quad (21)$$

which is the Jacobi solution, whilst

$$(L + D)_{n+1}\hat{\lambda} = - U_n q \quad (22)$$

is the Gauss-Seidel method. Given the sparsity of the matrices this might provide a superior solution to those mentioned.

There is now quite a literature on the solution of sparse equation systems, including direct solution by Gaussian elimination (see Duff, Erisman and Reid (1986), Pissanetsky (1984), or Reid (1971) for examples). The computer package Matlab<sup>(8)</sup> includes a sparse matrix capability, which is extremely fast in execution and obviates the necessity of Fortran or C loops to encode the sparsity.

However, an alternative approach which is more Bayesian (or mixed regression) in its inspiration is the following. If the objective function is written as

$$(x - \hat{x})' V^{-1} (x - \hat{x}) + (Gx - h)' W^{-1} (Gx - h) \quad (23)$$

where  $V$  defines the reliabilities of the initial estimates of  $x$  and  $W$  defines the degree of precision required of the restrictions (presumably very tight), then the first order conditions are

$$V^{-1}(x - \hat{x}) + G'W^{-1}(Gx - h) = 0 \quad (24)$$

which yields

$$[V^{-1} + G'W^{-1}G]x = V^{-1}\hat{x} + G'W^{-1}h \quad (25)$$

The parallels with Theil and Goldberger's (1962) mixed regression estimator are obvious. As  $W$  approaches zero this converges to the Lagrangian solution (see Brook and Wallace (1973)). The sparsity of  $G$  and the diagonality of  $V$  and  $W$  point to the possibility of a simple direct solution based on a number of possible factorisations. However, in its simplicity it presents no particular advantage over the Lagrangian solution already given.

Providing the restrictions are of full rank the conjugate gradient algorithm should converge and the three alternative solutions presented above offer no great advantage.

Two particularly good papers on the balancing problem are due to Harrigan and Buchanan (1984) and Harrigan (1990). The first paper addresses the failure of equality constraint Lagrangian methods to deal with inequalities and proposes a quadratic programming based on

(8) Matlab (1993), *The Mathworks, Inc.*, Natick, MA, USA.

Bachem and Korte's (1978) version of Hildreth's (1957) algorithm. Harrigan and Buchanan represent the problem as

$$\min z = \frac{1}{2} b'Wb + b'q \quad \text{st } Eb \geq e \quad (26)$$

where the row and column adding up restrictions are presented as reverse inequalities and all non-negativity conditions on  $b$  are handled in the same way. The optimising solution for  $b$  is

$$b = -W^{-1}(q - E'w) \quad (27)$$

where  $w$  are the Lagrangians. The dual representation is

$$\min z^* = -\frac{1}{2} w'EW^{-1}E'w + w'(e + EW^{-1}q) \quad \text{st } w \geq 0 \quad (28)$$

The first order conditions are

$$EW^{-1}E'w = (e + EW^{-1}q) \quad (29)$$

which are then solved by a modified Gauss-Seidel method with zeros inserted if any  $w$  becomes negative. Convergence to Lagrangians which minimise  $z^*$  and hence  $z$  is assured and the algorithm can be modified to handle blocks of restrictions. The authors report computational experience on two smallish problems with roughly 100 equality constraints and 1200-1500 variables and the time taken was just a few seconds on an IBM370.

One of the present authors (Byron (1992)) has developed an algorithm from portfolio analysis which could easily be applied in the present context. The algorithm uses exterior point methods in minimising a quadratic form unencumbered by inequality restrictions but subject to equality constraints. If inequalities are violated in a cycle they are imposed as a block and the quadratic form re-minimised subject to the additional (or reduced) set of restrictions. The first order conditions on the Lagrangians are solved using conjugate gradients and the process is repeated until convergence, which typically takes a few cycles. Such procedures are known as exterior point algorithms and are becoming quite well known in the literature (see Bertsekas (1982a, 1982b), Dunn (1981, 1987), Calamai and Moré (1987) and Burke and Moré (1988)). Recently, Moré and Toraldo (1991) have provided two theorems proving that, for a strictly convex non-degenerate problem, the algorithm converges to the global optimum in a finite number of steps. The SAM problem, properly defined, satisfies these conditions.

Essentially, both the exterior point and Hildreth-Gauss-Seidel solutions point to the absence of any real computational difficulties in applying quadratic programming to the SAM problem.

Harrigan's second paper explores the possibility of using information theory to provide the objective function for the SAM problem. This approach has the advantage of automatically satisfying simple non-negativity constraints. The objective function may be written as



$$f = x' \ln(x./x_0) \quad (30)$$

using Hadamard product notation ( $\cdot$ ). Kullback's information gain is the surprise in  $x$  given  $x_0$ . Kullback's information integral requires a normalisation condition of the form  $i'x = 1$ . Harrigan provides a solution and a good discussion. One of the major problems with this approach is that the adjustment relates directly to the size of the initial estimates  $x_0$  and does not force the unreliable estimates to absorb the bulk of the required adjustment. Minimisation of  $f$  means that the bulk of the (downward) adjustment will probably take place on the variables with large initial estimates. The RAS procedure is an application of the application on the Kullback information integral.

### 3.2. Computational considerations

The original programme was written in Lahey Fortran<sup>(9)</sup>, taking advantage of Fortran's ability to access multidimensional arrays. The intermediate trade and final demand arrays were expressed as  $(10 \times 27 \times 27 \times 10)$  and  $(10 \times 27 \times 6 \times 10)$  respectively. The structure of the balancing restrictions meant that expressing the matrices in stacked (tensor) form had computational and storage advantages. Furthermore, when the conjugate gradient algorithm was applied to the equations,

$$GVG'\hat{\lambda} = q \quad (31)$$

the algorithm, listed in equation (15), can be most effectively implemented by performing the multiplications  $GVG'\lambda$  as separate steps. Thus  $GVG'\lambda = (G\{V[G'\lambda]\})$  exploiting the sparsity of  $G$  and the diagonality of  $V$ . The product always remains a vector, and the full matrix  $GVG'$  is never stored. The only reason for dwelling on this, is that the Fortran programme was quite fast, much faster than the same programme written in Matlab. The problem took 119 seconds on a 50mhz IBM Thinkpad, a relatively slow 486 SLC/2 notebook computer. This could easily be reduced to 10 seconds or less on a modern RISC workstation.

Matlab 4.0 has a sparse matrix capability which suggested it might be advantageously used on this problem. Gilbert, Moler and Schreiber (1992) outline the sparse matrix techniques available in Matlab, some of which could be adapted to the problem of solving equations of the form  $GVG'\hat{\lambda} = q$ . For example, if the initial step of equation solution is  $LU$  or  $LDL$  decomposition, then one advantage of initial sparsity is that  $L$  and  $U$  remain sparse. Despite this, the original form of solution proposed by Byron (1978) still appears the best approach; indeed Gilbert, Moler and Schreiber provide Matlab code for such a solution. Our initial efforts with Matlab were discouraging, but that situation could well be reversed with more skilful programming.

(9) Lahey, F77L-EM/32 Fortran, Lahey Computer Systems, Incline Village, NV, USA.

There is a considerable literature on the solution of sparse equation systems and there are even Fortran subroutine packages such as SPARSPAK (see George and Liu (1981)), the Yale Sparse Matrix Package (see Eisenstat *et al* (1981)), the Harwell Subroutine Library (see United Kingdom Atomic Energy Authority (1988)) and Numerical Recipes by Press *et al* (1987).

As mentioned, *LU*, Cholesky and *LDL* factorisations of sparse matrices yield sparse matrices; permutation matrices can be used to achieve factorisation more conveniently; matrices can be re-ordered into block triangular form to facilitate computation of a direct inverse and parallel processing methods are also available on array processors (see Ashcraft *et al* (1987)). Despite all this, in view of the speed of the conjugate gradient algorithm and the extreme sparsity of the *G* and *V* matrices, it seemed more advantageous to pursue improvements to the existing Fortran conjugate gradient algorithm rather than to apply sparse technology to the solution of  $GVG'\lambda = q$  because  $GVG'$  will not be as sparse as *G*.

There are also a number of variations on the conjugate gradient formulae which apply in the context of minimising nonlinear functions (see Polak (1971)), but it appears unlikely that they will be useful in solving linear equations (corresponding to an underlying quadratic function), since the formulae are all equivalent if the function is quadratic. Finally, some of the features of Matlab relating to sparse matrix storage, which were so advantageous in the read-write phase, could be incorporated into a final version of the Fortran programme.

### 3.3. Balancing results and summary measures

The results are illustrated in Tables 5 and 6. The percentage differences between the initial pre-balancing process values of the domestic production account and the balanced values are shown in Table 5. The differences are zero for the official State domestic production account, because of the high reliabilities applied. They are also zero for all but the trade items in the consolidated regional domestic production accounts, because the initial estimates of the domestic production accounts of the regions, and their input-output disaggregation, were estimated congruent with the *Queensland State Accounts*. The trade data, however, were estimated separately, and, as can be seen in Table 5, the adjustments were quite substantial. Imports from overseas and inter-State were lowered in total by 10.1 per cent, and exports to overseas and inter-State were correspondingly lowered by 12.1 per cent. The adjustments for the intra-State trade elements ranged from -22.6 per cent to +23.3 per cent. Overall, the balance of the consolidated domestic production account decreased by 3.9 per cent, which was felt by the statisticians to be tolerable, given the quality of the initial regional estimates.

Table 6 indicates the average absolute adjustment for each of the matrix elements of the regional domestic production accounts. This was measured by calculating the mean absolute per cent adjustments of all

Table 5 – Percentage adjustments in domestic production accounts for the Queensland Statistical Divisions

	Brisbane Moreton	Darling Downs	Fitzroy	Wide Bay- Burnett	Mackay	Northern	Far North	South West	Central West	North West	Sum	Queen- land
Wages, salaries and supplements	0.0	0.0	0.0	0.0	0.1	0.0	0.0	0.1	0.0	-0.2	0.0	0.0
Gross operating surplus	0.0	0.0	0.0	0.0	0.1	0.0	0.0	0.1	0.0	-0.1	0.0	0.0
Indirect taxes less subsidies	2.5	-1.9	-4.7	1.8	-3.6	-4.1	1.4	-3.3	-5.7	-19.2	0.0	0.0
Imports from foreign & interstate	-5.5	-12.5	-14.4	-18.5	-21.9	-18.8	-18.5	-19.2	-41.3	-16.6	-10.1	0.0
Imports from Brisbane-Moreton	0.0	-15.5	-20.1	-15.5	-23.1	-21.6	-16.4	-4.0	-18.9	-24.8	-18.5	0.0
Imports from Darling Downs	-11.3	0.0	-6.7	24.7	-6.8	-9.3	-11.5	-11.0	33.5	-24.3	-9.4	0.0
Imports from Fitzroy	2.4	-18.1	0.0	0.0	-0.6	-25.1	-22.9	-17.4	-16.8	-11.2	-3.0	0.0
Imports from Wide Bay-Burnett	2.9	-19.0	-18.5	0.0	-7.8	-1.1	-9.1	-9.0	-16.3	-16.7	-0.8	0.0
Imports from Mackay	-25.2	-19.9	-13.6	-25.6	0.0	-27.6	-31.6	-28.1	28.7	-4.4	-22.1	0.0
Imports from Northern	-18.5	-5.8	-16.3	-16.3	-11.1	0.0	23.6	-13.3	-15.6	-32.3	-13.7	0.0
Imports from Far North	1.8	-9.5	-26.8	-15.9	-16.5	-22.4	0.0	-19.8	-33.0	3.9	-11.1	0.0
Imports from South West	18.7	21.4	-23.2	-14.7	-34.3	-26.0	-23.7	0.0	13.5	-20.0	18.8	0.0
Imports from Central West	25	1.4	-25.4	-14.5	57.7	-25.4	-25.3	-22.8	0.0	-14.6	23.3	0.0
Imports from North West	100.9	75.5	38.6	72.5	61.4	12.0	7.3	99.2	-47.3	0.0	20.2	0.0
TOTAL OUTGOINGS	-1.5	-4.3	-6.6	-5.3	-8.2	-7.4	-6.6	-5.9	-12.0	-9.6	-3.9	0.0
Private consumption	2.1	-2.3	-1.9	-4.9	-7.8	-2.1	-4.6	-6.4	-4.5	5.8	0.1	0.0
Government consumption	3.0	3.2	-2.6	-4.3	-2.9	-4.3	2.1	-17.0	-38.4	-42.6	0.0	0.0
Private investment	1.2	-3.9	-6.8	-2.8	-4.7	-1.6	0.9	-7.6	-13.8	-13.4	0.0	0.0
Public investment	8.5	8.8	-12.4	-2.1	-10.2	2.8	3.6	-12.2	-0.3	3.9	0.0	0.0
Government investment	4.2	5.5	0.0	-4.7	-1.7	-9.4	-1.3	-10.5	-13.0	-5.6	0.0	0.0
Increase in Stocks	0.6	6.1	8.2	-0.4	2.7	14.1	6.2	-196.4	-35.9	-5.4	0.0	0.0
Exports to foreign & interstate	-8.6	-11.6	-10.7	-15.9	-8.1	-15.6	-21.6	-18.6	-21.4	-23.2	-12.1	0.0
Exports to Brisbane-Moreton	0.0	-11.3	2.4	2.9	-25.2	-18.5	1.8	18.7	25.0	100.9	1.6	0.0
Exports to Darling Downs	-15.5	0.0	-18.1	-19.0	-19.9	-5.8	-9.5	21.4	1.4	75.5	-13.9	0.0
Exports to Fitzroy	-20.1	-6.7	0.0	-18.5	-13.6	-16.3	-26.8	-23.2	-25.4	38.6	-18.4	0.0
Exports to Wide Bay-Burnett	-15.5	24.7	0.0	0.0	-25.6	-16.3	-15.9	-14.7	-14.5	72.5	-12.9	0.0
Exports to Mackay	-23.1	-6.8	-0.6	-7.8	0.0	-11.1	-16.5	-34.3	57.7	61.4	-18.3	0.0
Exports to Northern	-21.6	-9.3	-25.1	-1.1	-27.6	0.0	-22.4	-26.0	-25.4	12.0	-14.6	0.0
Exports to Far North	-16.4	-11.5	-22.9	-9.1	-31.6	23.6	0.0	-23.7	-25.3	7.3	-12.7	0.0
Exports to South West	-4.0	-11.0	-17.4	-9.0	-28.1	-13.3	-19.8	0.0	-22.8	99.2	-5.2	0.0
Exports to Central West	-18.9	33.5	-16.8	-16.3	28.7	-15.6	-33.0	13.5	0.0	-47.3	-15.7	0.0
Exports to North West	-24.8	-24.3	-11.2	-16.7	-4.4	-32.3	3.9	-20.0	-14.6	0.0	-22.6	0.0
TOTAL INCOMINGS	-1.3	-4.3	-6.6	-5.3	-7.9	-7.4	-6.6	-5.9	-12.0	-15.8	-3.9	0.0
Intermediate transactions	9.6	15.5	17.5	13.7	17.9	14.3	9.8	34.0	14.3	-0.1	11.3	0.0
<b>TOTAL</b>	<b>-1.4</b>	<b>-4.3</b>	<b>-6.6</b>	<b>-5.3</b>	<b>-8.1</b>	<b>-7.4</b>	<b>-6.6</b>	<b>-5.9</b>	<b>-12.0</b>	<b>-12.8</b>	<b>-3.9</b>	<b>0.0</b>

**Table 6 – Average absolute percentage adjustments in elements of the domestic production accounts for the Queensland Statistical Divisions**

	Brisbane Moreton	Darling Downs	Fitzroy	Wide Bay- Burnett	Mackay	Northern	Far North	South West	Central West	North West	Sum	Queen- land
Wages, salaries and supplements	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
Gross operating surplus	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
Indirect taxes less subsidies	1.4	1.5	1.5	1.5	1.6	1.5	1.6	1.5	1.7	1.6	1.5	0.0
Imports from foreign & interstate	1.6	1.6	1.6	0.3	1.6	1.6	1.6	1.6	0.3	1.7	1.4	0.0
Imports from Brisbane-Moreton	0.0	0.3	1.3	0.2	1.4	0.3	0.3	0.3	0.3	0.4	0.5	0.0
Imports from Darling Downs	0.4	0.0	0.5	0.4	6.2	0.7	0.7	0.4	0.6	0.1	0.7	0.0
Imports from Fitzroy	0.4	0.1	0.0	0.3	0.4	0.6	0.5	1.2	0.4	0.6	0.4	0.0
Imports from Wide Bay-Burnett	0.3	0.4	0.4	0.0	2.8	0.7	0.6	0.6	0.8	3.3	0.9	0.0
Imports from Mackay	0.3	0.7	0.3	0.3	0.0	0.4	0.4	1.0	0.4	0.8	0.5	0.0
Imports from Northern	0.9	1.0	0.6	0.9	0.4	0.0	0.4	1.3	0.2	0.5	0.5	0.0
Imports from Far North	0.4	0.6	0.7	3.9	0.5	0.4	0.0	0.7	0.5	0.6	0.7	0.0
Imports from South West	5.4	11.3	4.9	4.9	4.9	4.9	4.9	0.0	8.9	4.9	7.0	0.0
Imports from Central West	49.7	18.0	11.4	3.0	20.2	3.0	3.0	1.5	0.0	1.2	12.4	0.0
Imports from North West	7.0	4.7	9.5	7.0	19.8	0.4	1.2	4.7	2.1	0.0	3.4	0.0
TOTAL OUTGOINGS	1.1	0.9	0.9	0.6	1.5	0.6	0.6	0.6	0.9	0.7	0.8	0.0
Private consumption	1.1	1.1	1.1	1.1	1.1	1.2	1.1	1.1	1.2	1.2	1.2	0.0
Government consumption	6.3	6.8	6.8	6.8	6.8	6.8	6.8	6.8	6.8	6.8	6.7	0.0
Private investment	0.2	0.2	0.2	0.2	0.2	0.2	0.2	0.2	0.2	0.2	0.2	0.0
Public investment	1.2	1.2	1.2	1.2	1.2	1.2	1.2	1.3	1.7	1.6	1.3	0.0
Government investment	1.1	1.1	1.1	1.1	1.1	1.1	1.1	1.1	1.6	1.4	1.2	0.0
Increase in Stocks	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.2	0.1	0.0
Exports to foreign & interstate	2.1	2.3	2.1	0.4	2.3	2.2	2.4	2.5	0.5	2.4	1.9	0.0
Exports to Brisbane-Moreton	0.0	0.1	0.1	0.0	0.1	0.2	0.1	0.3	2.0	7.0	0.2	0.0
Exports to Darling Downs	0.1	0.0	0.2	0.0	0.3	0.2	0.1	1.2	1.5	4.7	0.2	0.0
Exports to Fitzroy	0.1	0.1	0.0	0.0	0.1	0.1	0.2	4.9	2.0	3.5	0.2	0.0
Exports to Wide Bay-Burnett	0.1	0.1	0.1	0.0	0.2	0.2	0.4	4.9	3.0	7.0	0.2	0.0
Exports to Mackay	0.1	0.3	0.1	0.1	0.0	0.1	0.1	4.9	2.0	4.7	0.2	0.0
Exports to Northern	0.1	0.1	0.2	0.1	0.2	0.0	0.1	4.9	3.0	1.2	0.2	0.0
Exports to Far North	0.1	0.2	0.1	0.1	0.2	0.1	0.0	4.9	3.0	1.3	0.2	0.0
Exports to South West	0.1	0.1	0.3	0.1	0.4	1.3	0.7	0.0	1.5	4.7	0.2	0.0
Exports to Central West	0.1	0.1	0.1	0.1	0.2	0.1	0.5	0.4	0.0	0.5	0.2	0.0
Exports to North West	0.1	0.1	0.3	0.1	0.3	0.1	0.1	4.9	1.2	0.0	0.2	0.0
TOTAL INCOMINGS	0.2	0.4	0.5	0.4	0.6	0.5	0.6	1.5	1.7	1.9	0.5	0.0
Intermediate transactions	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
<b>TOTAL</b>	<b>0.5</b>	<b>0.6</b>	<b>0.7</b>	<b>0.4</b>	<b>1.0</b>	<b>0.5</b>	<b>0.6</b>	<b>0.9</b>	<b>1.0</b>	<b>1.0</b>	<b>0.7</b>	<b>0.0</b>

$n$  non-zero cells of the parts of the RAM comprising the domestic production account element  $j$ , for region  $r$ :

$$MAPA_{j,r} = \frac{\sum_{i=1}^n \left( \frac{|X_{i,r}^B - X_{i,r}^U|}{|X_{i,r}^U|} \right)}{n} \cdot 100 \quad X_{j,r} = \sum_{i=1}^n X_{i,r} \quad (32)$$

and provides an indication of the adjustment activity within each of the various domestic production account components of the RAM. While the average absolute adjustment for, say, indirect taxes less subsidies in the North West Statistical Division was 1.5 per cent, the adjustments were predominantly downwards, and cumulated in an overall adjustment of total indirect taxes less subsidies in the North West of  $-19.2$  per cent. On the other hand, the average absolute per cent adjustment to individual elements of Government consumption expenditure in Brisbane-Moreton was 6.3 per cent, but offsetting effects meant that the overall adjustment was only 3.0 per cent.

### 3.4. The statistician's assessment

The experience of balancing the regional tables in a RAM framework was felt by the GSO statisticians responsible for the production of the tables to be very worthwhile. This was not only because it ensured that the individual regional tables consolidated to the Queensland table, but also it was felt that the balancing process itself contributed in a useful and important way to the data construction process.

The first balancing run of the RAM generated a significant number of negative cells, especially in the inter-regional trade flows, together with some negative flows in the individual input-output tables. This was followed by undertaking an iterative construction process, in which individual cells and ranges of cells were examined, checked against primary information and judgement and re-determined in levels and reliability. Three examples of areas where the balancing feedback provided important information to the statisticians on areas requiring revision were:

- A number of trade flows in the electricity industry in the inter-regional trade matrix were found to be incorrectly specified. These data were adjusted, with new estimates inserted for the inter-regional flows of electricity and corresponding new, higher reliability weights.
- It was also realized that the output of the livestock industries had been specified incorrectly, and had not been grossed up to include inter-regional trade in livestock, to allow for correct aggregation to the State table.
- It was only when balancing feedback was available that it was appreciated that the measurement and treatment of indirect taxes was incorrect in the North-West Statistical Division, a mining intensive region.

The question of non-negativity is an interesting one. Input-output tables may in fact have true negative numbers, eg, if stockbuilding is specified explicitly. In that case, it is more than useful to have a technique which can accommodate negative data, or at least handle inequality constraints. In this regard, Harrigan and Buchanan (1984) have developed an interesting technique for handling balancing using a quadratic programming framework.

However, our experience suggests that the emergence of negative numbers in the solution process, while conceptually incorrect, did in fact provide an important contribution to the process of edit checking. We regarded the presence of an inappropriate negative value in the solution as indicative of a serious error in the input data, which could then be manually investigated.

Other problems may be revealed by major revisions to cells. All such major revisions should be examined, although the choice of how significant a major revision is in practice is largely arbitrary and judgemental. We found evidence of data error and unforeseen conceptual problems and felt that this proved to be a useful part of the construction process.

The iterative construction process that this requires is certainly resource intensive. However, the required resources are largely human, and the algorithms and hardware which balance the tables are so fast that they are no longer a serious resource constraint. If the tables were balanced in the first run, using a technique which enforces balancing with theoretically correct signs for all cells, this valuable edit checking process would be restricted to evaluation of cells showing major revisions. While this may prove satisfactory, it may be more useful to apply the quadratic programming technique at a late stage in the iterative construction process, to correct remaining intractable or small negative signs, if any. Our (admittedly *ad hoc*, but so far successful) approach was to address the negative numbers, either individually or in groups, with statisticians converting them to positive values, with higher reliabilities. It was our experience that the data revisions introduced to effect the elimination of large negative numbers by the statisticians resulted in general reductions of the number of other negatively incorrect cells. It would certainly be interesting to compare the results of the Stone technique with the quadratic programming technique. We intend to pursue this comparison in future work.

RAS, by comparison, certainly does not seem to be as suitable as these techniques. As Harrigan and Buchanan (1984, p. 341) point out, the RAS technique is traditionally the main method used in balancing input-output tables, especially since *the information minimand of the RAS problem precludes negative solution values*, even though, when negative solution values are appropriate, *the information minimand of RAS is problematic and computational difficulties will be encountered with iterative row and column sum balancing methods*.

As Harrigan and Buchanan (1984, p. 357) have reminded us:

*As Lecomber (1975) has observed, the greater is the capacity of a simulation or estimation method to absorb and represent information,*

*the less is the opportunity to reject its results as implausible, for such a judgement must inevitably rely on information so far ignored.*

While a substantial amount of "superior" data had been inserted into the initial regional tables, the balancing process, with its additional information, nevertheless did result in substantial adjustment to the individual regional domestic production accounts, as shown in Tables 5 and 6 above.

While the editing and production processes of the official inter-regional tables have not yet been completed <sup>(10)</sup>, our experience has convinced us that these tables will be superior to regional tables produced in isolation from a consolidated state or national table. We believe that this process should be recommended for the development of regional input-output tables, whether as parts of a province, nation or world system, as it substantially increases the use of data. Not only is the individual datum value used, but a statement of the reliability of the individual datum, taking into account its source, is also incorporated into the balancing process. To our knowledge, current procedures to balance not only inter-regional, but single region, tables do allow for the insertion of "superior" data, but do not allow these data to be further adjusted in the balancing process. The adjustments made by the balancing process were judged to be sensible and informative. We can only agree with the words of Stone:

*It is hard to see how the adjustments to which [this method] gives rise could have been otherwise predicted and yet when they are made they seem on the whole plausible. (Stone (1981, p. 19))*

#### **4. Implications and conclusions**

##### **4.1. Regional hierarchies and the scale of the problem**

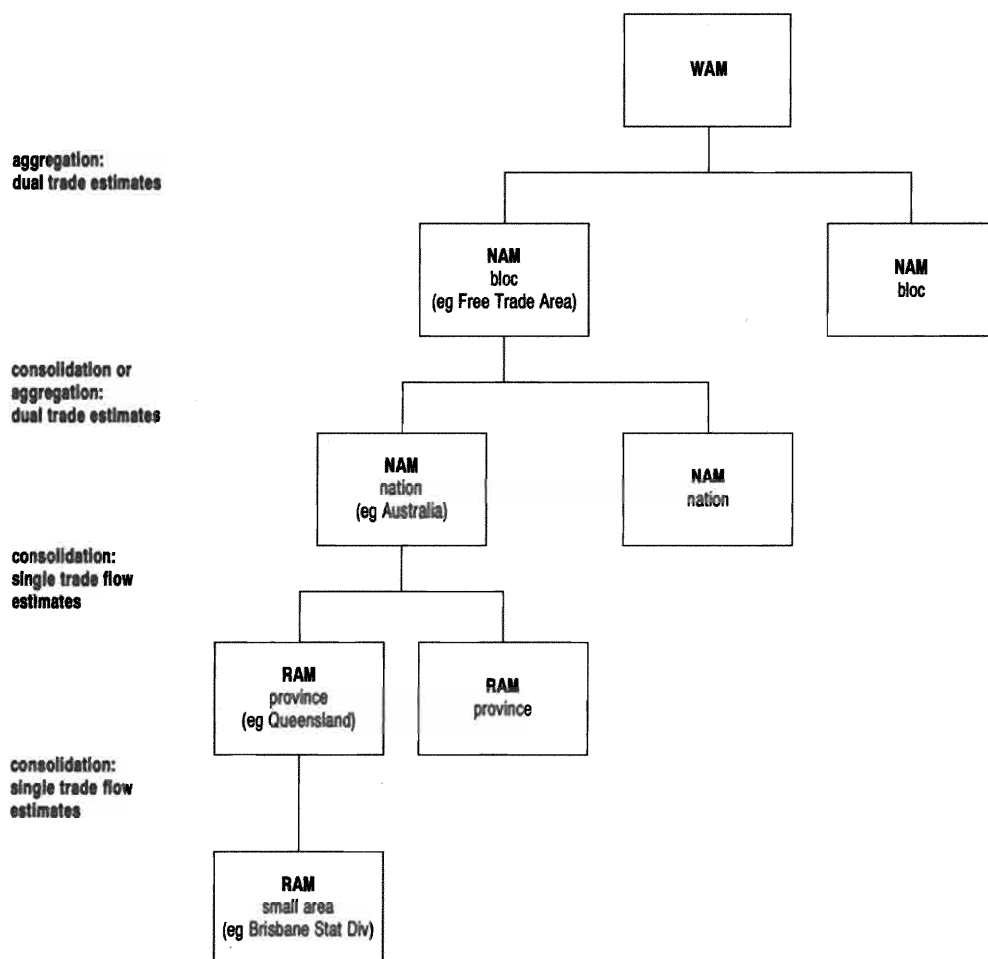
The technology outlined above provides an important answer to the general problem of size in estimating or simply adjusting large accounting systems. Many of these systems have been estimated and used as single region systems. As world trade grows, as further regions of the world develop and interact with the mainstream trading economies, as trading blocs emerge and develop, and as questions of intra-national regional development become more insistent, then there will be a growing need to estimate inter-relating systems of economic accounts.

The problem can be thought of as being the specification and estimation of a large set of inter-related and hierarchical regions. This would extend from, in absolute aggregate, the world accounts, including international trade flows, down one step in the hierarchy to supra-national federations (such as the EC), down another step to

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(10) It is expected that the official tables will be published late in 1993.

Figure 4 – Hierarchies of accounting matrices



national accounts (Australia), down another to provincial regional accounts (Queensland), and down once more to, perhaps, local regional accounts (eg, for local government, such as the Brisbane City Council, or for statistical regions, such as the Brisbane-Moreton Statistical Division). A complete estimation of a complete specification of this system defies our imagination as statisticians, although partial specification and estimation is certainly possible. A hierarchical approach, which might see coordinated downwards completion of the tasks, in parallel across the world, would seem to be an ambitious, but still feasible, way of proceeding with the estimation process.

#### 4.2. Regional and national balancing

It is intriguing to speculate that, if modern computational procedures and facilities had existed in the 1940s, then statistical discrepancies



might never have appeared in the national accounts. Similarly, it may not be inappropriate to suggest now, that single region SAM estimation, especially in sub-national regions, should be replaced by this multiple region simultaneous estimation. It is suggested that any provincial input-output table or SAM, assuming there exists a corresponding national table or SAM, should always proceed by specifying at least a two region estimation procedure or framework. Estimates of sub-provincial regional accounting matrices should also then proceed in this way. There could be various classifications or divisions. Examples include: urban and rural; metropolitan and rest of State; or, region of interest and balance.

As an aggregate region is sub-divided, the degree of openness of the constituent regions increases. Usually, the primary focus of interest of analysis of a regional economy is the extent and nature of the interaction of the region with the rest of the state or nation or world. The estimation of this disaggregation should be handled in a consistent way, integrated and congruent with the measurement of the aggregate economy.

This has, of course, already been done for many regions. Three examples are: D'Antonio *et al* (1988) [for the estimation of two regional accounting matrices comprising Italy: the Mezzogiorno and the Centre-North], Pyatt and Round (1985b) [for East and West Malaysia] and Skinner (1993) [for Queensland and the rest of Australia]. However, none of these examples are very large. The orders of the RAMs were for only two regions in each case, and, for accounts, 39, 11 and only 7 respectively, and the inter-regional trade flow matrices were highly aggregated. While the availability of data, and other resource constraints and analytical considerations will impose limits on the size of matrices, there is no real computational obstacle to the construction of much larger RAMs.

Certainly, in regional economics, there has never appeared to be any real difficulty in generating many, reasonably large, input-output tables, for individual regions. Not only is it time for regional input-output tables to evolve into SAMs, but it is also time for their individual estimation to be replaced by simultaneous estimation with related hierarchical RAMs.

There is also a trend to estimating combinations of national accounting matrices or NAMs, usually for contiguous countries, or for trading blocs. An example (which as far as can be determined did not use the Stone method for balancing) is the paper by Cohen and Husted (1985), which estimated a combined Canada-US SAM.

The increasing importance of SAMs which has characterized the last decade, and which has seen expanded use of SAMs in, for example, CGE models, micro-simulation models and for other analytical and descriptive purposes, may be reasonably expected to continue <sup>(11)</sup>. This will mean that questions of the balancing and

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(11) Note the chapter on SAM's in the new national economic accounting system standards, United Nations (1992).

congruence of accounts, including hierarchical congruence, will also be of increasing importance.

### 4.3. WAM balancing

The issue of reconciling the data on world trade requires the use of a world accounting matrix (WAM) and will be seen as an extension of the inter-regional SAM examined here. The authors were only able to access a few papers on the subject at the time of writing, most notable (Weale (1984), Baras (1993) and James and McDougall (1993)). The issue is relevant to project LINK and a committee (involving the World Bank, the IMF, the UN, the OECD and GATT) also has the matter under consideration. The UN has a TESSY model for trade reconciliation, but we were unable to get a description of it.

Weale (1984) describes a WAM based on 10 countries (or country groupings) and 5 sectors where the equations allow for price effects and consumption propensities. However, the information is not laid out in a simple mathematical way which is easy for the uninitiated to follow.

Baras's (1993) paper is essentially an algorithm description. The estimation mechanism is the same as that outlined here, trade flows are treated as elements of a multidimensional matrix, *where each dimension represents a characteristic of the flow; i.e. partner, commodity, year*. The estimates of these cells can arise from different sources and techniques, the basic problem then is one of reconciliation or finding a unique solution. As Baras reports it, the problem is essentially one of size – 200 countries, 3000 SITC codes, involving 600,000 unknowns, but only 3200 equations. Preliminary analysis of the data led Baras to the conclusion that he was only dealing with 30,000 unknowns and 400 equations as not all commodities are exported to all countries. The problem is therefore manageable.

An interesting example of a large adjustment or data reconciliation undertaking is that by the Australian Industry Commission in the construction of the database for the SALTER model (James and McDougall (1993)). The SALTER model, developed for the federal Department of Foreign Affairs and Trade, is a multi-region multi-sector model of the world economy, designed for policy analysis, in particular for analysing trade and industry policies in an international context. The database involves a multi-regional input-output table and an international trade database. The input-output tables had to be adjusted to agree with external data for international trade, industry assistance and macroeconomics. The adjustment and balancing procedures adopted were modifications of allocation techniques and the RAS method modified by applying Theil's (1965) application of information theory to the prediction of international trade flows.

The size of the SALTER problem was larger than the Queensland problem, with the world divided into 16 regions and with 37 industry/commodities, compared with 10 regions and 27 industries. There was, of course, no overall hierarchical dimension to the problem, although the specification of the EC and the rest of the world as regions

does suggest that some consolidation problems are potentially present. This world project of course does include the need to balance the set of dual trade flows, so the number of restrictions for the SALTER estimation would be larger than for the Queensland problem.

Nevertheless, the problems are similar, and there is no reason why the problem of the balancing of world trade flows, together with input-output flows, cannot be solved in a similar manner to this Queensland example, given appropriate specification of restrictions and reliabilities. The restrictions need to include individual country input-output table restrictions, consolidation restrictions<sup>(12)</sup> and dual trade flow restrictions.

#### 4.4. Lessons and future directions

##### 4.4.1. Information improvement

In the discussion following van der Ploeg's paper in 1981, the suggestion was made by statisticians from the UK Central Statistical Office that issue should be taken with the claim that *mechanical removal of accounting discrepancies contributes to improving and refining the data. It is not a substitute for finding the reasons for the errors, and correcting them. It does not necessarily contribute to quality.* (van der Ploeg (1982, p. 189))

This concern is arguably misplaced, given the demonstration of the gain in information provided by data reconciliation procedures by Harrigan (1991), and indeed by the experience of this project, which incorporates the Stone procedure into the statistical construction process.

The information gain means that the balancing procedure does not provide only a cosmetic smoothing at the end. The RAS procedure has been used for many years to eliminate discrepancies in input-output tables, generally at the end of the process, and it does seem odd that the Stone procedure, which uses otherwise largely wasted information and which can assist the statistician throughout the construction process, should continue to attract this concern.

It may be noted that the trend towards integration of the input-output and national income and expenditure accounts of national statistical agencies is gathering momentum, with, over the last decade, Statistics Canada, the UK Central Statistical Office and the New Zealand Department of Statistics adopting the policies long pursued by other countries (eg the Netherlands) in ensuring congruence in their national accounts systems. The Stone method is ideally suited to this application. In fact, the Italian Central Institute of Statistics has the distinction of

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(12) The world input-output table must balance, although, since there is no independent estimate of the consolidated world SAM, this is of course trivial if the input-output tables and trade flows are balanced, since the construction of the world SAM becomes a simple aggregation exercise.

being the first official national statistical agency to adopt the method to *balance the [Italian] 1982 input-output table and to revise the national accounts at current and constant prices from 1970* (Antonello (1990), p. 158).

#### 4.4.2. Future directions

It seems clear that the next decade will see increasing numbers of projects on specifying and estimating large world systems, national and federal systems, and regional systems. The undertaking of these projects will be expedited by using Stone's method, which generates internal congruency, and, importantly, which provides quick and instructive practical feedback for statisticians engaged in the construction and editing processes of the accounting system.

Stone (1981) himself, in addressing the concerns raised by the speakers from the UK Central Statistical Office in their discussion following van der Ploeg's 1981 presentation of his 1982 paper, and after noting that while *adjusting the accounts does not explain why some of their components were wrong in the first place*, made the point that,

*However, if the variance matrix is on the whole informative rather than misleading, adjustment does provide some plausible hypotheses about the kind of explanations to look for. This should be helpful in trying to improve the initial estimates: do not look for improvement in one group of entries; look for the reasons for upward or downward bias in a second; recognize that the indicators used in a third are erratic; and so on. Perhaps it is not possible to get very far on these lines but any success would have the desirable result of diminishing the adjustment problem.* (Stone (1981), pp. 21-22).

We believe that the experience of this paper, with the concurrent efforts in the construction of the Queensland regional accounting matrix, further contributes to the increasing body of evidence that suggests that Stone's last sentence is too pessimistic, and that the Stone balancing technique should certainly no longer be a neglected stage in measurement.

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## Appendix A

### The statistical regions of Queensland

**Figure A1 – Map of Queensland and its component Statistical Divisions**



Note : Divisions based on the 1991 Census Boundaries

Source: Queensland Government Statistician's Office.

**Table A1 – Unbalanced domestic production accounts of the Queensland Statistical Divisions** (All values in \$million, AUD, current prices, 1985-86)

	Brisbane Moreton	Darling Downs	Fitzroy	Wide Bay- Burnett	Mackay	Northern	Far North	South West	Central West	North West	Sum	Queen- land
Wages, salaries and supplements	10510.2	969.5	1246.9	824.1	819.3	1190.7	912.6	166.9	90.6	388.3	17119.0	17119.0
Gross operating surplus	7219.2	914.8	1503.6	888.3	1133.9	819.9	741.6	388.9	110.2	416.5	14137.1	14137.0
Indirect taxes less subsidies	1850.7	195.1	279.6	170.2	203.4	210.2	173.7	57.9	20.9	83.6	3245.3	3245.0
Imports from foreign & interstate	9707.0	558.2	1191.9	552.0	857.2	984.4	862.8	270.3	76.1	477.8	15537.6	13962.0
Imports from Brisbane-Moreton	0.0	349.8	603.8	341.3	435.8	364.1	292.4	168.6	45.9	180.9	2782.7	0.0
Imports from Darling Downs	193.1	0.0	13.3	9.9	8.3	7.5	6.6	18.9	2.1	4.0	263.7	0.0
Imports from Fitzroy	298.3	5.3	0.0	43.3	56.9	41.4	15.7	1.0	29.2	19.8	510.8	0.0
Imports from Wide Bay-Burnett	362.5	35.5	23.1	0.0	11.2	13.0	8.4	7.7	1.0	2.5	464.8	0.0
Imports from Mackay	68.5	2.2	14.6	3.2	0.0	6.3	1.5	0.7	2.2	2.0	101.4	0.0
Imports from Northern	91.1	4.2	15.4	4.0	17.1	0.0	32.8	1.3	2.4	43.3	211.6	0.0
Imports from Far North	133.4	4.9	105.4	2.1	6.6	25.0	0.0	1.6	1.4	20.3	300.8	0.0
Imports from South West	255.2	16.6	0.1	0.1	0.1	0.1	0.1	0.0	0.2	0.1	272.4	0.0
Imports from Central West	43.1	6.2	0.1	0.0	2.2	0.0	0.0	0.0	0.0	0.1	51.8	0.0
Imports from North West	1.3	0.3	28.8	0.1	7.4	114.6	1.7	0.0	0.1	0.0	154.4	0.0
<b>TOTAL OUTGOINGS</b>	<b>30733.6</b>	<b>3062.6</b>	<b>5026.6</b>	<b>2838.7</b>	<b>3559.5</b>	<b>3777.3</b>	<b>3049.9</b>	<b>1083.8</b>	<b>382.3</b>	<b>1639.0</b>	<b>55153.3</b>	<b>48463.0</b>
Private consumption	13207.7	1229.6	1402.7	1176.9	891.6	1244.0	1159.4	208.3	98.3	247.8	20866.1	20878.0
Government consumption	3702.8	358.0	294.2	326.9	169.2	450.6	330.6	78.0	60.2	110.5	5881.1	5880.9
Private investment	4512.2	239.5	257.1	220.3	205.3	407.6	487.9	52.5	17.4	26.0	6425.9	6425.0
Public investment	738.0	73.9	369.5	137.7	184.3	97.4	67.9	61.2	12.6	14.6	1757.1	1757.0
Government investment	745.5	81.4	113.1	126.8	76.3	182.7	93.5	41.1	30.4	35.1	1526.0	1526.0
Increase in Stocks	91.7	7.9	9.8	8.0	6.2	9.5	8.6	1.8	0.9	1.7	146.1	146.0
Exports to foreign & interstate	4882.7	809.0	2069.3	377.2	1913.2	1173.9	601.2	368.4	110.6	1168.8	13474.3	11850.0
Exports to Brisbane-Moreton	0.0	193.1	298.3	362.5	68.5	91.1	133.4	255.2	43.1	1.3	1446.5	0.0
Exports to Darling Downs	349.8	0.0	5.3	35.5	2.2	4.2	4.9	16.6	6.2	0.3	424.9	0.0
Exports to Fitzroy	603.8	13.3	0.0	23.1	14.6	15.4	105.4	0.1	0.1	28.8	804.6	0.0
Exports to Wide Bay-Burnett	341.3	9.9	43.3	0.0	3.2	4.0	2.1	0.1	0.0	0.1	404.1	0.0
Exports to Mackay	435.8	8.3	56.9	11.2	0.0	17.1	6.6	0.1	2.2	7.4	545.7	0.0
Exports to Northern	364.1	7.5	41.4	13.0	6.3	0.0	25.0	0.1	0.0	114.6	572.1	0.0
Exports to Far North	292.4	6.6	15.7	8.4	1.5	32.8	0.0	0.1	0.0	1.7	359.2	0.0
Exports to South West	168.6	18.9	1.0	7.7	0.7	1.3	1.6	0.0	0.0	0.0	199.8	0.0
Exports to Central West	45.9	2.1	29.2	1.0	2.2	2.4	1.4	0.2	0.0	0.1	84.5	0.0
Exports to North West	180.9	4.0	19.8	2.5	2.0	43.3	20.3	0.1	0.1	0.0	272.8	0.0
<b>TOTAL INCOMINGS</b>	<b>30663.6</b>	<b>3062.6</b>	<b>5026.6</b>	<b>2838.7</b>	<b>3547.4</b>	<b>3777.4</b>	<b>3049.9</b>	<b>1083.8</b>	<b>382.2</b>	<b>1759.0</b>	<b>55190.8</b>	<b>48462.9</b>
Intermediate transactions	10673.6	1017.8	1147.1	897.6	678.0	934.4	919.8	144.2	34.9	513.5	16960.9	21656.8
<b>TOTAL</b>	<b>61396.7</b>	<b>6125.3</b>	<b>10053.2</b>	<b>5677.4</b>	<b>7106.9</b>	<b>7554.6</b>	<b>6099.8</b>	<b>2167.6</b>	<b>764.5</b>	<b>3398.1</b>	<b>110344.1</b>	<b>96925.9</b>

**Table A2 – Balanced domestic production accounts of the Queensland Statistical Divisions (All values in \$million, AUD, current prices, 1985-86)**

	Brisbane Moreton	Darling Downs	Fitzroy	Wide Bay- Burnett	Mackay	Northern	Far North	South West	Central West	North West	Sum	Queen- land
Wages, salaries and supplements	10510.0	969.4	1247.1	824.3	820.1	1190.3	912.6	167.1	90.6	387.6	17119.0	17119.0
Gross operating surplus	7218.8	914.8	1503.6	888.3	1134.6	819.5	741.7	389.4	110.2	416.1	14137.0	14137.0
Indirect taxes less subsidies	1896.7	191.4	266.5	173.3	196.1	201.6	176.1	56.0	19.7	67.6	3245.0	3245.0
Imports from foreign & interstate	9169.7	488.3	1020.2	450.0	669.4	799.4	703.4	218.4	44.7	398.6	13962.0	13962.0
Imports from Brisbane-Moreton	0.0	295.6	482.2	288.5	335.0	285.6	244.5	161.9	37.2	136.1	2266.6	0.0
Imports from Darling Downs	171.3	0.0	12.4	12.3	7.7	6.8	5.8	16.8	2.8	3.0	239.0	0.0
Imports from Fitzroy	305.6	4.4	0.0	43.3	56.5	31.0	12.1	0.8	24.3	17.5	495.5	0.0
Imports from Wide Bay-Burnett	372.8	28.7	18.8	0.0	10.4	12.9	7.6	7.0	0.8	2.1	461.1	0.0
Imports from Mackay	51.3	1.8	12.6	2.4	0.0	4.6	1.0	0.5	2.9	1.9	79.0	0.0
Imports from Northern	74.2	3.9	12.9	3.3	15.2	0.0	40.6	1.1	2.0	29.3	182.6	0.0
Imports from Far North	135.9	4.4	77.2	1.8	5.5	19.4	0.0	1.3	0.9	21.0	267.5	0.0
Imports from South West	302.9	20.2	0.1	0.1	0.1	0.0	0.0	0.0	0.2	0.0	323.5	0.0
Imports from Central West	53.9	6.3	0.1	0.0	3.4	0.0	0.0	0.0	0.0	0.1	63.9	0.0
Imports from North West	2.6	0.5	39.9	0.3	12.0	128.4	1.8	0.1	0.1	0.0	185.6	0.0
<b>TOTAL OUTGOINGS</b>	<b>30265.4</b>	<b>2929.6</b>	<b>4693.6</b>	<b>2687.9</b>	<b>3266.0</b>	<b>3499.6</b>	<b>2847.2</b>	<b>1020.3</b>	<b>336.4</b>	<b>1481.1</b>	<b>53027.2</b>	<b>48463.0</b>
Private consumption	13485.5	1200.7	1376.2	1118.8	821.7	1217.3	1106.7	195.0	93.9	262.2	20878.0	20878.0
Government consumption	3813.9	369.5	286.6	312.8	164.3	431.1	337.5	64.7	37.1	63.4	5880.9	5880.9
Private investment	4566.0	230.2	239.8	214.3	195.5	401.0	492.2	48.5	15.0	22.5	6425.0	6425.0
Public investment	800.6	80.4	323.6	134.9	165.6	100.2	70.4	53.7	12.6	15.2	1757.0	1757.0
Government investment	777.0	85.9	113.1	120.8	75.0	165.5	92.3	36.8	26.5	33.2	1526.0	1526.0
Increase in Stocks	92.3	8.4	10.6	8.0	6.4	10.8	9.1	-1.7	0.6	1.6	146.0	146.0
Exports to foreign & interstate	4463.6	715.5	1848.3	317.3	1758.7	991.0	471.6	299.8	86.9	897.4	11850.0	11850.0
Exports to Brisbane-Moreton	0.0	171.3	305.6	372.8	51.3	74.2	135.9	302.9	53.9	2.6	1470.3	0.0
Exports to Darling Downs	295.6	0.0	4.4	28.7	1.8	3.9	4.4	20.2	6.3	0.5	365.7	0.0
Exports to Fitzroy	482.2	12.4	0.0	18.8	12.6	12.9	77.2	0.1	0.1	39.9	656.2	0.0
Exports to Wide Bay-Burnett	288.5	12.3	43.3	0.0	2.4	3.3	1.8	0.1	0.0	0.3	352.0	0.0
Exports to Mackay	335.0	7.7	56.5	10.4	0.0	15.2	5.5	0.1	3.4	12.0	445.9	0.0
Exports to Northern	285.6	6.8	31.0	12.9	4.6	0.0	19.4	0.0	0.0	128.4	488.8	0.0
Exports to Far North	244.5	5.8	12.1	7.6	1.0	40.6	0.0	0.0	0.0	1.8	313.5	0.0
Exports to South West	161.9	16.8	0.8	7.0	0.5	1.1	1.3	0.0	0.0	0.1	189.5	0.0
Exports to Central West	37.2	2.8	24.3	0.8	2.9	2.0	0.9	0.2	0.0	0.1	71.2	0.0
Exports to North West	136.1	3.0	17.5	2.1	1.9	29.3	21.0	0.0	0.1	0.0	211.1	0.0
<b>TOTAL INCOMINGS</b>	<b>30265.4</b>	<b>2929.6</b>	<b>4693.6</b>	<b>2687.9</b>	<b>3266.0</b>	<b>3499.6</b>	<b>2847.2</b>	<b>1020.3</b>	<b>336.4</b>	<b>1481.1</b>	<b>53027.2</b>	<b>48462.9</b>
Intermediate transactions	11703.3	1175.7	1348.2	1021.0	799.6	1068.3	1009.7	193.1	39.9	513.1	18872.0	21656.8
<b>TOTAL</b>	<b>60530.9</b>	<b>5859.2</b>	<b>9387.2</b>	<b>5375.8</b>	<b>6532.1</b>	<b>6999.2</b>	<b>5694.4</b>	<b>2040.5</b>	<b>672.9</b>	<b>2962.1</b>	<b>106054.4</b>	<b>96925.9</b>



# ACCOUNTING FOR HOMO OECONOMICUS

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## 1. Introduction

In an earlier paper [Pyatt 1990] I set out to explore some potential linkages between the System of National Accounts [SNA; see UNSO 1968] and that part of the System of Social and Demographic Statistics [SSDS; see UNSO 1975] which describes the structure of population and its activity status. Here I will try to take that discussion a step further by providing an analytic formulation of the interface between the monetised and non-monetised parts of an economy as seen from the point of view of an individual to be known here as *homo oeconomicus*. Building on these foundations it will then be possible to discuss the role of imputations in the measurement of domestic product and, hence, some of the problems of measurement which have to be addressed if the creation of product in the non-monetised sector is to be treated comprehensively within the national accounts.

Richard Stone was not enthusiastic about proposals to extend the production boundary of the SNA because of the attendant problems of measurement. He was less concerned, I suggest, that there might be some confusion if new aggregates were introduced. Indeed, Stone became less and less interested in aggregates over the years, and more and more concerned with structure. I expect, therefore, that he would have seen no necessary harm in extending the production boundary as proposed in this paper provided that series consistent with earlier definitions could be maintained and that a multiplicity of alternative concepts was avoided. Here I want to propose just one alternative to the SNA definition of the domestic product which has a natural claim on our attention because it corresponds to the broadest definition of production which can be entertained. And I also want to suggest that there are interesting possibilities for measuring this concept.

Some conceptual foundations for the discussion in this paper are provided by Becker [1965]. In its simplest form his theory of z-good production envisages that an individual derives satisfaction by the consumption of goods and services which are purchased from personal sector and then transformed within it into something useful or desirable.

Thus a meal is a z-good and so is shelter. The former requires inputs of food of different kinds, the time and skills of a cook, and the necessary utensils. The latter requires the coordinated combination of heat, light, space and furnishings. In this same spirit, the supply of labour services by an individual can be seen as requiring inputs of time, not only on the job but also for commuting, as well as the purchase of necessary clothes and tools. Accordingly the supply of labour services can be seen as the supply of a z-good.

From these basic notions of transformation within the personal sector a formulation of individual behaviour can be constructed which starts with individual endowments of real and financial assets, plus the time which each individual has available, which must necessarily be 24 hours per day for everyone. Given these endowments, the typical individual allocates some of their time to working for wages or in producing goods and services which the individual does not produce themselves but wants to have in order that they can then proceed to generate z-goods for their personal satisfaction and welfare.

This general conception of individual behaviour is elaborated in the next section of this paper. It implies a symbiotic relationship between the cash or monetised economy and the non-monetised sector. Within this framework, the issue which concerns national accountants as to where the production boundary ought to be located can be translated into questions about the extent to which activity in the non-monetised sector ought to be valued. The suggestion to be developed in this paper [as in Pyatt 1990] is that all such activity should be valued because it all has an opportunity cost. And here it can be noted that any other approach is in danger of giving a gender bias to the national accounts because a disproportionate fraction of the time of women is absorbed in non-monetised activities.

The formulation of personal behaviour in relation to z-good production and the connections between monetised and non-monetised activities are set out in the next section of this paper, section 2, first as a flow chart, which is Figure 1, and then as a social accounting matrix (SAM), which is Table 1. These alternative presentations serve, *inter alia*, to clarify the links between individual behaviour and that of other institutions, including other individuals. Then, in section 3, the relationship between the present formulation and the SNA definition of domestic product is discussed. The motivation of individual behaviour is taken up in section 4, with particular attention being given to the motivation for transfers. This then provides a basis for discussion in section 5 of the problems of valuation which need to be resolved if the comprehensive coverage of non-market activity, as discussed in this paper, is eventually to be implemented.

## 2. A Conceptual Framework

The structure of the individual economy which is envisaged in this paper is illustrated in Figure 1. This should be read in conjunction with Table 1 which translates the diagram into a SAM format. The relationship between the two is that a flow in Figure 1, represented by an arrow, translates into a non-zero entry in Table 1. By convention, the origin of

Table 1 – The accounts of *home oeconomicos*

	Cash flow (1)	Goods & services n.e.s. (2)	Labour market (3)	Production and distribution (4)	Financial assets (5)	Real assets (6)	Income (7)	Totals
(1) Cash flow	*	Cash sales of goods & services n.e.s.	Wages received in cash	0	Borrowing + interest & dividends received	Rental income received in cash	Transfer income n.e.s. received in cash	Total cash receipts
(2) Goods & services n.e.s.	Purchases of goods & services n.e.s.	*	Wages received in kind	Goods & services n.e.s. produced for cash sale	0	0	0	Total supply of goods & services n.e.s.
(3) Labour market	Wages paid in cash to employees	Wages paid in kind to employees	*	Labour services supplied to others	0	0	0	Wages paid and received
(4) Production and distribution	0	Absorption of goods & services n.e.s.	Labour services hired	*	0	Contribution to personal production activities	Personal time + transfers in kind received	Total inputs
(5) Financial assets	Lending + interest paid	0	0	0	*	0	Net interest & dividends received	Lending + interest & dividends received
(6) Real assets	Maintenance & investment paid for in cash	Net increase in stocks	0	DIY maintenance & investment activity	0	*	Surplus	Gross return on real assets + investment expenditures
(7) Outlays	Transfers paid in cash	0	0	Consumption expenditure + transfers in kind to others	Net lending	Investment expenditure	*	Appropriation of income
Totals	Uses of cash received	Uses of goods & services received	Wages paid and received	Uses of products	Lending + interest & dividends received	Gross return on real assets + investment expenditures	Total income	

an arrow defines the column in which an entry is to be made in Table 1, while the destination defines the row. Thus the flow from "Goods and services, n.e.s...." to "Real assets" in Figure 1 corresponds to the entry "Net increases in stocks" in Table 1. This latter entry is located accordingly in the row for real assets and the column for goods and services n.e.s. One implication is that there are as many arrows in Figure 1 as there are non-zero entries in Table 1.

There are six nodes in Figure 1, all of which are both a point of origin and of destination for particular flows. For each of these nodes there corresponds a specific account in Table 1, i.e. each of these nodes has its own account in the SAM, and the flows between them define the internal structure of the personal economy. All other flows link the personal economy to the rest of the world. They are therefore entered in the final row and column of Table 1, which is the income and outlay account of the individual in relation to their environment as defined by the assets they possess, their transactions with others, and their final consumption of z-goods.

With these explanations, the interpretation of Figure 1 is relatively simple. It can conveniently start with the cash flow implications of transactions in financial assets, which are that interest is both paid and received (as current transfers) while financial claims are both bought and sold (capital transfers). The specific details imply a figure for net interest received and another figure for net lending. When these are entered in Table 1 in the income and outlay account it follows that the SAM account for financial assets must balance, i.e. the sum of the row entries must be equal to the sum of the column entries.

The structure of transactions relating to real assets is somewhat more complicated. One reason for this is that the possibility of increases in stocks must be allowed. Otherwise, an essential complication is that real assets can be bought or they might be produced in the non-monetised sector on a do-it-yourself (DIY) basis. Similarly, maintenance services may be bought or they may be produced as an output of DIY. The other side of the coin is that the services of real assets may be hired out for cash, as when a house is rented, or they may be employed in z-good production, which is the typical case for consumer durables and housing which is occupied by the owner.

The net difference between the value of services provided by real assets and the costs of maintaining them define the surplus or gross profit they generate. It therefore follows that the account for real assets in Table 1 must balance.

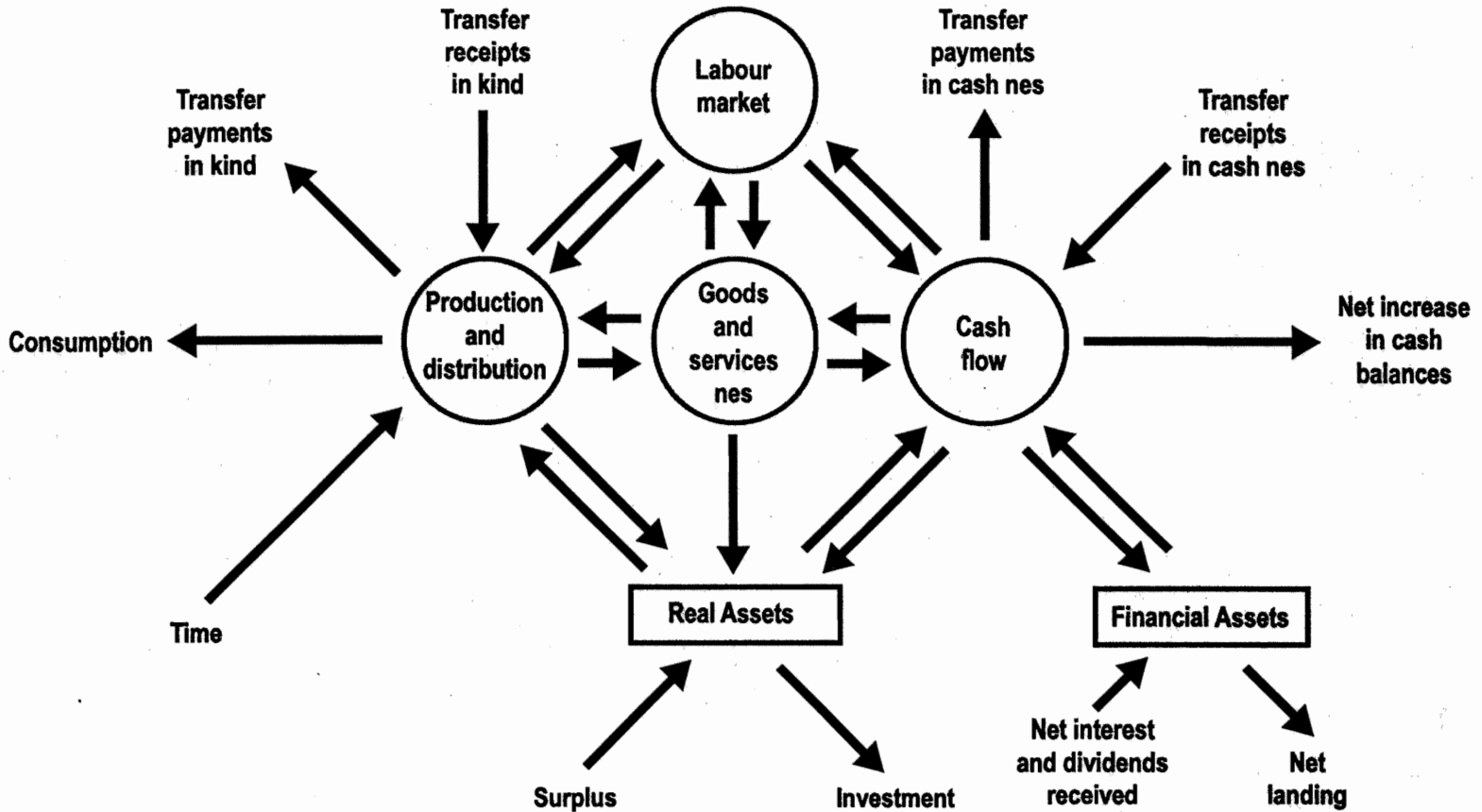
The cash flow implications of hiring labour or of hiring out one's own labour services are included in Figure 1, as are the implications of payments in kind, as opposed to cash, which add to or detract from the total supply of goods and services n.e.s. Otherwise, goods and services are bought and sold for cash.

It should be noted that the cash flow account must balance because of the explicit recognition of a balancing item, which is the net increase in cash balances.

Similarly, the goods and services account must balance because the



Figure 1 – The personal economy of homo oeconomico



net increase in stocks is a balancing item after due account is taken of cash transactions, wages paid in kind and the absorption of basic goods and services into personal production and distribution.

And the labour market account must also balance if the value of labour services hired is defined as being equal to wages paid to others in cash or kind, and the value of services provided to others is similarly defined as being equal to wages received in payment for such services.

The three nodes representing the labour market, goods and services n.e.s. and real assets define the interface between the cash and non-cash economies in Table 1. And the fact that the accounts for all three nodes must balance is, as we have seen, essentially a matter of definition. There are, therefore, no critical problems of valuation up to this point in the narrative.

Turning now to the non-monetised side of the framework, the central feature is evidently the balance of transactions through the intermediation of personal production and distribution activities. The balance of the account for these activities requires that the value of goods and services produced (including labour services) should be equal to the cost of the resources used, which include the value of personal time plus the cost of any labour services which are hired. Any system of valuation which meets this criterion would be acceptable. However, the system which is likely to have most appeal is to value goods and services which are not exchanged for cash at the opportunity cost of the inputs that are necessary for their production. This implies that, once shadow prices have been determined for the use of personal time and domestic durable goods, then the z-goods which are generated by the private economy should be valued in terms of these prices at the opportunity cost of their production. In this sense, the problem of valuing outputs is shifted back onto the problems of valuing inputs. The logic of this approach is discussed in section 4.

With the adoption of suitable valuation procedures as just described it follows that all six of the accounts in Table 1 which correspond to nodes in Figure 1 must balance. Hence the final account in Table 1 must balance because Table 1 is a matrix, which means that the external relationships of the personal economy as defined in Figure 1 must similarly be in balance. This latter balance can now be written as:

$$\begin{aligned}
 & \text{Value of personal time} \\
 & \quad + \\
 & \text{Surplus earned by real assets} \\
 & \quad + \\
 & \text{Net current transfer income} \\
 & \text{(including interest and dividends)} \\
 & \quad = \\
 & \text{Consumption expenditure} \\
 & \quad + \\
 & \text{Investment in real assets} \\
 & \quad + \\
 & \text{Net lending} \\
 & \text{(including increases in cash balances)}
 \end{aligned}$$

This is, of course, the basic relationship underlying the national accounts treatment of the personal sector: or, rather, it would be if all activity was monetised. The fact that it is not leads to some problems for national income statisticians which are addressed in the next section of this paper.

### 3. The National Accounts

Rather than attempt a full valuation of all the non-monetary flows in Table 1, the SNA adopts some compromises and short-cuts in the interest of simplicity.

Compromises are possible because, if  $T$  is a square matrix such that its corresponding row and column totals are equal, then

$$(T' - T)i = 0$$

and this same property will hold for the sum of any two matrices if it holds for each of them individually. Since the property holds for Table 1 (given appropriate rules for valuing non-market transactions) it follows that the property can hold for some modified forms of Table 1 which can be obtained by adding or subtracting a second matrix which also satisfies the above restriction.

As an example of the modifications which are admissible, consider the following. The entry in row 4, column 6 of Table 1 assumes that some value is imputed to the services rendered by all real assets employed in the non-cash economy – private motor cars, refrigerators, domestic furniture, etc. – as well as by owner-occupied housing. It would not be an impossible task to value all the services rendered by such consumer durables but national accountants have so far balked at the task, restricting their attention to owner-occupied housing and ignoring the rest.

If the value of services rendered within the non-cash economy by domestic durables other than owner-occupied housing is denoted as an amount  $x$  then the approach of the SNA can be characterised as a modification of Table 1 by subtraction of a second matrix which has three elements equal to  $x$  and all other elements equal to zero. The three non-zero elements are located in row 4, column 6, in row 6, column 7, and in row 7, column 4 which implies that both the surplus earned by real assets and final consumption expenditure are also to be reduced by  $x$  while the overall balance of row and column totals in the original matrix is not disturbed.

To complement this adjustment, the SNA treats expenditure on domestic durables as consumption, not investment. This calls for appropriate deductions from row 6, column 1 and row 7, column 6; with offsetting additions in row 2, column 1, in row 4, column 2 and in row 7, column 4. There is therefore no aggregate effect on the income and outlay account (account 7) but consumption goes up and real investment goes down by the same amount.

By implicitly making these adjustments to Table 1, national accountants are able to avoid the need to measure the value of services rendered by real assets (other than housing) in the non-monetised economy. Similarly, they are able to avoid the valuation of time irrespective of how it is used and to restrict their coverage to some specific activities. This can be achieved by subtracting an amount,  $y$ , from both the entry in row 4, column 7 and that in row 7, column 4. Hence, for example, it has been proposed by Hill [1979] that the coverage of time-use in the national accounts might be restricted to those activities which, in principle, one person could perform for another i.e. to the theoretical limits of the cash economy. In support of this proposal it is argued that it is only by staying within this limit that a surrogate market transaction can potentially be identified which could be used as a measure of the value of each non-market transaction for which an imputation is to be made. This argument can be countered by the suggestion that there are other methods of valuation which might be used in practice while, in theory, all time has an opportunity cost irrespective of how it is used.

This last point is to be developed in the remaining sections of this paper. But, before coming to it, a prior point should be noted, viz. that the effect of not valuing some uses of time or some uses of real assets within the non-cash economy is invariably to lower the resulting measure of domestic product because it lowers the aggregate value of time in one case and the surplus earned by real assets in the other. It follows that if all non-cash activity was valued then the domestic product would be bigger. And, as previously noted, because women are disproportionately engaged in the non-cash economy, the effect of present practice is to under rate their contribution. This is one serious implication. Another is the tendency to under-rate the importance of that part of the economy which is not monetised. The effectiveness of policies adopted in the cash economy will inevitably depend on responses within the total system described in Figure 1. It follows, in particular, that labour supply responses to price incentives will be conditioned by custom and practice in the non-monetised sector.

#### 4. The Motivation of Individual Behaviour

Final consumption, transfers to others, investment, lending and increases in cash balances are all leakages out of the system described in Figure 1. None would take place unless each was deemed to be desirable in some sense by *homo oeconomicus*. Accordingly, some statement of motivation is necessary in order to explain these various flows.

Perhaps the simplest formulation of motivation is to assume that each individual is concerned to maximize the present value of their own welfare, which depends on consumption and that all other leakages are driven indirectly by this selfish motivation. This is consistent with the

notion that investment in real assets and lending are motivated by the expectation of future returns, while cash balances are held for transaction reasons only, given that questions of uncertainty are to be set aside here. It then remains to be explained why transfers occur between individuals or between an individual and some other institution outside the personal sector.

Transfers define the way in which institutions are connected, one with another, other than through markets with the receipts of each individual being payments made to them by some other institution. In aggregate, therefore, transfers net out. But, at the micro level, they are essential to the characterisation of the non-market economy. They describe, for example, crucial aspects of the relationship between husband and wife, mother and child, and the individual and the state.

Transfer payments by an individual to the state are mainly in the form of taxes, which are compulsory. Transfers by the latter include the provision of basic services such as public and personal health care, education and social services. They therefore condition individual behaviour but cannot normally be influenced by it.

Transfers between individuals are another matter. Their motivation would seem to be of two kinds. In the first instance, transfers can take place within a family, say, because there are gains to be made from trading according to comparative advantage. For example, one member of the family may have a comparative advantage in gardening, while another has their comparative advantage in cooking. Such possibilities are a part of the explanation as to why there can be economies of scale associated with household size, and why single parent families may sometimes need public assistance in order to get by. Accordingly there are efficiency reasons why transfers may take place and these reasons will encourage and support the aggregation of individuals into family units whose behaviour might be characterised as a cooperative game.

If people are totally selfish then the gains from trade may be the only justification for transfers between them. However, such an explanation cannot explain the transfers from a mother to their child. An additional argument is required, therefore. Its most obvious form is to allow that the welfare of individuals can be interdependent and hence to generalize the notion that welfare depends only on personal consumption by allowing that welfare can also depend on certain types of transfer as well. Such an extended formulation of the determinants of welfare provides the minimal assumption necessary to motivate behaviour as described in Figure 1. However, further generalisation is clearly possible. It would be possible, for example, to allow that individuals might enjoy some uses of their time more than others. The point here is only that such generalisation is not essential for a sufficient formulation of individual motivation.

Given that a personal objective function can be defined in terms of individual consumption and those transfers to others which enhance personal welfare, it follows from Figure 1 that individual behaviour will depend on endowments, technology and relationships. Some comment on each of these is therefore in order.

Endowments refer firstly to the real and financial assets of the individual, which will obviously effect the level of consumption and transfers which can be achieved. Beyond these, and the most important endowment in many respects, is the time which each individual has available each day.

Next, technology is important in defining the efficiency with which an individual can turn inputs into outputs, the structure of the markets with which they can interact, and the access which each individual may have to social services.

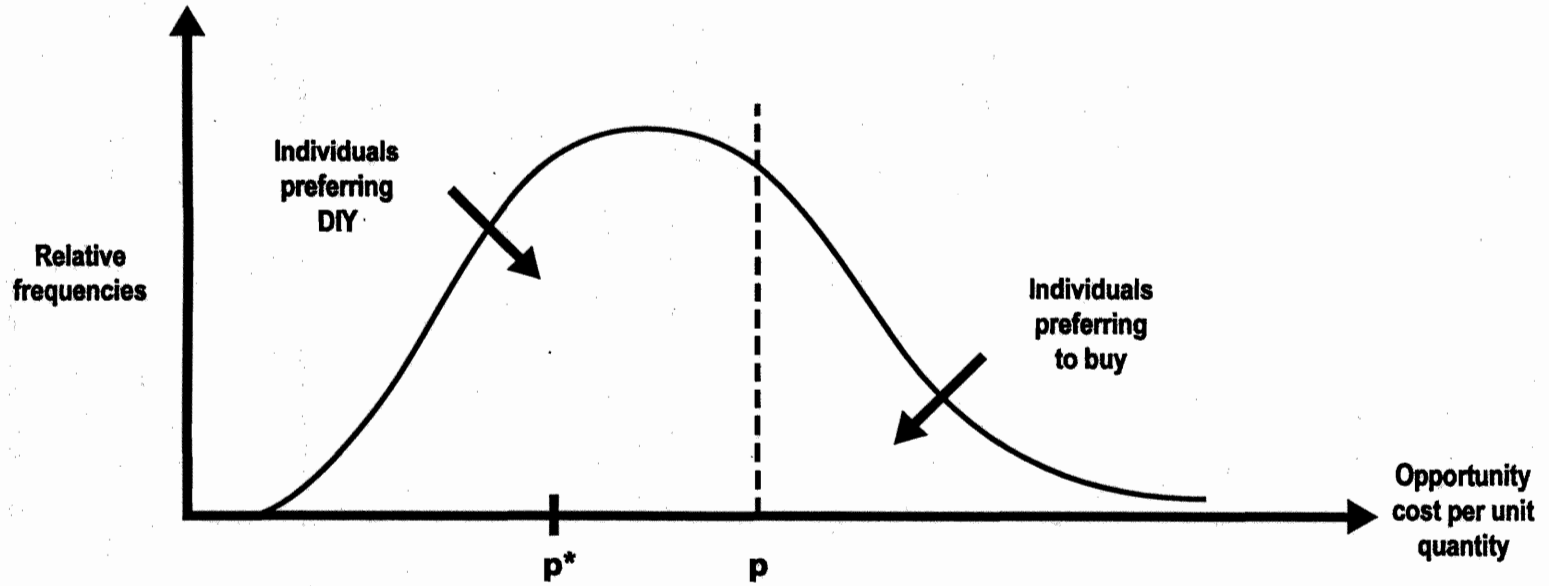
And, finally, relationships define the possibilities for transfers to and from other people and the extent to which there are social support networks which can be relied on.

Given these considerations, the behaviour of an individual will emerge from their efforts to maximize welfare. In particular, the opportunity cost of their time and the services of any real assets which they own will emerge as the shadow prices of the respective endowments, which will in turn depend on their wealth in various forms, their efficiency, and their social context as defined by their family and social setting. Hence, in theory, the problem of measuring the value of resources employed in the non-cash economy can be solved with reference to the shadow prices of resource endowments. And the corresponding valuation of the goods produced in the non-market sector will be their marginal opportunity cost.

## 5. The Valuation of Time and Products

The literature on imputing value to non-market production suggests that there are two basic approaches. The first, which can be referred to as the market surrogate or  $p \times q$  approach, suggests that the correct procedure is to identify a closed market surrogate for any good or service produced by DIY and to adopt the price of the surrogate as providing a suitable estimate of  $p$ . This estimate can then be multiplied by actual quantity produced. It has been suggested earlier that this approach is inadequate because it cannot cover those uses of time for which no market surrogate can exist. A second objection can now be added with the help of Figure 2. This shows the hypothetical distribution across individuals of the opportunity cost of self-sufficiency in producing some good or service which has a close market surrogate priced at  $p$ . It is to be expected that those individuals for whom the opportunity cost of self-sufficiency exceeds  $p$  will buy the surrogate. For the rest, i.e. those who will not buy the surrogate but will rather depend on their own efforts, the average opportunity cost is  $p^*$ , which is less than  $p$ . Hence the use of the market surrogate price  $p$  over-estimates the value of DIY activity as measured by its opportunity cost for those who practice self-sufficiency. However, in this connection it should also be noted that the over-estimation of  $p^*$  by  $p$  will disappear to the extent that an individual can in fact market the proceeds of their DIY activities which

**Figure 2 – A hypothetical distribution of the opportunity cost of self sufficiency among Individuals and their revealed preferences**



need not, therefore, be limited to the supply of personal requirements. This is because, in the absence of obstacles to marketing, an individual who is intent on maximising welfare will pursue each DIY activity which has a market surrogate, to the point at which the marginal opportunity cost, i.e.  $p^*$ , is equal to the price of the market surrogate which is  $p$ . Hence the difference between  $p^*$  and  $p$  will vanish. And the quantities produced according to this criterion will then be consumed by the individual themselves or else they will be traded on the market to the extent that DIY production exceeds personal requirements. Hence it follows that  $p^*$  can be less than  $p$  for those goods which have a market surrogate only to the extent that any surplus generated by DIY production cannot be sold at the market price  $p$  without incurring some marketing costs. To the extent that the implications of such costs can be ignored, the  $p \times q$  approach can be adopted with some confidence over the complete range of activities which lie within the theoretical limits of the monetary economy.

To go beyond the theoretical limits of the monetary economy it is necessary to adopt the alternative to the  $p \times q$  approach which is to attempt to estimate opportunity costs directly. To the extent that such costs are purely labour costs, this approach may be referred to as the  $w \times t$  approach. It revolves around the estimation of the time taken to perform some task and the identification of a suitable wage rate for valuing time.

Both parts of the  $w \times t$  approach are suspect. Measures of time taken in DIY activities are fraught with complications over multiple time use and the pace of work. And the choice of  $w$  is equally hazardous, with alternative suggestions ranging from the wage rate for professionals engaged in the task in question to that of generalists or unskilled labour. However, none of these suggestions corresponds to the measure implied by discussion in this paper, which is that the appropriate  $w$  is the average shadow price of time for those who choose to perform a particular task for themselves. Moreover, the implication of the present analysis at the macro level is that the amount of time taken on particular tasks is not an issue since everyone has the same amount of time in aggregate, viz. 24 hours per day. Rather, the problem at the macro level is simply to measure the shadow price of time on average for everyone or, more ambitiously, for different groups in the population, such as the groupings which can be defined with reference to age, sex and level of education.

Stated in this way, the problem of estimation which needs to be addressed is not very different from that of estimating an individual earnings function, except that here the dependent variable is a measure of the shadow wage rate. This might be estimated by the actual hourly wage rate received by those who are paid an hourly rate and are free to vary their hours of work. However, for those who are not so situated (which is the vast majority) a series of problems arise, some of which may be more or less tractable. For example, it may be possible to measure the marginal revenue product of labour among an account workers via a production function approach. But, in general, it may not be easy to measure the opportunity cost of time accurately, and the first



approximation which is provided by average wage per hour of work may be very crude.

Within the generality of measurement problems which arise, particular attention can be given to the problem of selectivity bias. This problem was recognised originally in the form that individuals are unlikely to take a job if their reservation wage is above the wage rate offered to them. Hence those observed to be working are a subset of the total population which is biased towards those individuals for whom the reservation wage is low.

The bias which arises in this case reflects a particular example of the revealed preferences which will emerge within the framework of Figure 1. This framework implies a whole spectrum of things which an individual may or may not do for themselves – laundry, decorating, cooking, etc. – depending on how they value their time relative to the cost of buying substitute services. Similarly, the response of *homo oeconomicus* to price incentives may be to buy labour-saving goods such as washing machines and convenience foods.

To the extent that individuals differ only in the opportunity cost of their time, their behaviour will yield a common ranking of activities from menial tasks which all would hope to avoid, through to rest and recreation and various forms of self-indulgence. Revealed preference will locate each individual on such a list and therefore provides an important insight into the value that each individual places on their time. The problem of estimating the shadow price of time can therefore be supported by the additional information which is contained in the distribution of the population according to its revealed preferences along the spectrum of those activities which it may or may not choose to undertake. The fact that, in practice, the ranking of activities may be similar for most people but not the same complicates the problem of estimation but does not change its essential character.

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# FOR NATIONAL ACCOUNTS EXTENDED TO SOCIODEMOGRAPHIC AND ENVIRONMENTAL DATA

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## **Introduction**

Since the late 60's, economic decision-makers have been concerned with wider preoccupations. Alongside the stabilization and development objectives which still remain important, other considerations have arisen concerning environmental deterioration, the depletion of natural resources, the "disamenities" of urban life, distribution and employment issues and finally the distribution of economic welfare closely linked with the standard of living and the satisfaction of human needs (4) (1).

These supplementary dimensions of economic policy require more diversified information than that to be found in traditional national accounts. They probably imply that the accounting system should be less exclusively focused on firms -a greater part being devoted to household analysis.

This adjustment of the SNA obviously does not mean that the corporate sector or market output have become less essential than before. The theme of the crisis of the protective state, which in the stagflationist context of the 70's superimposed itself on the preceding welfare theme, indicates that it is necessary to treat simultaneously the economic and social domains which are two intertwined aspects of one and the same reality.

Consequently, the information system we need should try to "integrate" economic and non economic information (sociodemographic and environmental statistics) to permit a better analysis of socioeconomic reality and a better coordination of economic and social policies.

So, the national accountants are faced with a fundamental issue: must they limit their field of observation to that which is susceptible of being measured in a reasonable way i.e in monetary terms or must they try to supply economic decision-makers with an extended "instrument

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(1) Figures between brackets refer to the bibliography supplied in the appendix.

panel" ("tableau de bord") including non strictly economic data (sociodemographic or environmental)?

I am inclined to believe that Richard Stone's message tends toward an extension of the traditional field of observation.

In his Nobel Memorial lecture (41), Professor Stone strongly emphasized that the study of a society must rest on three – economic, sociodemographic and environmental – pillars which must be "accounting" pillars as well. Indeed, in a previous study (39), Richard Stone had already commented that "There is no reason why the application of accounting ideas should be restricted to concepts expressible in terms of money; concepts expressible in any other homogeneous unit will do equally well."

Part of the distance to the articulation of the economic and the non-economic could be covered by establishing extended economic accounts, using, alongside the central framework, supplementary accounts and satellite accounts allowing a certain flexibility. This recourse to economic accounts has the advantage of allowing us to benefit from the qualities of consistency of economic accounts.

However, this approach may be considered as somewhat reductive and if we go too far in this direction, we risk caricaturizing the non-economic. Thus it would be useful to have an independent set of sociodemographic accounts which could be connected with the economic accounts. Likewise, environment and natural patrimony raise extremely complex questions which would be better treated within the framework of a system of information supplementary to the traditional economic accounts but which could be linked to them.

For some time now (47), it has been stressed more and more that perhaps the most specific contribution of national accounting is conceptual normalisation, the "common language" aspect, at least as much as the articulated language aspect. Hence the "federative" role which it has begun, it appears, to play in the organization of a system of economic and non economic information. A well-developed multi-dimensional central framework prolonged by satellite accounts and intermediate systems is likely to supply possibilities of linking economic national accounts and various sets of statistics, including social statistics and at least part of statistics on the environment.

In brief, what we are looking for is a set of interrelated systems. Each of them would be internally fully consistent. The systems might differ from the others but would remain compatible with them in the loose sense of the term. In this set, traditional economic accounts which have now become a well established practice all over the world would be the main piece.

This economic accounting and its socio-demographic and ecological extensions are committed to serving knowledge, action and social dialogue.

## **1. Extended Economic Accounts**

Although it is quite essential to have a measure of market output in order to analyse problems associated with market disequilibriums such

as unemployment and inflation, on the other hand in order to measure long term growth (above all if we are aiming, beyond the measuring of output, at an "indirect" measure of certain elements of well-being) and for international comparisons, we need a relatively extended production concept, the measure of which must be independent of social organization and its evolution (the so-called "invariance" principle).

Further more, in order to measure the costs and benefits of redistribution activities and of the public spending of governments for a better analysis of that which is known as household economy, the accounting system should be less exclusively focused on firms and a greater part devoted to the analysis of households.

To develop monetary accounting toward this extended "Tableau de bord", the conventions of the Central System can be altered to include broader capital and income production concepts, within the spirit of the work of Kendrick and Eisner (7). However to avoid overburdening the structure of the central system, one is often led to address some problems "outside" the central framework, especially within satellite accounts.

### 1.1. Introducing broader concepts in the central framework

Although "integrated", the central system is not necessarily "unidimensional". It may include several approaches; various choices can be made as regards the extension given to the system aggregates. For a long time, the SNA has included non market government output and the SNA 93 will introduce a concept of actual individual consumption covering all goods and services (private or public) available for individual consumption. Eventually, by retaining a definition of "productive" activities more extended than that of the present SNA -ie the "third person" criterion recently advanced by Professor Hill (14) – we could integrate household output into the central framework, if it were not for the measurement issue...

Nevertheless, "integrated" means that the central system must be fully consistent, must contain no contradiction whatsoever neither conceptually nor, at least in principle, as regards figures. Every aggregate belonging to the central system must respect those consistency conditions.

Concerning the introduction, in the central framework, of broader concepts, Stone's attitude seems to have been both cautious and open. He noticed that it would be interesting to know whether our econometric models would be better or worse if the extended definitions were used instead of the traditional ones.

From this first method of extending the field of monetary accounting come the propositions for "corrected aggregates", the extended accounts of Kendrick and Eisner and the works concerning household output. The "under-ground economy" poses slightly different problems.

#### 1.1.1. *The corrected aggregates of the GNP*

The inadequacies of the GNP per capita indicator have been noticed for a long time now. They have appeared especially in international

comparisons. This being so, several attempts have been made to adjust th GNP.

In the early 70's, two American economists, Nordhaus and Tobin and certain Japanese economists attempted to adjust the GNP, more particularly with "regrettable necessities" in order to make it a better measure of economic well-being.

However interesting the compilation alongside the GNP of an adjusted aggregate reflecting economic well-being, this approach in general has not been retained by statistical offices for both theoretical and practical reasons. Practitioners will always be somewhat reticent about propositions which are likely to alter a proved system. However aside from this practical reason, there are two fundamental conceptual difficulties: the questionable nature of the "regrettable necessity" concept and the feasibility of measuring well-being (1). Some have pointed out, with some humour, that except for some people who live to eat rather than eat to live, food is a regrettable necessity...Further more, how could we create a single generally acceptable indicator of well-being when we lack a widely accepted ponderation system allowing to combine the various components of well-being.

Other adjustments of GNP have been proposed to take its distribution into account. Kuznets, Ahluwalia and Chenery (6) have suggested that the GNP growth rate is in itself a misleading indicator of development because it is highly weighted by the income shares of the rich. If, as is the general rule in developing countries, the income proportion of the richest 40% of the population represent around 3/4 of the total GNP, the GNP growth rate mainly measures the growth of the income of the richest 40% and is hardly affected by what happens to the income of the remaining 60% of the population. That is why it has been suggested to assign the same social utility to a 1% growth of the income of anybody or even to weight more the growth rate of the income of the poorest.

Obviously, there too, the choice of ponderation raises the problem of the collective preference function. But if there is no agreement in "the ideal distribution", a consensus should be reached more easily on the point that, for the same total given income, a distribution with fewer poor families would be better. In any case, economic policy makers will take decisions of the basis of implicit ponderations.

Another way of proceeding has the advantage of not putting the emphasis on the distribution of income, a politically sensitive issue in many countries: it consists in calculating the "level of absolute income" of the poorest 40% as an indicator of the satisfaction of primary needs. However, progress in the reduction of poverty cannot be assessed except if the income of the poor can be compared to a minimum reference, the so-called "poverty-line". Defining this line poses some difficulties.

### *1.1.2. The "extended accounts" proposed by Kendrick and Eisner*

For a sounder basis for intertemporal growth calculations and for income and product international comparisons, Kendrick and Eisner

introduced "imputations" in order to cover all types of significant non-market activities and not only market activities.

National accounts have always included some imputations but Kendrick and Eisner go much further in that direction. Thus Kendrick ascribes a value not only to household output and to the services provided by the durable goods at the disposal of households and governments but also to non profit activities, to the work of students and to frictional unemployment representing some job-seeking activity etc... However, Kendrick does not include "non economic" leisure-activities in his accounts and make no exclusions for regrettable necessities.

With a few exceptions, Eisner uses the same imputations as Kendrick.

These extended income and production concepts are linked with a broader definition of capital. starting from Irving Fisher's definition of capital. Kendrick retains a "Total capital" concept which includes as well intangible as tangible capital. Intangible capital includes expenditures for Research and Development, education and training, health and mobility. Tangible capital includes not only produced and non produced physical assets but also all of the costs of child rearing. The intangible capital is incorporated into tangible capital whose quality and productivity it improves. This total capital concept was introduced by Kendrick in an accounting framework which is a modified version of the American national accounts.

The interest of the American economy accounts, established by Kendrick over the 1929 – 73 period and by Eisner over the 1946 – 76 period, is to give a much wider vision of economic activity than that provided by the official accounts which, above all, reflect the market sphere. The national incomes and products calculated by Kendrick and Eisner are about 50% higher than the official estimates which do not retain these imputations. Both the household sector and government represent a much bigger and growing share of the national economy. Finally, the Eisner and Kendrick figures throw doubt upon what is suggested by American national accounts: an excessively rapid growth of consumption to the detriment of investment. The accumulation of total capital – tangible and intangible – appears to have increased considerably more rapidly than consumption.

The merit of these extended accounts is to remind national accountants that the market transactions which they are mainly concerned with are only a partial description of economic activity.

However, it seems that the Kendrick and Eisner propositions have not been looked upon with favor by statistical institutes. This may be surprising given the importance, for example, of human capital and R & D as development factors.

But admittedly "human capital" raises difficult theoretical and practical problems of identification and measurement. Should human capital be measured by its historical costs or its present value? Numerical results are quite different. Which expenditures should be classified as human capital? How do we evaluate the depreciation of this capital? How do we evaluate its services? How do we calculate its rate of return?

Moreover, this concept can't be introduced without dramatic changes in the usual concepts, in the border between consumption expenditures and capital expenditures, between labour income and capital income etc..

On the other hand, Research and Development does not seem to pose so many problems. For some time, we would have thought that the revised SNA extended gross capital formation concept would include it. But, for not too obvious reasons, it will finally remain excluded from it.

Another proposition made by Kendrick and Eisner, which has not yet been implemented, is the inclusion of the value of non-market household production in the national accounts.

### *1.1.3. The measurement of non-market household production*

The present national accounting system does not take the value of housework into account. This exclusion is justified by the fact that not only is household production not exchanged on the market but that its production factors (especially household labour) are not marketable.

However, this accounting convention is more and more criticized in the wake of a conjunction of socio-political (women's liberation), economic, scientific or technical factors (Becker's theory of time utilisation and of the household as a utility producer, the movement in favour of an attempt to measure well-being, settling of time budgets).

Household work has been the subject of theoretical reflexion and sometimes of estimates not very reliable yet.

The measurement of domestic and personal services raises two problems: the definition of what should be measured and the way to measure it.

Time-budgets indicate the various uses made of the twenty four hours of the day. Even if we put aside the time devoted to professional work, the remaining time, obviously is not devoted to household work only; we must take account of free time and the time devoted to the satisfaction of biological needs (eating and sleeping for instance).

However, the third person criterion permits the settling of this problem of definition. We can have our meals prepared or our children looked after by a third person; it is a household "production". On the contrary, we cannot entrust another person with eating, sleeping or going to a concert for us.

There remains the most delicate problem, i.e. how to measure this household work. It can be evaluated in physical units (the number of workers or that of working-hours). According to time-use surveys, the time devoted to household labour is about the same as that devoted to paid activities. However, attempts have been made to impute a money value to non-market household production; but here, further difficulties arise.

The first attempts in this direction have aimed at imputing a money value to the work input based on opportunity cost (the amount of income households would have earned had they performed paid work in the



market) or on the market-rate (the average wage of a general housekeeper, a cook, a nanny etc.).

These various evaluation methods, especially that using opportunity cost, have not proved very satisfactory. One hour spent on gardening by a government minister will be worth  $n$  times an hour of his driver even if the latter is more talented at handling a spade...

A method, more in accordance with the spirit of national accounting, consists, when it is possible, in calculating the added value of household work using the market - prices of goods and services equivalent to those produced in the home (9).

Although the results vary enormously depending on the evaluation methods (3 and 5), it seems that the value of household production should roughly represent between 25 and 40% of the GNP and a higher level still (between 30 and 50%) in the developing countries characterized by a larger subsistence sector.

Since those household activities obviously contribute to the standard of living of households in a significant way, it seems desirable that some information about this "out market" be supplied, from time to time, at least. This could be done, without modifying the production frontier or adding supplementary imputations, in the shape of a satellite account.

#### *1.1.4. National accounting and the underground economy*

Unlike household production, the underground or hidden economy does not pose, strictly- speaking, a problem of extension of the national accounts. It is merely an aspect of the task that consists in insuring a comprehensive character for the statistics of national accounts. Indeed the term – as it is used by national accountants – covers activities which are supposed to be included in the GNP but which in practice may well be left out since one or several of the parties concerned try to conceal them from public authorities.

National accountants seem to have reached a consensus on an operational definition of the underground economy. It includes legal activities not declared to the tax authorities, illegal production of goods and services and undeclared payments in kind. On the other hand, domestic or charitable labour (the third sector) is excluded for reasons of coherence. If we want to be able to measure the extent of the underground economy as a percentage of the GNP, we must include in the underground economy only those elements susceptible of being entered into the GNP as defined today.

Undeclared production, the main component of the underground economy, corresponds to what is sometimes called "the black economy" (fraud and tax evasion; black labour). Incidentally its being concealed from tax authorities does not imply necessarily that it escapes national accounting altogether since national accountants introduce corrections in order to take under-declarations into accounts.

Under this heading of the production of illegal goods and services come the drug-traffic, prostitution, smuggling, etc. Let it be noted in passing that in national accounting there is no place for moral

considerations. The description of flows must be as consistent and exhaustive as possible.

Numerous methods to measure the underground economy have been proposed including e.g. in France "sensitivity" analyses of the GDP estimates to various hypotheses about the size of the "black economy". In 1988, an extra equivalent of 4% of the GDP was integrated into the French national accounts for the black economy (54). However, those various methods lead to an extremely wide range of numerical results. That means that the figures regarding the underground economy are of limited reliability for the moment.

## 1.2. The household sector accounts

The national accounting systems have tended to focus all too exclusively on the measurement of the production of the enterprises. At present, financial and non-financial corporations are analysed in detailed input-output and financial tables. There is no equivalent for the analysis of the households. The household account is global whereas measuring the costs and benefits of redistribution and public spending government activities supposes deconsolidating the household sector. Besides, social policies often require rather detailed data on the economic, demographic and social characteristics of the population.

Some progress has been made, these last years, in the direction of a better analysis of the problems of distribution and redistribution, of the consumption of "divisible" public services and of a breakdown of households in socio-professional categories as described below.

### 1.2.1. *Income distribution*

The state of the distribution is a significant element in the measurement of well-being. A description of the distribution phenomena is outlined in the central framework of national accounting with the traditional dichotomy between primary distribution and redistribution. The 1993 SNA will even analyse the distribution phenomena in careful detail underscoring the "mixed income" of self-employed workers, redistribution in kind etc. (13).

However we may wish to go further and, for example, in complementary analyses introduce the concept of "discretionary income" freely assigned to consumption or to saving. The disposable income includes a part which is not really disposable... either because its use is pre-assigned (salaries and transfers in kind for instance), or because the right to this income is subordinated to an expense or to an accident (insurance compensation for example).

### 1.2.2. *The extended consumption of households (20, 28, 22 and 51)*

The development, at the international level, of the concept of "extended" or "total" consumption has been dominated by the concern of comparing the standards of living of countries with planned economies and those with market-economies which required a notion of individual

consumption indifferent to the institutional variations of the financing methods of the expenses for health, education etc. ("invariance principle").

The concept of extended consumption sets a clear distinction between two aspects: the "effective" consumption of the beneficiary of the outlay and the financing of the expense.

Under the consumption aspect, it is definitely an extended concept since we add to the consumption of households (defined in the traditional way) the final consumption of government and non profit institutions serving households in education, health, social welfare, recreation, cultural and sports activities (non-market), i.e. what we call "divisible" public services as opposed to "pure" collective services.

The second and fundamental aspect of the concept is that of the financing of the expense. Private consumption can be either borne by households or not (social transfers in kind, subsidies which can be very significant, in particular in some East-European countries). The divisible public consumption is financed with budgetary resources.

This second face of the extended consumption permits to bring to the fore its "rate of collective financing" and to understand better the consumer behaviour of households according to what they pay, really spend (the idea of definitive charge).

The concept of extended consumption with its two aspects of "effective consumption" and "financing of the expense" is now well defined methodologically. It will be retained in the future SNA for 1993 introducing the distinction between final consumption expenditure and actual final consumption (12). The qualifier "total" in respect of consumption has been avoided as it might have suggested an illusory exhaustiveness (53). Indeed, not to mention environmental services, some services provided free of charge to households (e.g., that part of television and press services financed by advertising) will not – for various reasons – be incorporated in this actual consumption. Moreover, another question is raised i.e. whether the amount of some subsidies for food products, railway or housing services should be included.

However a social policy, concerning redistribution in particular, cannot be circumscribed by global measures. It is important to know who the beneficiaries of the services provided by the governments free of charge, are. We ought to be able to establish comparisons between socio-professional groups and even to study the breakdown within these categories.

### *1.2.3. Deconsolidating the household sector*

In the UN SNA of 1968, the household account was global. Consequently it could not be used to study distribution and redistribution in detail. This represented a serious shortcoming which some recent works, in particular by Graham Pyatt and his colleagues of the ILO and of the World Bank, on "Social Accounting Matrices", have tried to remedy. Beside the production account broken down into industries, they

include a factor income account and accounts for institutional sectors with a household sector disaggregated into sub-sectors.

The future SNA 93 will integrate these works. Richard Stone stressed the need to complement the SNA on this point (42 and 43) and effectively the breaking up of the household sector is part of the current revision of the SNA.

The deconsolidating of the household account ought to prove easier for the current accounts – as household surveys provide data for them – than for capital accounts.

Numerous classifications have been proposed to break up the household sector: by income-size, demographic composition, by age and profession of the head of the household, by region etc. according to the various targets.

We seem to be turning towards a classification by socio-economic groups. The professional classification of the ILO could provide a starting point. The future SNA is likely to supply a broad classification where households will be allocated to sub-sectors according to the socio-economic characteristics of the reference person, supposed to provide the main source of income. The task of elaborating detailed classifications will be left open to countries.

In France, the breaking up of the household sector is scheduled in the complementary analyses of the SECN and, every five years, the household accounts are broken down according to the socio-professional category of the "reference person".

Despite its certain interest, this exercise remains limited since it gives no information about the distribution inside each category because the statistical unit is the average household in each social category and not the basic household taken individually as in the surveys.

#### *1.2.4. Micro-data on the households*

A direction proposed by Nancy and Richard Ruggles (26) to go deeper into the analysis of the distributions within the various socio-professional categories, is the conception of a model of micro-data, concerning the household sector, integrated into the demographic aggregates and into the household sector of national accounting. The possibility of basing the macro-data on appropriate micro-models represents a practical way of linking together the macro-data and characteristics and attributes which are difficult to aggregate or for which the systems of units (monetary, demographic etc.) are not suitable at the aggregate level. The complementary utilization of the macro and micro-data permits, for example, to interpret the global household saving ratio by taking account of the age and socio-economic structure of the population, its rural/urban distribution.

However, the integration of macro-and micro-data requires some harmonization as regards classifications, definitions used in national accounts and in household surveys to ensure compatibility between micro-data and macro-data. This is a difficult task, because it is a

permanent trade off between the need for homogeneity and the respect of the richness of specificities adapted to their own objectives.

### 1.3. Satellite accounts

In order to avoid over-weighting the structure of the central system, national accountants have often chosen to treat certain problems "outside" the central framework. We have already had one example of this practice with the "complementary analyses" concerning households. However the complementary analyses give a rather limited freedom to manoeuvre: these analyses can never contradict neither conceptual logic nor the conventional choices of the central framework. On the other hand, the second instrument of extension of the Central System, i.e. satellite accounts (46) presents more possibilities in this respect, for they are only "semi-integrated"

The French technique of satellite accounts seems to have gained widespread acceptance since some have expressed the wish to have a chapter on satellite accounts introduced in the future UN SNA.

Satellite accounts were born in France to meet the need – for both the managers of the public sector and national accountants – to use approaches permitting, in the same time, some freedom with regard to the central framework and "dialogue with the macroeconomy". This dialogue is required when global economic evolution highly constrains the observed field (social protection, education, health....) or when the field concerned has a large impact on the global economy (energy, transport, telecommunications....)

In fact there can be several families of satellite accounts: they can be concerned as well with economic activities which are not exhaustively or not explicitly covered in the central framework (agriculture, transportation, tourism, trade...) as with the comprehensive analysis of economic and social functions (research, social protection, education, health-care...) only partly represented in the central framework.

But if each satellite account has its own history and its own characteristic since the initiative of its introduction is generally taken by the competent ministry or management service, it seems possible to gather a number of characteristics all the satellite accounts have in common.

A satellite account is a framework designed to gather the information which concerns one given "domain". (functions, economic activities). This information, expressed in monetary or non-monetary terms, is presented in a form adapted to the specific characteristic of the domain and often in greater detail than in the central framework. In order to permit to place the analysis of the domain within the aggregate economic equilibrium, it is organized so that it can be synthesized in more homogeneous aggregates in connection with the central framework.

The objective of satellite accounts is triple:

- to analyse the organization of the production and its financing in the domain considered, given that the production can have a wider field than that retained in the central framework;

- to measure the global financial flow allocated; from this view-point, the procedure of the satellite accounts constitutes a functional approach generalized to the whole national economy. A satellite account permits to calculate national spending and domestic spending concerning a domain;
- to improve and structure the data on the results and the beneficiaries of the programmes set up in the domain concerned.

So satellite accounts answer the three fundamental questions: who finance? who produces? what is the result of this expense and who benefits by it? But the relative importance of the three questions and then the developments which are devoted to them, vary according to the satellite accounts. The answer to the third question is probably the most difficult. For example, the beneficiaries of health and education expenditures are households but enterprises too. Designating the beneficiary is more difficult still in the case of environmental expenses, for which finally "nature" has been retained. Moreover, it would be interesting to connect beneficiaries with socio-demographic data, for instance to classify, in the education satellite account, the students who benefit from education services according to their education level and their socio-professional origins, which has not been completed yet.

Inserting – as we desire – satellite accounts into the studies and the decision-process of the public services implies a stronger adaptability than that of the central framework. Then these accounts require to be completed, developed, indeed modified more frequently than the central framework. It still remains that their objective is to produce long series. See the 1962-85 series for the Eurostat accounts for social protection, the OECD 1960-83 series for health etc. The series are essential for the development or forecasting models using satellite accounts.

#### 1.4. Extended "Tableau de Bord" (Control panel) and SNA (12 and 53)

In the future SNA, economic authorities who are more and more desirous to have an extended instrument panel to clarify their decisions will be partially satisfied with the introduction of a disaggregated household account, of a concept of household actual final consumption, totalizing market goods and services and "divisible" collective services, of an environment satellite account. The capital formation concept ought to be somewhat extended and to integrate all expenditures on mineral exploration, the expenses on software and the literary or arts works (film production, book writing).

On the other hand, after many reversals and for not fully convincing reasons, it seems it has finally been decided to continue to treat expenses on R and D as intermediary consumption.

The frontiers of production will not be modified and household production will still be excluded from it.

Despite limitations, partly unavoidable (for the moment, themes as crucial as environment and human capital are not fully controlled) the SNA for 1993 will represent an appreciable improvement compared to

that for 1968, even if the progress may well be less substantial than that had been expected, in particular as regards the treatment of R and D. It will present a general conceptual framework enabling to cover, in principle, all the types of situations and economic problems, not only those of the industrialized countries with market-economies but also those of the economies "in transition" and those of the developing countries.

## 2. The Social Accounts

We have seen that it may be possible to introduce some social and demographic characteristics as criteria of classification in the economic accounts; but there is a limit to what can be done in this way.

Then it would be useful to have a whole independent set of social and demographic accounts which could permit to take into account the fact that the behaviour of "homo economicus" is affected by all kinds of non-economic factors, and which could be linked to the economic accounts as and when appropriate (44).

### 2.1. Socio-demographic accounts

In his work, starting in the 1960s, within the framework of demographic accounts, then of socio-demographic accounts, Richard Stone showed his intention of embracing all the economic and social phenomena, transforming the system of statistic information so as to cover all the diverse aspects of social life in a coordinated way.

In order to find a means of coordinating, within a single framework, a variety of demographic and social statistics which can be used analytically and connected with the national economic accounts, Stone's original intuition was the idea of a possible extension of the application of the accounting method which had proved so successful in the economic domain. To demonstrate this, he referred to a second set of data as coherent and as well structured as the national accounts, namely the demographic accounts, the methodology of which he had elaborated a few years before.

The demographic accounts can be represented by a stock/flow demographic matrix where the opening stock is constituted by the survivors of the preceding year, the closing stock by the survivors of the next period and the flow of the year by the entrants (births and immigrants) and the leavers (deaths, emigrants).

Stone's idea was to extend the field of demography into a "social demography" and to transform the demographic matrix into a social and demographic matrix in which classifications relate to social and demographic concepts (age, sex, health, family-size, social class, occupation etc...) instead of economic concepts and where the unit of account is not a monetary unit but the human individual being.

Hence his proposition, in 1976, of a System of Social and Demographic Statistics (SSDS) (38). From the start, Stone laid stress on

the fact that what he wanted to build was a "system" and that in such a definition, the essential notion was that of connection, of inter-relation.

The need to establish connections is one of the leitmotives of the SSDS. Not only does it propose to organize the integration of the statistics of stocks and flows of individuals, of socio-demographic and economic data, but it also recommends to indicate the main connections between the various social domains (sub - systems).

Indeed, to take only one example, health, education and housing conditions can influence the careers of individuals. Then it is desirable that they be susceptible of being brought together.

The diverse aspects of social life covered by the SSDS are the following: size and structure of the population, births, deaths and migrations; formation and composition of families and households, saving and wealth; housing and its environment; use of time and leisure time; social protection; education and training; occupation; health; public order and security; delinquency.

This demographic and social information can – it is one of the most original points of the Stone system – be presented in the form of demographic and social accounts of the stock/flow type. Its lay-out is somewhat similar to the input – output table of economic accounts but its unit of account is the individual human being.

The population can be classified by states. The simple example of a state is the combination of sex and age. But to these basic distinctions there can be added other characteristics (classification by activity, medical classification, marital status etc.) according to the aspect of society we want to represent. For instance, we can have an "active" sequence dividing the total life – expectancy into school, work, leisure, retirement or a "marital" sequence with single, married, divorced, widowed.

The socio-demographic accounts show how the distribution of the population over states at the beginning of a period changes during a timeperiod because of two types of flows:

- "outside world" flows: births, deaths, migrations, immigrations;
- "inter-state" flows (moves from one state to another).

From the "inter-state matrix" which contains all the people who have survived in our country from one period to the next, we can derive a set of coefficients which show in what proportions the number in each state has been redistributed over all states in the course of the period. Many interesting calculations can be made with these "transition proportions" and especially that of the life expectancy of an individual from the moment he enters a state and the break down of this life expectancy into the mean times spent thereafter in different states. However, this requires the "probabilist's interpretation" of the transition coefficient matrix and the calculation of the Leontief inverse of this matrix (35).

An independent system of social and demographic data, the SSDS should, when necessary, be in a position to be connected to economic accounts. In fact the SSDS has two main connecting links with the



economy. The former concerns the costs of the services provided to meet such preoccupations as education, health – care, etc.. For instance, the life table information of the educative sequence could be combined with data on the cost of spending time in the various educative states, supplied by the education satellite account. The latter relates to the welfare benefits and payments in kind of certain groups. To be able to carry out an analysis of the beneficiaries of economic benefits, the national accounts need a breakdown of the "households" agent according to the diverse classifications: social and occupational categories, income-brackets, regions, etc... This will be provided for by the SNA 93.

Of course these two sets of economic and socio – demographic data can be connected only provided economic, demographic and social classifications and definitions are at least easily reconcilable.

Demographic accounts can also be linked with data on the use of time. This is very useful because a person is not just a student or a worker; beside studying or working, he or she is engaged in many other activities in the course of the day. In order to take these many activities in consideration, data on the use of time (time-budgets) need to be integrated.

The propositions of the Stone report have been submitted to the various national statistical agencies for experimentation and we have passed, progressively, from "System" to "Systematisation", then to "integration framework". Let us have a brief survey of the reasons for this evolution (2).

Stone's idea was to create, in the social domain, something akin to what the SNA is in the economic realm. The first studies focused on the shifts in population, on education and employment programmes (the active sequence of the SSDS), a type of studies the approach in terms of demographic and social accounts is particularly well-suited to. But, as the studies extended to numerous other sectors, the system revealed its difficulty in incorporating all the aspects of social development. Indeed, while social accounts have a homogeneous unit of account, the individual human being, on the other hand the attributes and characteristics of human beings cannot be easily assembled or added up like the monetary values. Social and demographic attributes are necessarily pluri-dimensional and the life expectancy concept, proposed by Dudley Seers (31) as an integration factor, does not solve all the problems.

Besides, aggregating requires keeping only the common elements. But a socio-demographic description is interesting only if it sufficiently respects the diversity of the attributes and characteristic of human beings.

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(2) In his interview by Pesaran (Economic Theory, 7,1991), Stone points to some reasons why official statisticians except for the Netherlands (10) may have been discouraged – introduction not gradual enough, indigestible mathematics – and insists on the SSDS as only a help. No country is expected to cover all the topics in the report.

That is why, instead of trying to elaborate a series of sub-systems within a unified system, finally it has been proposed rather to identify a number of statistical domains, unified by common classifications and other connections and to group social statistics within this whole.

In order to indicate the new orientation, it has been decided, following Claus Moser's suggestions, to substitute the term "framework" for "system" which seemed to lead to overrigid interpretations and the term "domains" for "sub-systems".

An application of the principles of data-structuration to the whole range of social data, that is what the "revised and corrected" SSDS would be.

We have discussed the works of Richard Stone at length because they seem to highlight the arduous search for a pluridisciplinary information system even if some initial notions had to be amended. We shall be more concise on time-budgets.

## 2.2. Time – budgets

We have already alluded to the interest of relating demographic and social accounts with the data about the use of time.

The time-budget technique has been developed by Szalai (45). The period covered by time budgets is generally the twenty-four hours of the day or the 168 hours of the week. The main objective of this research is to determine which part of the whole studied period is devoted to such or such activity, using a list of the time-uses or of activities. The way people use time varies not only according to the diverse community groups but also with the season and the various types of days such as working days, week-ends and other holidays.

The poll surveys on time-tables represent a type of micro-data collected and used more and more frequently to complete more traditional economic and social statistics. Despite the numerous difficulties entailed by the collecting and processing of these data, they prove very useful to account for some activities very important for economic and social analysis but which cannot be fully traced by ordinary statistics or for which the ascription of monetary values can only be arbitrary or controversial: household labour, commuting-time to go to work, more exact estimates of unemployment etc.. International and intertemporal comparisons of the time-tables can also shed a new light on the comparative ways of life between various countries and various epochs. The increase of leisure-hours can be considered as an increase in welfare which the other statistics cannot fully convey.

## 2.3. Social accounts in working-time (15)

Unlike time-budgets, the working-time accounts are not "non monetary" accounts representing a direct recording of the working-times in every industry, at least in France. In fact, they are conversions into working-time of the monetary estimates contained in the input-output tables.

The working time accounts based on a transposition of the input-output tables, consist in evaluating, from these tables, the quantity of global work incorporated in the production of the various industries. They take into account both direct work (also called "live work") directly involved in the process of production of each industry and indirect work (or "dead work") involved in the intermediary consumption of the branch and in the use of its fixed capital.

These studies have met a number of difficulties because some statistical data are not available but also because of hard methodological choices for the evaluation of the work-content. While direct work is easy to grasp – even then it is not easy to take the differences in the qualifications into account – evaluating the time contained in the fixed capital or in the imports is much more problematic. For example, in some French research-works, the solution which has been retained to evaluate the work content in imports has been to assume that one franc of import, whatever the nature of this product, corresponded to the average quantity of work needed for one franc of export during the same period.

Despite this, work-time accounts may constitute a useful analytical instrument -probably more so for retrospective structural analysis than for forecasting studies.

### **3. Environmental and Natural Patrimony Accounts**

In spite of its importance, the treatment, in national accounting, of environmental issues and of the relations between the economy and nature, is still in infancy.

The difficulties experienced in measuring, at a macroeconomic level, the impact of economic activity on the environment and to integrate these measures into the national accounts are due to the conventions which govern national accounting, to the fact it is supposed to make an inventory essentially of economic activities involving monetary exchanges of scarce resources (many environmental products are not exchanged for money and some resources may have appeared "unlimited"). Moreover rigorous criteria govern the inclusion of capital goods in national accounting and environmental assets do not always satisfy them.

However, now that the environment is in the heart of our preoccupations it has become more and more embarrassing that national accounting, a reference instrument for economic leaders, should ignore it.

The problems to be faced are more or less difficult. The phenomena of pollution seem partially susceptible of an accounting treatment within the expanded system of Leontief or with the satellite accounts. On the other hand, the environment and the natural patrimony pose much more complex questions which had better be treated within an information system, complementary to the conventional economic accounts.

### 3.1. An input-output approach to some environmental issues

As early as the 70's, Leontief (18) had shown the potentialities of his system, the so-called Leontief system, to relate economic activity to the natural environment in which it takes place. In several studies, he proposes to have his open input-output model enlarged to permit to foresee, at the national level, the generation of pollutants as well as the effects of the decisions taken to reduce them.

Leontief starts from the basic idea that the technical interdependence between the levels of desirable production and of undesirable production (of pollutants) can be described with structural coefficients similar to those which are used when we are looking for the structural interdependence between all the ordinary sectors of production and consumption.

The extension of the system consists in introducing, in the input-output table, additional rows containing the emissions of pollutants which appear among the cost elements of the various industries and an equal number of additional columns containing the cost-structures of the treatment services.

This extended input-output table, which can integrate final users too, especially households and may be transformed into an "enlarged" open Leontief system, offers interesting possibilities to analyse environmental issues. It brings out the most polluting sectors and permits to measure an "environment pressure" indicator related to production and consumption structures. Besides, the use of the well known Leontief inverse matrix allows to answer numerous questions: increase or reduction of the pollutant output linked either to variations in final demand (volume and structure) or changes in the technical structure of such or such industry, or to a combination of the two factors; consequences of the fixing, at a given level, of a standard of pollution or of the modification of this standard.

Finally, this system permits to calculate the costs of treatment services and their repercussions on the structure of relative prices. Besides, various scenarios can be envisaged to cover the costs of the pollution-eliminating sector, either with taxes paid by households, or by increasing the prices of the goods sold to households in the case of an environmental action initiated by the industrial sector.

While it offers many possibilities of application, the Leontief approach has its limits. The enlarged Leontief system treats essentially primary environmental damage which can be traced back to a precise source. It ignores that which is not imputable to a particular agent. The system does not permit to follow the evolution of the wastes once they have been rejected into the environment. Moreover pollution is essentially regional; we ought to have regional tables. Environmental phenomena often present threshold effects, irreversibilities which the Leontief "linear" model will prove incapable of treating. Finally, the important question of how much of each pollutant would have to be removed in order to strike a balance between the gains from reducing pollution and the the loss of regular goods and services which the cost

of this treatment would entail cannot be answered without a complementary cost-benefit analysis.

Then an environmental policy could not be based only on the data obtained with an enlarged input-output model. It needs a more comprehensive model of which the input-output model would only be a part.

That is why, for some years now, national accountants have been trying to set up something even larger than the "enlarged" system of Leontief. They have started elaborating environmental satellite accounts and even, namely in France, natural patrimony accounts, though at an experimental stage for the moment.

### 3.2. Environmental satellite accounts

As early as the 70's, French national accountants have become aware that the environmental information system could be set up only through structural information. As the methodology of the French "SECN" comprises the notion of satellite accounts, as early as 1977 it has seemed appropriate to take this course to build an environmental account. The French example seems to have been contagious, since on the occasion of the current revision of the SNA, the setting-up of an environmental satellite account would be proposed.

#### 3.2.1. *The environmental satellite accounts of the French INSEE*

Like all the satellite accounts, the environmental accounts have been developed to meet the specific needs of the environmental managers., permitting them, at the same time, to place their action within the global framework of the national economy.

Initially, the approach has been experimented in three directions: the management policy of water, of regional and national natural parks.

These first experiments have shown that it was preferable to set up a set of accounts, based on a common logic but adapted to the characteristics of each style of management. As a result we have not elaborated one environmental satellite account but six different accounts since 1977 (16) Besides the first three accounts we have already quoted, three supplementary accounts were added between 1983 and 1985: waste cleanup and collection, hunting, conservation of maritime areas. At the moment, other sub-accounts are being submitted to an experimental figuring or are planned; they include fauna and flora, fishing, green areas, noise, air.

So French environment satellite accounts are not interested only in pollutants but more widely in environment management. As any satellite account, environment satellite accounts answer the three usual questions: who produces, who finances and who are the beneficiaries, although the answer to these three questions is not perfectly satisfying yet. The identification of the "characteristic activities" of the environment field is the most delicate issue. But the interpretation remains difficult too

as regards the financiers and the beneficiaries. Moreover, we ought to be able to calculate efficiency indicators.

However, these accounts allow to assess the domestic and national expenditures for environment protection in absolute value and even, what is more adequate for international and intertemporal comparisons, in relative value (environment expenditures, taking account of environmental pressure). Besides, they combine both physical and monetary data allowing the calculation of diverse ratios.

### *3.2.2. The planned environmental satellite account for the revised SNA*

The current revision of the SNA was an opportunity to be seized to try to fine-tune national accounting according to the "Nature - Economy" relation (2). It has been agreed that the environment would be subjected to a satellite account. The advantage of an environmental satellite account is that it permits to treat ecological and economic problems taking liberties with the conventions of the central framework of the SNA. The central framework takes account of only "economic" non produced natural assets. For instance, discoveries or depletion of the sub-soil are recorded in "other changes in volume of assets accounts". It would be possible, in the satellite account, to enlarge the frontier of natural assets to the non economic ones.

However, even within the more flexible framework of a satellite account, the treatment of the environment poses numerous conceptual and practical problems. In particular, national accountants are still considering whether something ought to be added to the consumption of fixed capital or to stock-depletion for the exhaustion of mineral resources resulting from mining-activities. Moreover, the propositions made by "environmentalists" of "adjusted aggregates" like the "sustainable product and income" which unlike the conventional National Income indicator take account of environmental degradation, of the depletion of natural resources and of "defensive expenses" such as the health-care costs related to environmental damage, hardly seem to fill national accountants with enthusiasm (54).

### 3.3. Natural patrimony accounts

Natural patrimony accounting poses even more crucial problems than those evoked previously. At the international level, there has not even been an agreed framework for the organization of environmental statistics. A first step has been taken with the "A framework for the Development of Environmental Statistics" proposed by the UNSO in 1984 (50, 51), recently followed par the European System of environmental statistics (SERIEE)(8).

Natural patrimony accounts raise difficult issues especially because of two characteristics: the need of measuring quality as well as quantity; the frequent use of physical measures. However, some countries have boldly forged ahead; France is one of them which, in 1986, published national patrimony accounts (17). In this pioneering work, French

experts have come up against a series of difficulties in defining natural patrimony and in attempting to "assess" it.

### *3.3.1. Defining national patrimony*

The patrimony concept may seem debatable when natural goods are concerned, because if some of them are appropriated, many others are not. However it has the advantage of stressing the fact that the goods in question have been passed on to us and that it is our responsibility to hand them down in good condition to the future generations.

As for the qualifying term of "natural", it makes it clear that the goods in question result from the activity of Nature, even if they are subjected to modifications through man's doing.

Then the natural patrimony will consist of only a part of the constituent elements of Nature (or the ecosphere). We shall exclude both the permanent elements of nature (relief of the terrestrial globe) which are not man's responsibility and the artificial patrimony which does not result from natural activity and belongs to economic patrimony.

Consequently, the main components of natural patrimony are continental and maritime waters, the soil, the air, raw materials and energetic resources, wildlife and vegetal species.

National patrimony is multi-functional, in the sense that it has various utilities: economic, ecological and socio-cultural. As we shall see, this makes its assessment difficult. After it has been defined, we are led to ask whether it can be "counted" and if so, "how"?

### *3.3.2. Natural patrimony assessment*

So far, natural patrimony has been excluded from national accounts for two reasons: there exist no institutional units, owners of this patrimony; often it is impossible to assess the monetary value of this patrimony. From this must we infer that the accounting method is unfit to answer the two basic questions: what is the state of our national patrimony and what is the effect of our management of this patrimony?

This goes against what the work of the French interdepartemental committee for the patrimony accounts suggests. On the contrary, these studies, confirming Stone's intuition of the applicability of accounting approach outside the monetary field, show that accounting can provide a mighty backing to answer both of these questions, on three conditions:

- not to want to use the only monetary standard to assess the value of the patrimony and its modifications <sup>(3)</sup>;
- to propose a concept analogous to that of "institutional unit";
- to conceive a system that would be different from conventional accounting and yet connectible to it.

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(3) But monetary evaluation which is likely to facilitate the taking into account of environmental problem in economic decision (Cf. Stahmer "SEEA of the UN", International Conference in Memory of Richard Stone) may be used when available.

### 3.3.2.1. *The unit of measure of value*

At the onset of its work, the interdepartemental committee insisted on the fact that the value of natural resources could not be adequately expressed in monetary terms and that the general rule should be an assessment using physical units – monetary evaluation being just a particular case. Indeed if the natural patrimony sometimes lends itself to exchanges, most of the time it is used without transactions and anyway its production is due essentially to the action of nature itself.

Hence the proposition to establish an accounting using physical units (surface area, volume, mass, length) which will express as well quantities as qualities, and incidentally a monetary accounting when we must trace the financial movements entailed by patrimony management (exploitation, repair, maintenance, surveillance etc.). Incidentally, let us note that this accounting per matter has already started...For a long time water has been measured as well in quality as in quantity; the forest is evaluated in square measure and volume, the quality of the atmosphere or of the sea is described using rates of pollutant concentration.

Despite its undeniable interest, this accounting in physical units poses a number of problems. First the duality of physical and monetary measures will generate two heterogeneous and not fully aggregative systems. However this may not be the most serious problem as they will be "connectible". The most difficult issue, which so far has only partially been solved, is whether an accounting synthesis of the physical data will ever be reached.

At this stage of the work on natural patrimony accounts, solutions have been offered to make some subsets of the natural patrimony aggregative. They include forestry, continental waters and wildlife. However the question remains open whether it will be possible to aggregate these various subsets together to obtain a brief answer and if possible only one answer to the question: "has our natural patrimony deteriorated or improved"?

Currently, the only aggregative unit we have is money and it is only marginally used to evaluate the natural patrimony.

Attempts to convert physical units into energetic units do not settle all the problems. The use of the multicriterion analysis has also been recommended to obtain synthetic indicators. However the multi-functional character of natural patrimony, it has been remarked, leads to postulating the impossibility of a single synthetic indicator and to affirm the existence of at least three irreducible evaluations: economic, ecological and socio-cultural. Consequently, several accounts ought to be envisaged, established according to different view-points.

### 3.3.2.2. *Statistical units*

The second problem that has been encountered in the setting up of the national patrimony has been that of the statistical units.

The economic patrimony accounts of national accounting are accounts of institutional sectors. Then we may well wonder if the



absence of institutions owners of the national patrimony is an insurmountable obstacle to the setting up of natural patrimony accounts.

In fact, such would be the case if the unit of measure of natural patrimony was money for it is exchanged only between institutional sectors. But if the unit of measure is physical then it is possible to choose accounts of natural entities (e.g. for the aquatic ecosystem) as the framework of description of phenomena.

Accounts of institutional sectors (accounts of agents) will only be used when dealing with human activity, with management activities concerning the national patrimony (exploitation, protection).

If we proceed in this manner, we will finally obtain a set of accounts different from those of monetary national accounting but which will be connectible to it.

### *3.3.3. The system of natural patrimony accounts*

The natural patrimony is a set of elements or ecosystems modified by flows or operations resulting either from natural activity or from human activity. All of this is formalized in a system of accounts.

Accounts are established for "natural elements", for "ecological sectors" and for agents.

For natural elements and ecological sectors, the objective is to describe both the state of the patrimony and the modification it undergoes as a result of natural and human activities. This simultaneous presentation of natural activity and human activities permits to tell whether we are wasting natural capital, considering the cumulated joint effects of natural activity and of our own activities.

Partially expressed in monetary terms, the agent accounts include, in particular, natural patrimony economic management accounts which ensure the connection of natural patrimony accounts with environment satellite accounts.

Natural patrimony accounts have been submitted to partial figuring in three domains: continental waters (the most complete figuring<sup>(4)</sup>), forestry and wildlife. The final objective is an assessment of the patrimony itself, of the resources we draw from this patrimony and even of the final services it procures (economic, socio-physical or social).

## **4. Conclusions**

In brief, what economic decision-makers need is a normalized system of information consisting of a macro-mesoeconomic "core" used as an instrument of consistency, and of peripheral instruments, connected with this core (satellite accounts) permitting to establish links

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(4) M. Braibant (from INSEE) "quels comptes de l'environnement?" paper forthcoming (5th General Conference of the French National Accounting Association, Paris, 12,13,14 december 1993).

with the other systems of socio-demographic and environmental information and also allowing the micro-macro transition which is increasingly demanded.

Of this pluridisciplinary information system, so far only the specifically economic part has been achieved in good conditions within the framework of the national accounts which aggregate and synthesize the monetary information thanks to the price-system.

Now, national accounting, in its conventional form, is a well-established practice through-out the world. It has reached a maturity stage, which incidentally does not mean that there is a perfect unanimity on the exact shape it should take as a descriptive and analytical instrument and that it cannot be improved even if next SNA, for 1993, presents an appreciable improvement compared with that of 1968.

As for socio-demographic accounts, they are far less advanced and have hardly gone beyond the methodological stage. There exist many demographic and social statistics but so far no country, it seems, has organized these data within a framework relating stocks and flows. The only existing demographic accounts have been built mostly in the university sphere, especially by Richard Stone.

However, international organizations, notably the OECD and the UNSO, have encouraged the development of these accounts and coming years may well see a partial realization of this second aspect of the information system. It was Richard Stone's forecast or hope in an article in 1986 (44). Although, this does not mean that all the aspects of the social reality must necessarily be "forced" within a framework relating stocks and flows....But it would be interesting to have some structured social information, perhaps by means of a socio-demographic "core" coupled with flexible modules, as in economic accounts....

On the contrary, in spite of a number of interesting experimentations, environmental accounts are still in infancy. A part from pollution which should have a partial solution within the framework of national accounts, the other aspects, notably the so-called natural patrimony, raise difficult issues which require resorting to multi-criterion analyses and it is evident that the UN proposition, on the occasion of the revision of the SNA, of an integrated system of economic and environmental accounts may have seemed, to many national accountants, premature to say the least. Although it is a major preoccupation which will have to be answered in coming years.

In spite of the substantial progress made in the direction of this enlarged "control panel" at the disposal of international, national and regional decision-makers and of the general public, there remains much to be done by researchers and by the national accounting practitioners alike.

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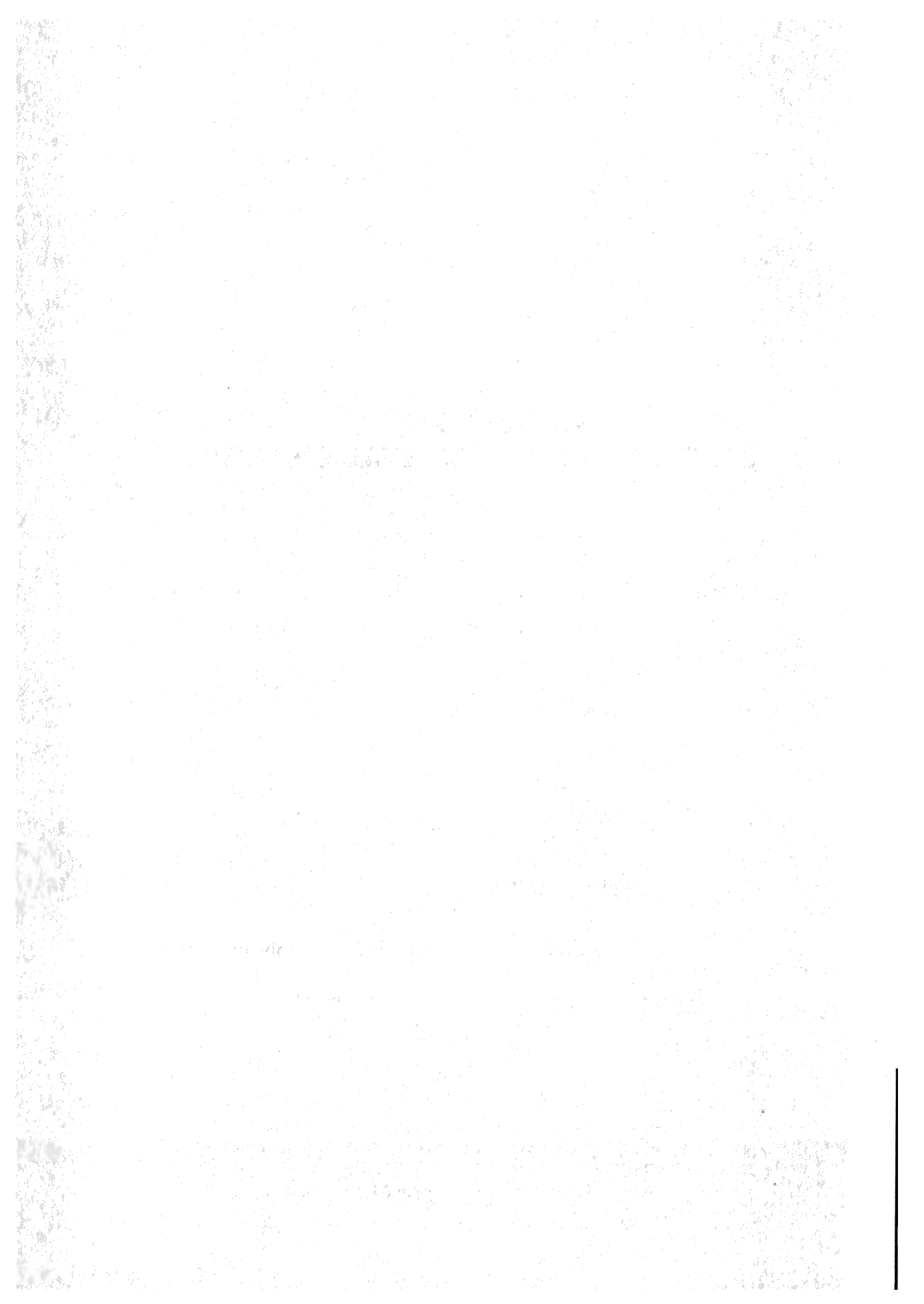
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**CONCLUSION**  
**STATISTICS FOR SOCIETAL CHANGE**





# STATISTICS FACING THE CONCERNS OF A CHANGING SOCIETY

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## **The Background**

The mid – 1990's find our societies in a state of prostration and instability not dissimilar from that of the mid-1940's.

The great hopes and expectations that accompanied the end of the cold-war, and the courageous steps taken by the developing world towards political democracy and the market economy are now confronting the complex realities of building and maintaining peace and security, promoting non-inflationary economic growth, providing jobs to the growing number of unemployed, carrying forwards the liberalization of trade and financial activities, promoting the enforcement of human rights, the alleviation of poverty and the improvement of living standards.

A common aspect of these diverse and wide-ranging processes is that they are profoundly affecting the way in which our societies work, the lifestyle of men and women, their visions and beliefs. Moreover they require the coordinated efforts and commitment of individuals, households, business and communities. For instance, the diffusion of technology and the circulation of information is shaping the pattern of consumption and production, having an impact on the distribution of earnings, status and power in society, on the way people form expectations about the future and take decisions. Furthermore, population aging, changing family patterns, labour mobility, greater equality of opportunities, the increasing demand for education and skills (and new skills) put pressure on, and redefine, the division of roles in society and the pact of solidarity between the young and the aged, women and men, rich and poor, abled and disabled, urban and rural communities, inner cities and suburbs.

Most of these issues are not new. However, at the end of this century, they are seen in a new light for at least two sets of reasons.

First, the collapse of the "second" world has reduced the cultural distance between rich and poor countries, between developed and developing nations, between the first and the third world. People are induced not to think of other people as belonging to different worlds: they

compare and judge, care or react, feel sensitive or share concerns. The dimensions of relevance are shifting and crossing national borders. The changing society is acquiring a global perspective in connection with international processes such as the technoglobalism or the liberalization of international trade. Different countries increasingly wish to exchange experience, identify the best practises and learn from one another about the issues of societal change, on the basis of what is perceived as common concern, opportunity or threat. There is an unprecedented demand for international dialogue over social issues, even though the contexts, the analyses and the policy solutions remain by-and-large tied to specific national frameworks.

The formidable progress in information and communication technologies is creating a societal context in which it is easier and quicker to inform, interact, exchange. The global village and the information society are gradually affecting the way we work, live, think and feel. At the same time, they evidenciate the increasing difficulty of understanding each other, the frictions created by the coming into contact of once separated cultures, the risk of confusion and disorientation, of impatience and intolerance, the temptation of shying away from the complexity into the comfortable familiarity of isolated national local or ethnic cultures.

Second, an increasing awareness is emerging of the close interdependence between social conditions and economic prosperity. The "new growth theories" have underlined the role of investment in education and training for sustained and stable economic development. The experience of transition countries has shown how important the setting up of effective social safety nets is. The social costs of industrial restructuring and the economic costs of joblessness or inactivity have been analysed. Greater coherence and coordination among the many separate domains of policy action in the social field, and idiosyncratic compartments of research disciplines, are considered an essential condition for success in structural reform (see OECD Social Policies Ministerial 1993).

The new climate of international attention to social issues has produced a host of initiatives at the national and international level. Within the range of activities for the celebration of the 50th Anniversary of the United Nations, special prominence is given to the World Social Summit to be held in May 1995 in Copenhagen. The various initiatives on the issue of unemployment, from the EU White Paper to the OECD study and the Detroit Seminar, have shown that much more is at stake than providing an income or any job for the unemployed: the complexity of the question derives from its wide-ranging societal implications.

### **The Contribution of Statisticians on Social Issues**

Statistics and statisticians have contributed considerably to the advancement of knowledge on social issues at the national and international level.

The range of topics dealt with in the international sphere is wide and differentiated:

- a) population and housing services;
- b) systems of civil registration and vital statistics;
- c) fertility, family surveys and demographic projections;
- d) human settlement and housing statistics covering city/urban data;
- e) migration;
- f) statistics on special population groups including children, the elderly, persons with disabilities, etc.;
- g) statistics on the role of women;
- h) labour, employment, underemployment, working time and income;
- i) the distribution of income and wealth;
- l) the measurement of living standards;
- m) poverty monitoring;
- n) education and training;
- o) health;
- p) culture and mass communications.

Activities under these headings are also varied and numerous: from the development of guidelines, standards and norms, classifications definitions and concepts, to the collection of data, compilation of compendia and publication of studies; from international cooperation to technical assistance and training; from support of specific policy programmes to development and diffusion of technologies and methodological applications; from development of statistical surveys to utilization of administrative records.

Many international organizations are active in this area: within the UN family, the Statistical Commission and the regional agencies, such as the Economic Commission for Europe; the specialized agencies such as FAO, ILO, UNESCO, the World Health Organization, UNIDO; the Bretton Woods institutions, i.e. the IMF and the World Bank; GATT and UNCTAD; the OECD; EUROSTAT; and many others.

### **The Shortcomings of Current Supply**

In spite of the considerable efforts that are being made, the demand for statistics in support of policy decisions in this area is so large and rapidly expanding, that the supply falls significantly short of requirements. On the whole, policy makers and citizens feel that the statistical community can do more for the analysis of societal changes, at least to the extent of matching the good results obtained in the fields of economic or demographic statistics.

The main reason for the existing shortcomings is the difficulty of developing conceptual frameworks capable of doing justice to the multidimensionality of social phenomena. The comparison with economic

statistics therefore should not be misinterpreted: following the path of economic statistics in the search for a single comprehensive framework, the use of monetary variables, the fine-tuning of precise quantitative indicators, does not necessarily take us far enough in social analysis. Indeed, the recent debate has shown that even in economic statistics adjusting to the complexity of the real world can be greatly advantageous, in spite of the inherent difficulties: relying uniquely on monetary measures may determine biased or arbitrary evaluations (e.g. in the evaluation of public administration activities in national accounts or in the use of figurative values). One might also put in question the fact that a single comprehensive statistical framework guides the understanding of economic events, and wonder whether this is still at all desirable or feasible. In some cases the simplicity of economic indicators proves to be deceptive, and possibly misleading for policy making (e.g. output in the services or standardised unemployment rates). It is undeniable however that the development of theoretical and conceptual frameworks for social statistics lags behind and hinders progress in the field.

But there are other more specific reasons why social statistics lacks visibility and needs to be enhanced.

1) First, new issues have emerged requiring additional, and better, data. The information available is not sufficient to support transition in former command economies, to promote sustainable development in developing countries, to address the growing issues of unemployment, inequality and social exclusion that plague advanced industrial countries. The global aspects of social change are acquiring an increasing focus: for instance, the social implications of post-conflict peace building, preventing conflict peace-keeping; or the social pre-requisites of structural adjustment programmes; or the welfare and social costs of protectionism, trade and financial barriers and the distortions caused by aid and subsidies.

2) Second, a great deal of social data is based on administrative sources (e.g. education, health, crime, social welfare, etc.), often under the direct and strict control of politicians. This makes social statistics particularly vulnerable vis-à-vis policy shifts, and dependent on the political climate, keeping it at a distance from central statistical offices and official statisticians. Progress in social statistics has been generally driven by the "visible hand" of political imperatives: in the U.S. for instance, the household consumption survey was started in the 1930's at the time of the New Deal; the Panel Survey dates from the late sixties, the years of the "war on poverty".

3) Social statistics has often had a "residual role", almost like a by-product of statistical instruments having a different objective, as in the case of Population Censuses, vital statistics, consumption surveys, etc. Only recently has social statistics emerged as a free-standing set of policy relevant information.

4) In some areas of social statistics, the linkage with the policy perspective has been weak and not sufficiently focused. Take for instance the case of international standards and definitions of migration,

or the lack of user-friendly and internationally comparable social indicators.

5) Finally social statistics and analysis has often been developed in isolation from the analysis of main economic and other trends. Attention was given in the 1970's and 1980's to the extension and revision of national accounts, the promotion of satellite accounts, and the development of frameworks for integration or coherence and for the analysis of linkages, as in the case of R. Stone's "System of social and demographic statistics". But not much has been achieved in operational terms.

The lack of operational frameworks for linking economic and social phenomena and for connecting within the wide and diverse realm of social policy different aspects such as health, education, labour markets has in turn affected the performance of social policy, often fragmented in a domain by domain approach. This has led sometimes to inconsistencies and overlapping, and has neglected by-and-large feedbacks and interdependences.

The perceived shortcomings in social statistics are creating obstacles to the development of activities, particularly in relation to resources: in fact the fragility of information infrastructures and the lack of statistical frameworks easily used for policy purposes might explain to some extent the reluctance of investors (be they public or private) to make resources available for social statistics.

### **Enhancing Work and Revisiting the Agenda**

The time is ripe for revisiting the agenda and enhancing work. The present context of increasing policy attention and demand for social statistics provides a unique opportunity; it also requires a timely, visible and concrete response from the community of statisticians.

Statistics is a powerful, and complex, instrument of communication. It is the language of informed rational science-based interaction, and should be fully developed and exploited for promoting dialogue over social issues.

There are three main directions in which the agenda needs to be reconsidered and priority action taken.

i) Work has to be pursued on the analytical models specifying *linkages* and *sequences*. The task is particularly arduous because the analytical tools operate at the intersection of different disciplines and scientific categories. Moreover it is difficult to operationalise integrated approaches and derive effective statistical concepts and definitions. The socio-economic linkage is of utmost relevance; but also the link between society on one hand and on the other the environment, natural resources, institutional reform, etc. The new SNA provides an opportunity to extend the accounting framework and to develop satellite accounts. The lessons learned through the experimentation on the "System of Social and Demographic Statistics" have shown how complex

and ambitious is the task of building "comprehensive" frameworks where society is seen interacting with the economy, the natural environment and the territory, rather than in isolation. However we are still quite distant, and will probably remain so for a long time to come, from putting into operation broader and more integrated "systems" or accounting frameworks.

But the original inspiration of that programme is still valid and timely. The analysis of explicit linkages and the development of integration frameworks represent a fundamental and largely unexplored component of the agenda.

ii) Social changes affect people, individuals and their families. The aggregative approaches that play a vital role in economic or demographic analysis often appear inadequate in social analysis and social policy monitoring and evaluation.

The use of *micro-data* offers far-reaching opportunities for *micro-modelling* and the *micro-simulation* of policy measures. The integration of survey and administrative data and the development of panel surveys currently allow unprecedented possibilities to understand better how society works, and how different individuals and social groups react to environmental changes.

Progress in this area has been constrained in the past above all by technological possibilities (and the costs associated with them) and by concerns over the safeguard of confidentiality. At the present time, the formidable advances in technological capacity, the improvements in the protection of human rights for individuals, and the search for greater economic freedoms create a new context for the promotion and use of micro-techniques.

Statisticians can contribute significantly to developing and supporting these statistical and policy tools, and to feeding them with the appropriate quantity and quality of data. But above all official statisticians can invest all their credibility and reputation in this sensitive area by providing guarantees against undue government interference, breach of confidentiality principles and non application of scientific standards. Statisticians can also support the international dialogue over micro-techniques, promoting as much as possible the cross-country comparability of results.

iii) There is a great need for simple policy-oriented and user-friendly *statistical indicators*, capable of guiding and putting on firmer grounds the policy dialogue over social issues. Efforts in the past, in various institutional quarters and on various occasions have met with mixed success.

It is well known that the complexity of reality and the sensitive nature of the issues involved highlight the risk of adhocery, excessive simplification, distortion and ambivalence in the underlying interpretation, dangerous jumping to unwarranted policy conclusions, not to speak of the danger of manipulation and opportunistic exploitation.

Nevertheless, the lack of sound statistical and quantitative indicators is having a damaging impact on the quality of the policy dialogue, overexposed to emotional upheavals, prejudice and passion, qualitative

value judgements, and wall-to-wall confrontation of interests. Progress in the economic dialogue has been greatly helped by statistical work on basic economic indicators, which – in spite of their shortcomings – provide guidance and orientation to markets and economic agents, and solid benchmarks for policy targeting and monitoring. This is not the case for issues like poverty and inequality, health and education which are essential for the assessment of both family and individual welfare and for the policy process affecting it. Indicators are particularly needed in relation to the on-going reform of social protection systems with the aim of improving the cost-efficiency and the effectiveness of those systems. The crucial, and most difficult, task here is to define "outcomes indicators", which involves complex conceptual and measurement issues. "Ultimate outcomes" in fact may significantly differ from first impact or intermediate (medium-term) outputs, considering the possibility of J-curve effects; for instance investment in higher education could produce in the short-run mismatch and unemployment, and only later following the lagged response of productive structures an increase in productivity. Moreover outcomes are often different from the explicit and contingent objectives or targets of policy. Therefore, the whole input-target-outcome nexus should be investigated. There is great scope for promoting work in this area.

This tentative identification of the main directions of work needs to be carefully examined and discussed in revisiting the Agenda for Social Statistics. The outcome of the exercise, however will have wider implications; the whole field of current work in fact has to be tapped on and involved in the enhancement of social statistics. Moreover, priorities should be assigned with the purpose of better focusing the revamping of the field. The following areas of current work appear deserving of particular attention:

- 1) regional and small-area social statistics;
- 2) definitions and classifications (age, skills, occupations, social conditions, family types, etc.);
- 3) data collection instruments: Censuses, multi-purpose or single-purpose surveys; cross-section and cohort analysis; regular and occasional surveys; administrative data, etc.;
- 4) social accounting matrices in specific areas, such as health, education, etc.

### **An Interactive and Flexible Approach**

Revisiting the Agenda and giving new impetus to activities in social statistics is not simply a technical question and cannot be treated as a routine task.

Three aspects need to be considered in designing the appropriate initiatives.

a) First, a close and deep involvement of the research community is required. Analysts and academics should pay much greater attention to

the issues of data production, and should cooperate closely with producers (National Statistical Offices *in primis*) in developing concepts, methods and definitions. On the other hand, official statisticians should rely on expert advice and link up with the centers of excellence in statistical research in the area of social studies. Closer interaction between research and official statistics can also strengthen the authority of National Statistical Institutes and provide greater recognition for their role in social statistics.

b) A much greater involvement is required of policy makers and public administrators working in the many and various government departments dealing with social issues either at the local level or in the community. This involvement should facilitate a better policy focus of statistical activities, and should also improve the statistical and analytical capacity of many relevant players. The vast and by-and-large unused information and data patrimony of policy departments should be fully exploited for statistical purposes. In the field of social policy, many government departments are producers of statistics; they should therefore be supported by statistical institutes to improve the quality of both statistical products and processes.

c) Finally, support and resources should be provided for statistical activities at the international level. International organizations have limited scope for mobilizing additional resources for social statistics in their tight budgetary frameworks. The challenges and statistical demands at the international level are new and growing. Many institutions are involved and many new programmes are being developed.

In conclusion, the opportunities ahead are rich and promising; but the tasks are paramount and official statisticians cannot proceed alone.

### **Proposals for Further Work**

The present phase of post-cold-war history presents many analogies with the extraordinary and difficult times of reconstruction and development in Europe in the late 1940's and the 1950's. Even though there are not presently the conditions for something like a Marshall Plan, an extraordinary effort is being made to provide support for reconstruction and development in the East and the South of the world.

The contributions of statisticians, both in national institutes and in academic circles was then fundamental: it suffices to mention the setting up of national accounts and their close finalization to the management of aid and policy support within OEEC and ECE. Cooperation with international organizations was intense; their role in guiding statistical progress and operationalizing it in policy terms was recognised and supported.

A group of scholars and students gathered in Siena at the initiative of ISTAT in October 1993 to honour the memory of Sir Richard Stone, a scientist who had greatly contributed to progress in statistical analysis in



the early post-war period, and afterwards. The meeting reviewed the perspectives opened by R. Stone's contribution in various fields of statistical and economic analysis and focused on his ambitious programme in social statistics and accounting which inspired the last part of his rich research trajectory and life. Official statisticians and international organizations participated in the meeting interacting fruitfully with researchers and academics. At the end of the Seminar, it was suggested to launch an initiative to reflect on and promote activities in social statistics in order to revisit and better focus the research Agenda, to support the advance of work particularly at the international level, to link it up closely with policy concerns and expectations, and to create a flexible and open forum for interaction among national statistical institutes, academic experts, international organizations and policy makers on the issues of a changing society.

Support should be given to the continuation and enhancement of work in each specific domain of the vast and variegated social field (education, health, crime, social welfare, etc.). There are many organizations and institutions at the national and international level already engaged in ambitious projects for developing indicators, measuring outputs and analysing the major factors influencing outcomes (e.g. the OECD project on education indicators; the WHO project in the area of health; ILO, EUROSTAT, OECD, IMF and the World Bank in their monitoring social policies and issues, etc.). Progress in these activities has been considerable even though uneven and sometimes not sufficiently coordinated. Therefore any new initiative should avoid duplicating and overlapping with on-going work. On the contrary, support and encouragement should be given to the many players active in this field.

However there remains considerable scope for a new initiative venturing into the complicated and relatively unexplored universe beyond the domain-by-domain approach, encouraging dialogue between different expertise, exploiting the synergies, promoting interactions and the analysis of interdependencies, standardizing as much as possible definitions, statistical concepts, classifications and methods.

The process has to move pragmatically and in an iterative mode both at the sectorial and at the intersectorial level, and between the two levels: that is, articulating hypothesis in each domain, developing data to test and redefine the hypotheses, improving data collection in the light of the obtained results; then establishing links between different domains, checking the consistency of definitions, classifications and methods, analysing the interaction between factors and outcomes, going back to sectorial hypotheses and data; and so on.

A promising way to start the process could be looking at different domains in pairs; for instance education and the labour market, or health and poverty, or family patterns and crime, or housing and internal migration, ecc. Another, more familiar, approach is to look at specific target groups in a multidisciplinary approach and across a broad spectrum of policy domains.

A new initiative is needed therefore to proceed beyond the domain-by-domain approach, develop an agenda of what is needed and

feasible, and make progress towards broader frameworks and more comprehensive perspectives.

The initiative should take the form of a working group, possibly of the Voorburg-type.

Statistics Sweden, jointly with Statistics Norway and Istat, is organizing a first meeting of the "Siena Group" in Stockholm in June 1994. At this meeting the use of social indicators, social accounting and microstudies in social reporting and social policy analysis is to be discussed. The idea is to create a link between different approaches in economic and social statistics. With combined contributions from different fields of statistics it will be possible to give a more comprehensive picture of the development and distribution of welfare than is possible today.

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